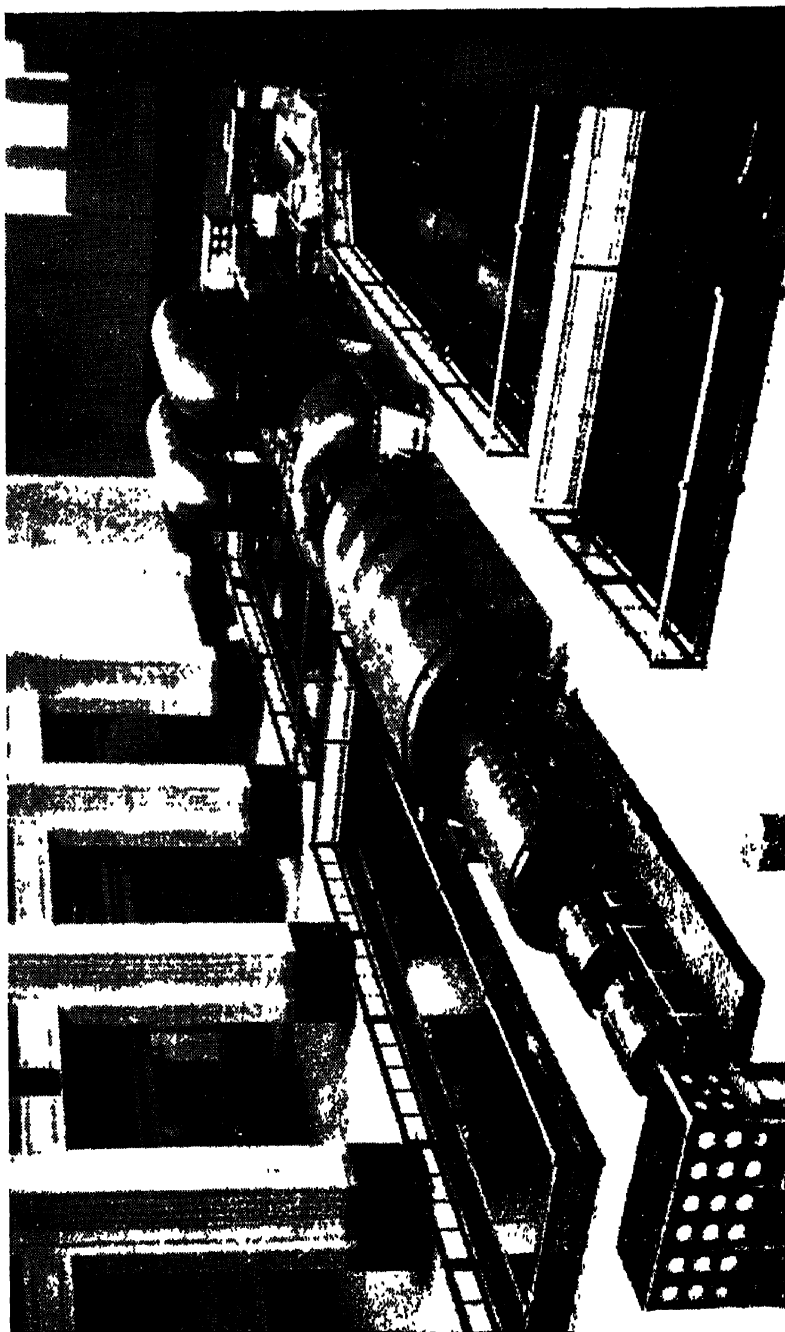


STEAM TURBINE
THEORY AND PRACTICE



105,000 kW Steam Turbo-alternator

1,500 r.p.m., Economical Load 84,000 kW. Steam Pressure 600 lb. per sq. in. gauge. Steam Temperature 850° F. Vacuum 29.1 in. Hg. at the economical load. Constructed by the Metropolitan-Vickers Electrical Co., Ltd.

Frontispiece

(T.5200)

STEAM TURBINE THEORY AND PRACTICE

A TEXTBOOK
FOR ENGINEERING STUDENTS

BY
WILLIAM J. KEARTON, D.ENG.

*Member of The Institution of Mechanical Engineers
Member of The Institution of Naval Architects
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University of Liverpool*

WITH 395 ILLUSTRATIONS
39 WORKED EXAMPLES
AND 133 EXAMPLES
WITH ANSWERS

SIXTH EDITION



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PREFACE TO THE SIXTH EDITION

MINOR corrections and changes have been made in the text, and the final chapter on Governors and Governor Gears has been enlarged by the addition of some notes on fluid pressure governors.

W. J. K.

LIVERPOOL
1950

PREFACE TO THE FIRST EDITION

It is hardly necessary to lay stress upon the very important position of the steam turbine as a prime mover to-day. Its high commercial efficiency, its great reliability, and the ease with which it can be designed for large powers are advantages that have placed it in a unique position. Admitting these to be the facts, there is still no excuse for another book on the subject were it not the case that many of the works that have appeared in the past few years have been over-developed along certain lines, and few have been generally useful to the engineering student. This book is written for the student, and it should prove useful to engineers and draughtsmen who are anxious to obtain a wider knowledge of the theory of the subject.

A feature of the book is the number of examples fully worked out and also the examples set at the end of the chapters dealing with the theory of steam turbines. Answers to these are provided at the end of the book. The examples are somewhat graded and it is hoped that the keen student will find these a means of testing his progress and increasing his confidence.

The various symbols used have been standardized and tabulated at the outset.

Free use has been made of the calculus, as most engineering students nowadays have a working knowledge of the subject sufficiently extensive to enable them to integrate the simpler functions and solve the differential equations commonly met with. Many points may seem to have been laboured. The justification for this is that such points are rarely understood or are insufficiently appreciated.

Much confusion exists in students' minds regarding the various

terms which are used to classify the different types of turbines. It is hoped that the descriptions and diagrams of Chapter I will do much to remove this confusion. The section of the work dealing with the properties of steam and with the entropy and Mollier diagrams is next introduced. In spite of the many ingenious steam diagrams which are devised from time to time, the one and only diagram, at least for impulse turbine calculations, is that of Dr. Mollier, in which total energy is plotted against entropy. This has been used extensively to illustrate the subject. The elusive subject of reheat factor or "*coefficient de récupération*" is explained at some length in Chapter IV.

The flow of steam through nozzles is dealt with in the usual manner, i.e. expansion is assumed to take place in thermal equilibrium. It is felt that for the student who is approaching the subject for the first time this is the simpler and easier method. In view of the theoretical importance of supersaturation, however, sufficient space has been devoted to it to explain the anomaly of excessive mass flow and also the supercooling effect.

The flow of steam through blades is treated along familiar lines, and in order to enhance the practical value of this section, a number of examples are worked out illustrating the usual methods of design employed in drawing offices.

The fundamental principles underlying the design of rotors are embodied in Chapters IX and X, dealing respectively with the strength and critical speed of rotors. Here one pays a tribute to the exceptionally valuable pioneer work of Dr. de Laval, who introduced the wheel of constant strength and the flexible shaft.

For permission to publish the equations relating to the strength of hyperbolic discs, and also an account of nozzle experiments, the author is indebted to Professor A. Stodola. Practical examples relating to the design of discs and rotor shafts are worked out fully and should prove useful to the student and draughtsman.

In Chapter XI a few fundamental theorems relating to the determination of the principal dimensions of steam turbines are given, while the remainder of the book is devoted to illustrated descriptions of turbines and associated plant by the leading turbine builders.

In view of the importance of reduction gearing in marine work, attention is directed to Chapter XVII, in which the fundamental principles of involute gearing are set out. Every attempt has been made to bring the book up to date and a few of the recent developments are dealt with very briefly in the concluding chapter.

With respect to the preparation of the latter half of the book, the author wishes to acknowledge the cordial help he has received from the following firms, who have kindly supplied information and, in some cases, blocks—The British Ljungström Marine Turbine Co., Ltd., London; the British Thomson-Houston Co., Ltd.,

Rugby ; John Brown & Co , Ltd., Clydebank ; Brown, Boveri & Co., London ; the Brush Electrical Engineering Co., Ltd., Loughborough ; the De Laval Steam Turbine Co., London (Aktiebolaget de Lavals Angturbin) ; Fraser & Chalmers Engineering Works, Erith ; the Metropolitan-Vickers Electrical Co., Ltd , Manchester ; C. A. Parsons & Co , Ltd , Newcastle-on-Tyne ; Svenska Turbinfabriks Aktiebolaget Ljungström, Finspong, Sweden.

The author is also indebted to the editor of *Engineering* for permission to publish a brief description of the Larderello Natural Steam Plant and also a description of the 15,000 kW turbo-generator constructed by Messrs. C A Parsons & Co for the Lots Road Power Station, London ; to Messrs. T Stevens and H. M. Hobart, the authors, and Sir Isaac Pitman & Sons, Ltd , the publishers of *Steam Turbine Engineering*, for permission to use certain illustrations relating to the vertical Curtis turbine and the Rateau heat accumulator ; and to the Controller of H M. Stationery Office for permission to publish a drawing of the Brown-Curtis patented Diaphragm

In conclusion, in spite of the large amount of arithmetical work entailed in working the examples, it is confidently hoped that errors will be " few and far between " The author will be glad, however, to receive intimations of such errors and any helpful suggestions that may come along.

W. J. KEARTON

LIVERPOOL

August, 1922

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THE MEANING OF SYMBOLS USED THROUGHOUT THE WORK

THE number of symbols required is such that it has been necessary in some cases to employ the same symbol to represent different quantities. As this only occurs in quite different branches of the subject, no confusion should arise.

ENGLISH LETTERS

λ	=	cross-sectional area of nozzle, etc
λ_m	=	mouth or outlet area of nozzle
λ_t	=	throat area of nozzle
$\lambda h p$	=	adiabatic horse-power of a turbine stage, based on the total internal steam flow and the stage adiabatic heat drop
l	=	axial length of journal bearing
b	=	constant in Callendar's equation of state
b	=	axial width of turbine blade
C_w	=	specific heat of water
C_p	=	specific heat of superheated steam at constant pressure
$C_{p,m}$	=	mean specific heat of superheated steam at constant pressure
D	=	mean diameter of turbine blade ring, valve, etc
E	=	internal energy (and Young's modulus of elasticity)
E_s	=	internal energy of steam
E_w	=	internal energy of water
e	=	eccentricity of journal bearing
f	=	stress
f_b	=	bending stress at root of blade
f_c	=	centrifugal stress in blade (generally at root section)
f_r	=	radial stress in disc
f_t	=	tangential stress in disc
f_{tr}	=	hoop stress in a thin ring
H	=	heat of generation of steam (and heat supplied)
H_a	=	energy available due to adiabatic expansion, i.e. the adiabatic heat drop
H_c	=	cumulative heat drop
H_i	=	internal work done per lb. of steam
H_w	=	coupling work done per lb. of steam passing through the turbine
$H p_d$	=	horse-power loss due to disc friction
$H p_{bw}$	=	horse-power loss due to blade windage
$H p_{wf}$	=	horse-power loss due to wheel friction
h	=	sensible heat
h_n	=	heat drop in nozzles
h_b	=	heat drop in blades

I	= enthalpy
I_w	= enthalpy of water
I_s	= enthalpy of steam in any condition
I_x, I_y , etc.	= second moments of area about axes XX, YY, etc.
J	= Joule's mechanical equivalent of heat
K	= load on turbine in kW (also the Parsons Number for a turbine)
k	= ratio of relative velocity at outlet to relative velocity at inlet, in impulse turbine blades
L	= latent heat of vaporization
L	= circumferential length of effective bearing arc in journal bearing
l	= radial height of blades
l	= lift of throttle valve
m	= index of v in Pv^m for expansion with friction
m	= ratio of blade or nozzle height to mean diameter of turbine
N	= revolutions per minute
N_c	= critical speed in revolutions per minute
n	= index of v in Pv^n , generally for adiabatic and frictionless expansion
P	= absolute pressure, lb. per square foot
P	= load on journal bearing per unit width
p	= absolute pressure, lb. per square inch
p	= intensity of pressure in journal bearing
p_s	= absolute pressure of saturated steam
p_{ss}	= absolute pressure of supersaturated steam
p_{sv}	= steam pressure before stop valve
p_{gv}	= steam pressure before governor valve
p_o	= steam pressure in first-stage nozzle box
p_e	= exhaust steam pressure
p_m	= mean circumferential pitch of blades
R	= degree of reaction
R	= radius of journal bearing
$\bar{R}$	= gas constant
R_u	= universal gas constant
R	= reheat factor
R_n	= reheat factor for n stages
R_∞	= reheat factor for an infinite number of stages
R.h.p.	= rim horse-power, or the power developed by the steam on the blades at the rim of the wheel
r	= critical pressure ratio in nozzle flow
r	= radial clearance in journal bearing
S	= degree of supersaturation
S.h.p.	= shaft horse-power of a stage, i.e. the power transmitted by the wheel to the turbine spindle.
T	= absolute temperature
t	= temperature in degrees Fahrenheit
U_1	= relative velocity of steam at inlet to blades
U_2	= relative velocity of steam at outlet from blades
u	= mean peripheral velocity of blades

V	= velocity
V_w	= velocity of whirl
V_{ws}	= velocity of whirl of dry steam
V_{wm}	= velocity of whirl of moisture particles
v_s	= specific volume of steam, cubic feet per lb.
v_w	= specific volume of water, cubic feet per lb
W	= steam consumption of turbine, lb per hour
W	= work done
W	= work done in opposing friction
w	= steam flow through nozzles, etc , in pounds per second
w_i	= total internal steam flow through turbine
w_n	= steam flow through nozzles
w_g	= steam flow through diaphragm gland
x	= dryness fraction of steam
r	= radial displacement of turbine wheel at any radius r
Z	= coefficient of viscosity of lubricating oil

GREEK LETTERS

α	= nozzle or jet angle
β_1	= blade inlet angle
β_2	= blade outlet angle
γ	= absolute direction of steam leaving blades
Δ	= maximum static deflection of turbine spindle, inches
ΔH	= adiabatic heat drop in a stage
ϵ	= eccentricity ratio of journal bearing = $\left(\frac{e}{r}\right)$
η_c	= thermal efficiency of cycle
η_t	= thermal efficiency of turbine based on shaft work
η_r	= efficiency ratio = $\left(\frac{\eta_t}{\eta_c}\right)$
η_n	= nozzle efficiency
η_v	= nozzle velocity coefficient
η_b	= efficiency of blades
η_{gs}	= gross stage efficiency
η_s	= stage efficiency
η_i	= internal efficiency of turbine
η_g	= generator (alternator) efficiency
λ	= circumferential length of nozzle arc, or in other cases, the fraction of the circumference covered by active nozzles
λ	= percentage leaving loss, i.e the ratio of the residual velocity loss to the adiabatic heat drop
μ	= coefficient of friction in journal bearing
μ	= coefficient of viscosity of steam
μ	= number of rows of moving blades in velocity-compounded turbine

ρ	=	ratio of blade velocity to steam velocity
ρ_s	=	density of saturated steam
ρ_{ss}	=	density of supersaturated steam
σ	=	Poisson's ratio
ϕ	=	"carry-over" coefficient
ϕ	=	"attitude" of journal bearing
ϕ	=	entropy
ϕ_e	=	change of entropy during evaporation
ϕ_s	=	entropy of steam in any condition
ϕ_w	=	entropy of water

CHAPTER I

INTRODUCTION

1. THE steam turbine, in a crude form, is said to have been used many centuries ago, but its development as a really practical form of prime mover has only taken place during the last sixty years. To-day it stands as the most important prime mover in existence. It offers many advantages.

From a thermodynamic point of view the steam turbine occupies a favourable position, as it can translate into mechanical work a relatively large fraction of the heat energy rendered available by the expansion of the steam in the turbine. Its thermal economy is also fairly good, especially in turbines of large output and operating at fairly high pressures.

From the mechanical point of view, the turbine is ideal, because the propelling force is applied directly to the rotating element of the machine and has not, as in the reciprocating engine, to be transmitted through a system of connecting links which are necessary to transform efficiently a reciprocating motion into a rotary motion. Hence, since the steam turbine possesses for its moving parts rotating elements only, if the workmanship is good and the machine is correctly designed, it ought to be free from out-of-balance forces. In practice, a rotor weighing 20 tons or more and rotating at 1,500 revolutions per minute may be so well balanced that the motion, except for sound, is just perceptible. The significance of this statement may be appreciated when it is realized that if the same turbine rotor were out of balance to the extent of one pound at a radius of 4 ft., or in other words if the centre of gravity of the rotor were displaced only one-thousandth part of an inch from the axis of rotation, then the periodic force tending to produce vibration would be 3,070 lb. weight or 1.36 tons. When one considers that it is practically impossible to balance any reciprocating engine, the superior merits of the steam turbine in this respect are immediately apparent.

The impulses applied by the steam to the blades of a turbine come with such regularity and constancy that if the load applied to the machine is maintained constant, and therefore if the steam flow through the turbine is maintained at a steady value, the torque applied at the turbine coupling due to such impulses will be quite uniform. Therefore, if the turbine be employed to drive a machine which offers over a given period of time a sensibly constant resisting torque, the speed of the turbine and the driven machine will be very uniform.

Certain driven machines such as electric generators, centrifugal

pumps, centrifugal gas compressors, etc., when operating under steady conditions as regards output, offer a fairly constant resistance. Such machines are also essentially high-speed machines. Consequently the steam turbine is, from every point of view, eminently suitable as a prime mover for driving such machines.

The uniform turning moment and consequent uniform speed are also very desirable qualities for a prime mover required to drive certain textile machines.

A feature of considerable practical value is the entire absence of internal lubrication in steam turbines (excepting one or two small turbines). Two important advantages follow from this. In the first place, the exhaust steam is not contaminated with oil vapour and the condensed steam may be fed direct into the boilers without passing through filters. Furthermore, in certain industrial turbine plants where the whole or part of the turbine steam is used for some heating process, such steam is clean and may, in many cases, be used for direct heating. In the second place, there is a considerable saving in lubricating oil.

As the only parts of the turbine which require lubrication are the bearings supporting the rotor spindle, the governor shaft bearings and one or two gear wheels, and as there are no sliding parts, the amount of wear is negligible. The lubricating oil is circulated around the system, is cooled and filtered, and so may be used for long periods without replacement. Thus the cost of lubricating oil is negligible when compared with that for a steam engine of equal power.

The extensive use of electricity in the present age has brought into existence many large electricity generating stations. Economic considerations call for individual generating units of very large output. Similarly, the propulsion of vessels of large tonnage and at high speeds has called for machinery of such a power as would be quite beyond the possibilities of the reciprocating engine, steam or Diesel. These demands have been met with comparative ease by the steam turbine. The steam turbines of the ill-fated battle cruiser *Hood*, built in 1917, developed about 150,000 s.h.p. on 4 shafts and sent her 42,000 tons through the water at 31 knots. To-day, there are in central power stations turbines developing more than 100,000 kW, or nearly the total power of a modern battleship, on a single line of shaft.

2. Principle of Action of Steam Turbine. In the reciprocating steam engine, the pressure energy of the steam is utilized to overcome external resistances, and the dynamic action of the steam is negligibly small. Steam engines may be operated by using the full pressure without any expansion or drop of pressure in the cylinder. Such engines are said to work non-expansively. The steam turbine proper could not be operated in such a manner. The machine in which the pressure of steam is used for forcing round a rotating plate or similar device is a rotary engine and not a turbine.

This type of turbine is termed a "simple" impulse turbine because the expansion of the steam takes place in one set of nozzles only. Generally the nozzle has a form similar to that shown in the sketch, Fig. 2, i.e. having a rounded entrance leading into a

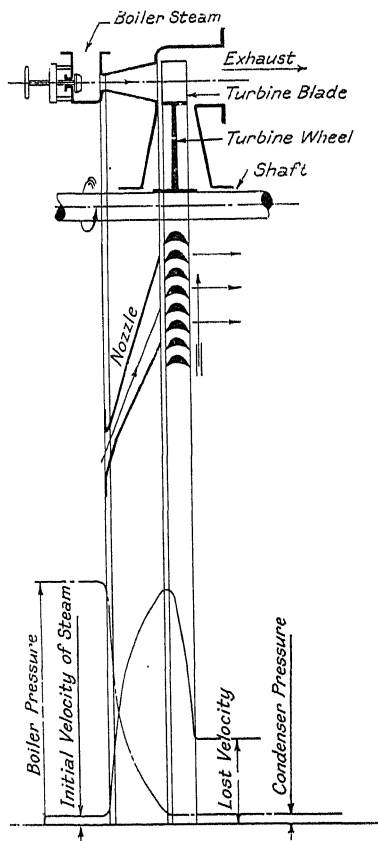


FIG. 1.—Diagrammatic Arrangement of Simple Impulse Turbine

constriction, termed the "throat," and a diverging outlet or mouth. The pressure of the steam falls from that in the steam chest to that existing in the condenser (or atmosphere, if the turbine is non-condensing), while the steam flows through the nozzle. Hence the pressure in the wheel chamber is practically equal to condenser pressure.

There is thus a relatively great ratio of expansion of the steam in the nozzles with the result that the steam issues from the nozzle

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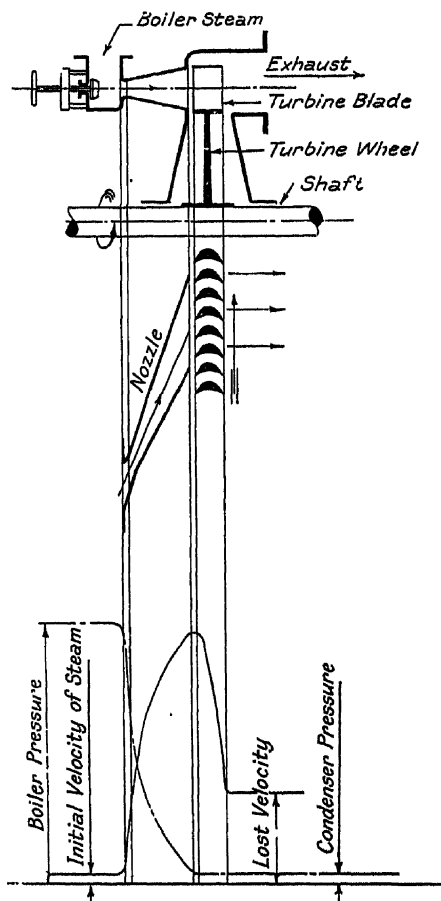


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outlet with a very high velocity, e.g. about 3,500 ft. per second. As will be shown later, the velocity of the blades for good economy should be about one-half of the steam velocity, i.e. about 1,700 ft per second. In practice, the maximum blade velocity reached in this type of turbine is about 1,400 ft per second, or about 950 miles per hour. For economic reasons—since this type of turbine is only employed for relatively small powers—the rotor diameter is kept

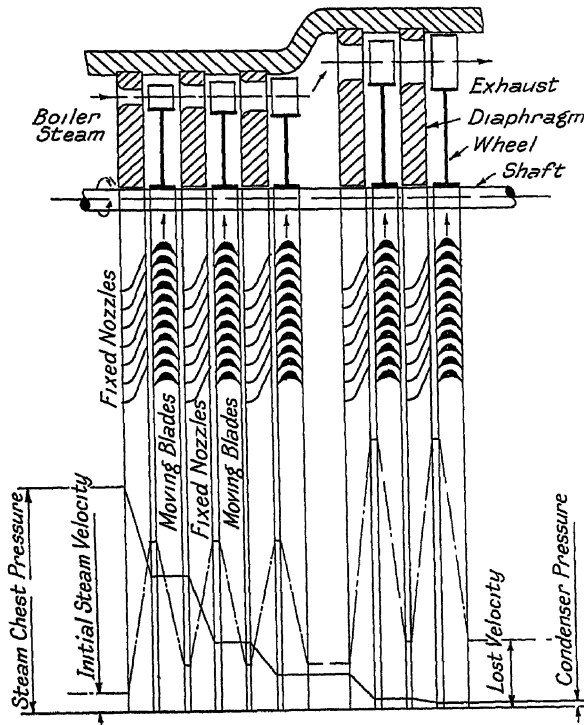


FIG. 2 —Diagrammatic Arrangement of Pressure-compounded Impulse Turbine

fairly small, and in consequence the rotational speed is high, reaching 30,000 r.p.m. in some of the smaller machines. Few driven machines require such a high speed, and it is usually necessary to reduce the speed by gearing.

It will be seen that the velocity of the steam leaving the moving blades is a large proportion of the maximum absolute velocity of the steam when leaving the nozzle. The exit velocity involves a loss of energy which is commonly termed the "carry-over loss" or "leaving loss." A small final velocity is necessary in order that the steam may escape from the turbine into the condenser. A

moderate outlet velocity would involve only a small loss, since the actual energy loss varies as the square of the leaving velocity. But where a relatively large leaving velocity is inevitable, the corresponding leaving loss may reach a large value. In actual de Laval turbines, the "leaving velocity" may amount to about 33 per cent of the nozzle outlet velocity. Hence the "leaving loss" may reach about $(0.33)^2$ or 11 per cent of the initial kinetic energy of the steam.

The characteristics of the simple impulse turbine are, therefore, a single expanding nozzle, or set of such nozzles, with the high-pressure steam confined to the steam chest and a casing subjected internally to the lower pressure only; high steam and blade velocities, with consequent high speeds of rotation; and, lastly, a high carry-over loss. The principal example of the type is the well-known de Laval turbine.

4. The Pressure-compounded Impulse Turbine. It is obvious that by arranging the expansion of the steam in a number of steps, we could arrange a number of simple impulse machines in series on the same shaft, allowing the exhaust steam from one turbine to enter the nozzles of the succeeding turbine. Each of the simple impulse machines would then be termed a "stage" of the turbine, each stage comprising its set of nozzles and blades.

This is equivalent to splitting up the whole pressure drop into a series of smaller pressure drops; hence the term "pressure-compounded."

The nozzles are usually fitted into partitions, termed "diaphragms," which separate one wheel chamber from the next. The wheels are mounted individually on the shaft or spindle, and carry the blades on their periphery. Expansion of the steam takes place wholly in the nozzles, the space between any two diaphragms being filled with steam at constant pressure. The pressures on either side of any diaphragm are therefore different, the greatest difference of pressure occurring in the first few stages. Hence, steam will tend to leak through the space between the bore of the diaphragm and the surface of the shaft or wheel hubs. Special devices are fitted to minimize these leakages, and are described in later chapters.

The pressure-compounding causes a smaller transformation of heat energy into kinetic energy to take place in each stage than in the simple impulse turbine. Hence the steam velocities are much lower, with the result that blade velocities and the rotational speed may be lowered. It is fairly clear that given a constant quantity of available energy per pound of steam, the speeds may be reduced at will simply by increasing the number of stages, but for very low speeds the number of stages may become excessive.

Although the leaving velocity may bear the same ratio to the steam velocity at the nozzle outlet as it does in the simple impulse turbine, yet the kinetic energy per stage is now only a fraction of

the total available energy. Consequently, the leaving loss in the pressure-compounded turbine is also only a fraction of that associated with the simple impulse turbine and is usually only about 1 or 2 per cent of the total available energy.

This type of turbine was developed by the late Professor A. Rateau, of Paris, on the one hand, and by Dr. Zoelly, of Zurich, on the other.

5. Simple Velocity-compounded Impulse Turbine. This type is shown diagrammatically in Fig. 3. It comprises a nozzle or set of nozzles and a wheel fitted with two or more rows of moving blades. The example shown in Fig. 3 has three rings of moving blades on the rotor and such a wheel is referred to as a "three-row wheel." There are also a number of guide blades, suitably arranged between the moving blades and set in the reverse manner. With a three-row wheel there are two rows of guide blades placed between the first and second and second and third rows of moving blades, respectively. These fixed guide blades need not necessarily extend around the full circumference of the casing. It is only necessary to arrange them roughly in front of the nozzle or nozzles but covering a somewhat larger arc of circumference than the nozzles themselves.

Steam entering the nozzle expands from the initial pressure down to the exhaust pressure. Thus, in general, the steam velocity is very high, as in the simple impulse turbine. The provision of two or more rows of moving blades, however, enables the blade velocity to be made appreciably less than would be necessary for the turbine having a single row of blades, it being assumed that the efficiency is the maximum in both cases. On passing through the first row of moving blades, the steam gives up only a part of its kinetic energy and issues from this row of blades with a fairly high velocity. It then enters the first of the two rows of guide blades and is re-directed by them into the second row of moving blades. There is a slight drop in velocity in the fixed guide blades due to friction. In passing through the second row of moving blades the steam suffers a change of momentum and gives up another portion of its kinetic energy to the rotor. It is re-directed in the second row of guide blades, does work on the third row of moving blades, and finally leaves the wheel in a more or less axial direction with a certain residual velocity. This velocity is comparatively small, and hence the leaving loss is small, being about 2 per cent of the initial available energy of the steam.

For a fairly large pressure drop, say from 200 lb. per square inch absolute to 2 lb. per square inch absolute, the steam velocity is theoretically about 4,000 ft. per second. It will be shown later that with the ordinary nozzle arrangements used in practice, the most efficient speed of the blades for this type of turbine is about 0.15 of the steam speed. Thus, in this case, the blade speed would be

about 600 ft. per second, which is still fairly high. Generally speaking, owing to its low efficiency, this type of turbine having three rows of moving blades is used largely for small turbines driving auxiliary machinery. The two-row wheel is appreciably more efficient than the three-row wheel and, as will be shown later, is often incorporated in turbines of other types with advantage.

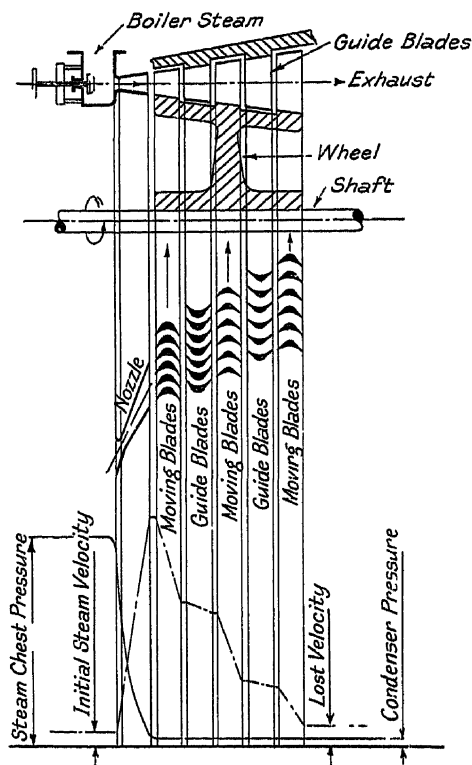


FIG. 3.—Diagrammatic Arrangement of Velocity-compounded Impulse Turbine

The turbine which has just been described and that to be described in the next paragraph were invented by the American engineer, Mr. C. G. Curtis.

6. Pressure-velocity-compounded Turbine. As has already been mentioned, the two-row wheel is more efficient than the three-row wheel. But for the best economy it may be shown that the blade speed would have to be about 0.23 of the steam speed, or about 920 ft. per second. The construction of the wheel required to carry two rows of blades at this velocity would present some difficulty but is more or less possible, depending on the length of the blades. But an obvious way to reduce the blade velocity would be to split up the

available energy by having two or more simple velocity-compounded turbines in series on the same shaft. The total pressure drop is then effected in as many steps as there are wheels on the shaft, and hence the turbine is pressure-compounded as well as velocity-compounded. As in other types of impulse turbines, the steam is

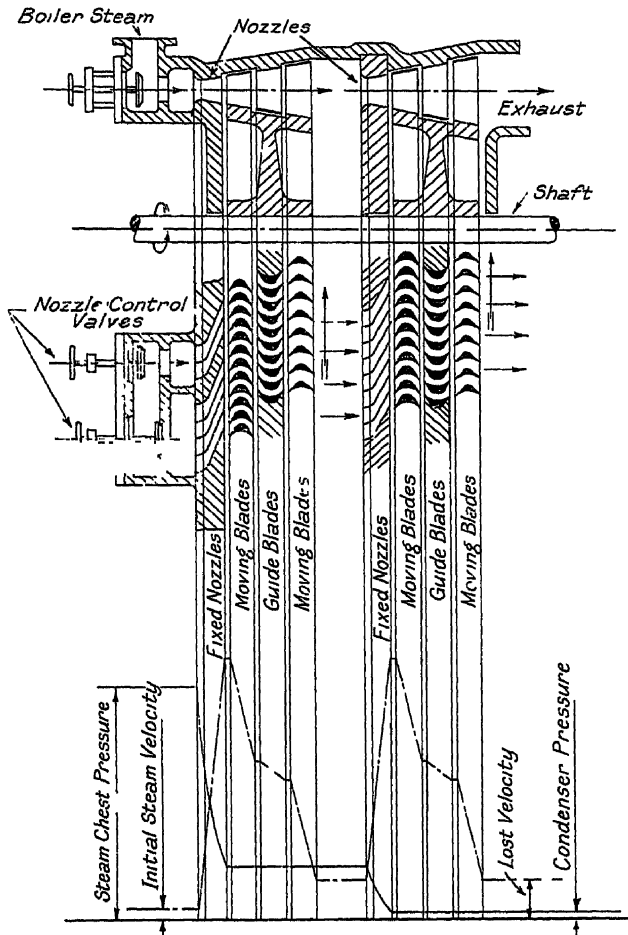


FIG. 4—Diagrammatic Arrangement of Pressure-velocity-compounded Impulse Turbine

expanded wholly in the nozzles, and the wheels rotate in steam at constant pressure. Thus ample clearances are permissible in the radial direction, while the clearances in an axial direction should, from the point of view of efficiency, be kept as small as possible.

The middle portion of Fig. 4 is intended to show roughly how this and other types of impulse turbine may be controlled so as to give

the best efficiency under varying loads. It is well known that for high efficiency the difference between the initial pressure after the governor valve and the condenser pressure should be as great as possible. If governing is effected by throttling the steam, then at light loads the efficiency will be considerably reduced. In the diagram, Fig. 4, two sets of nozzles are shown, one set having 2 nozzles and the other 4 nozzles. Thus there are 6 nozzles in all, and for the sake of illustration we shall assume that the total power of the turbine is 60 h.p. Moreover, in order to simplify the explanation, let it be assumed that the steam delivered by each nozzle under the full pressure drop is sufficient to develop 10 h.p. Actually this assumption is not justified.

Then at full load, all 6 nozzles will be delivering steam at full pressure and the turbine will operate at maximum efficiency. Similarly, at 20 h.p. only 2 nozzles will be open, and at 40 h.p. only the set of 4 nozzles will be delivering steam, in both cases at the full pressure. Now suppose the horse-power required is only 30. The valve controlling the supply of steam to the group of 4 nozzles would have to be open, but there would have to be a certain amount of throttling of the steam, since at full pressure only 3 nozzles are required. Yet the efficiency will be higher than if the whole of the nozzles were in one group and the steam supply to that group were controlled by a single throttle valve. At 10 h.p., only the valve controlling the set of 2 nozzles would be open and there would be a certain amount of throttling.

The pressure-velocity-compounded turbine is comparatively simple in construction and is much more compact than the multi-stage pressure-compounded impulse turbine. Unfortunately, its efficiency is not so high. At one time it was widely used in power stations, but is now an obsolete type.

7. Pure Reaction Turbine. This type, which is illustrated diagrammatically in Fig. 5, comprises two or more radial tubes rotating on and communicating with another pipe or trunnion through which steam is supplied to nozzles which are screwed into the ends of the radial pipes, and arranged with their axes in the tangential direction.

Steam expands in the nozzles and issues with a certain velocity which is represented by the vector AB. If CD represents the peripheral velocity of the nozzle, then ED will represent the velocity of the steam relative to a fixed point at a given instant. Due to the change of velocity which occurs, there is a thrust or *reaction* on the rotating tube and tangential to it. This reaction constitutes the driving force, hence the name.

The pure reaction turbine is not a practical type, although efforts to develop it were made by the late Sir Charles A. Parsons.

8. Axial-flow Impulse-reaction Turbine. In this type of turbine we have a joint application of the impulse and reaction principles

of operation. There are a number of rows of moving blades attached to the rotor and an equal number of rows of fixed blades attached to the casing. The fixed blades correspond to the nozzles mentioned in connection with the impulse turbines. Steam is admitted for the whole circumference, and thus there is what is termed "all-round or complete admission." In passing through the first row of fixed blades, the steam undergoes a small drop in pressure and its velocity is somewhat increased. It then enters the first row of moving blades and, just as in the impulse turbine, it suffers a change in direction

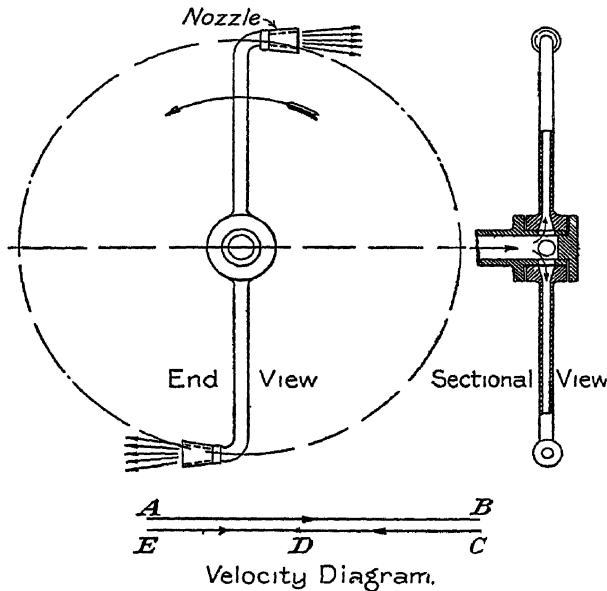


FIG. 5

and, therefore, of momentum. This gives rise to an impulse on the blades. During its passage through the moving blades, however, the steam undergoes a further small drop in pressure and, in consequence, there is a certain increase in steam velocity produced in the moving blade which gives rise to a reaction in the direction opposite to that of the added velocity. Thus the gross propelling force is the vector sum of the impulse and the reaction. This type of turbine is commonly termed a "reaction" turbine, although the term is not exact. It may here be reiterated that in the pure impulse turbine *no* expansion of the steam takes place in the moving blades.

The diagram given in Fig 6 shows roughly how the blade heights increase as the specific volume of the steam increases with reduction in pressure, also how the pressure falls gradually as the steam passes through the groups of blades. It will be observed from the diagram

and also from the foregoing remarks that there is a pressure drop across each row of blades, fixed and moving. This is of considerable practical importance, especially at the high-pressure end of the turbine where the pressure drops are greatest, because this difference of pressure tends to force some steam through the clearance spaces

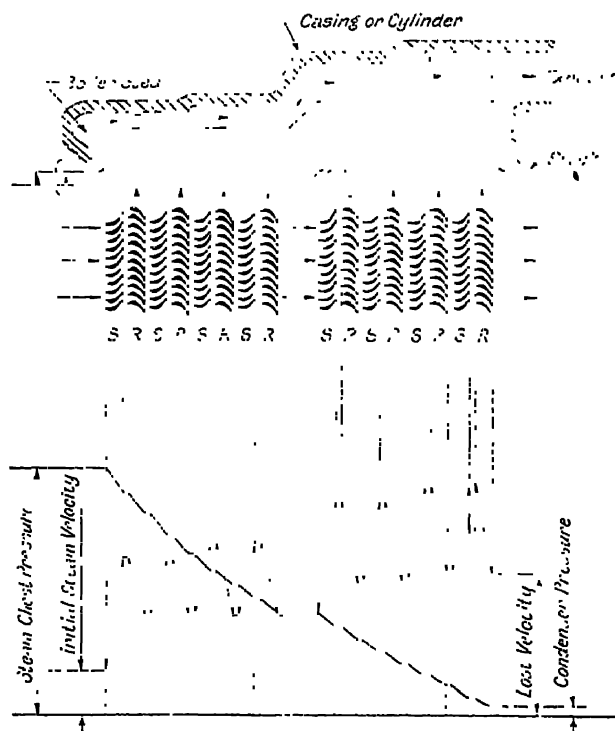


FIG. 6.—Diagrammatic Arrangement of Axial-flow Impulse-reaction or "Reaction" Turbine

between the moving blades and the casing and between the fixed blades and the rotor. The available energy possessed by this leaking steam is thus partly lost.

The steam velocities in this type of turbine are comparatively moderate, the maximum being, according to theory, about equal to the blade velocity. In practice, the steam velocity is commonly arranged to be greater than the blade velocity in order to reduce somewhat the total number of blade rows. The leaving loss for this type of turbine is normally about the same as for the multi-stage impulse turbine having single-row wheels. This type of turbine has been and continues to be very successful in practice. Its practical development was due to the genius and engineering skill of

the late Sir Charles A. Parsons. The turbine bears the name of its illustrious inventor.

9. **The Radial-flow Double-motion Reaction Turbine.** The turbines described hitherto, with the exception of the pure reaction turbine, have been of the axial flow type, i.e. the general direction

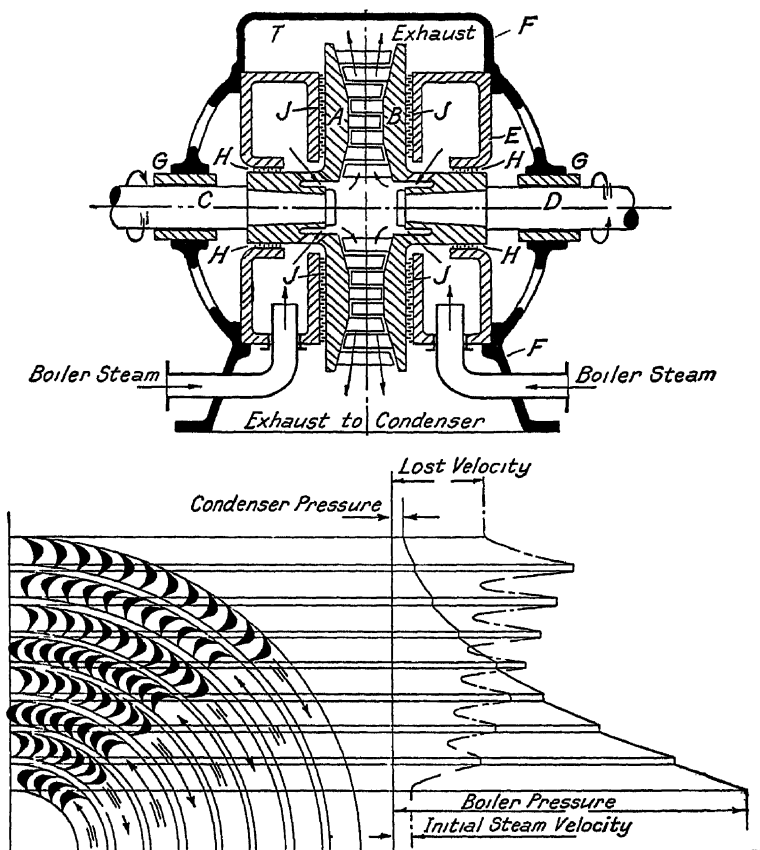


FIG 7—Diagrammatic Arrangement of Double-motion Radial-flow Reaction Turbine (Ljungstrom type)

NOTE—In each ring of blades the blades extend round the complete circumference

of steam flow has been roughly parallel to the turbine axis. It is, however, possible to arrange the groups of blades in concentric rings and attached to the sides of separate discs. If one set of blades is stationary, then we have a single-motion radial-flow turbine. The steam may be arranged to flow from the centre outward or from the outside towards the centre. Assuming that such a turbine operates as an impulse-reaction (reaction) turbine, then the most

economical steam velocity from the blades in any stage is approximately equal to the peripheral velocity of the blades in that stage, and usually a fairly large number of stages is required.

Suppose now that the disc to which the fixed rings of blades are attached is arranged to rotate in the opposite direction to the moving blades, then the action will be generally the same as before except that the effective blade speed is doubled. Hence the steam velocity may be doubled and the turbine may, for a given size, be made more efficient. The practical construction of such a turbine presents many difficulties and is dealt with in a later chapter.

The diagram, Fig. 7, will serve to explain the main features of this type of turbine. Discs A and B are attached to the shafts C and D respectively, and these rotate in opposite directions. When this type of turbine is employed for driving electric generators, each of the shafts C and D drives a separate alternator at the same speed, the alternators running in parallel. E is the steam chest and F is the exhaust casing, from which the bearings G are suitably supported. Steam enters the steam chest E through two pipes leading from the side of the exhaust casing. The steam passes through a number of holes in the hollow shaft, as shown, and thus enters the centre space of the turbine. There is an impulse-reaction effect in every ring of blades except the first one, in which there is a reaction effect only. Special means are adopted to prevent leakage of the steam from the chest E. These means comprise an elaborate shaft packing at H to prevent leakage of steam to the atmosphere, and also concentric rings of labyrinth packing at J to prevent leakage from the steam chest into the exhaust space.

The practical representative of this type is the Ljungström turbine.

10. Combination Turbines. Any multi-stage turbine may be a combination of two or more of the types described above. Such a turbine is commonly referred to as a "combination" machine. Certain advantages which accrue to the use of two different types of turbine in the same machine will be dealt with in later sections of the book.

The most common combinations are—

(a) Velocity-compounded impulse wheel (or wheels) followed by several single-row impulse wheels. This is a combination of the Curtis and Rateau types.

(b) Velocity-compounded impulse wheel (or wheels) followed by several stages of impulse-reaction blading (Curtis-Parsons type).

(c) One or more single-row impulse stages followed by several stages of impulse-reaction blading (Rateau-Parsons type).

CHAPTER II

PROPERTIES OF STEAM

11. Introduction. For a study of the steam turbine from the thermodynamic point of view it is necessary to understand the various thermal properties of steam and water and also to have access to the values of those properties. Both liquid water and the dry and saturated vapour are complex substances, and their thermal properties are not related in any simple way. They can only be determined by experiment. Much painstaking experiment and research by investigators in many countries has been done in the past and the results of this work are usually embodied in carefully laid-out tables, known as Steam Tables.

Several such tables have been published from time to time. In many cases, the data given for high pressures have been obtained by a fairly extensive extrapolation of existing experimental data at normal steam pressures. In consequence, the tables differed from one another at the high-pressure end of the scale, and in making reports of tests it became necessary to specify the tables which had been used in making the calculations. Many of the tables were not thermodynamically consistent, a calculation made by using the tables in one way would not agree with another calculation having the same end in view, but utilizing the tables in a different way. The late Professor Callendar was the first to introduce a thermodynamically consistent table of properties, based on his own experimental work.

In recent years, investigators working in Great Britain, the United States, Germany and Czechoslovakia have met at three International Steam Table Conferences for the purpose of extending international agreement on the properties of steam. Three such conferences have already been held: in London, 1929, in Berlin, 1930, and in Washington, 1935. New International Steam Tables (1)* were recently published.

The following steam tables have been published in recent years—

1. *The Mollier Steam Tables and Diagrams* (Extended to the Critical Pressure). By Dr. Richard Mollier. English Edition by Dr. H. Moss (1927). Published by Sir Isaac Pitman & Sons, Ltd. The two Mollier Diagrams are published separately in envelope.
2. *International Steam Tables* (In Centigrade or Fahrenheit Units). By the late H. L. Callendar, F.R.S., revised by his son, Mr G. S. Callendar. Third Edition published 1934 by Edward Arnold & Co. Unless otherwise stated, the problems set in the various parts of this book and involving the use of steam tables, will be solved by using these tables. A Fourth Edition of these tables has been published (May, 1939).

* For literature and other references see the end of the chapter

3. *The Revised Callendar Steam Tables* (1931), calculated by Dr. H. Moss from the new data and formulae of the late Professor Callendar. Published by Edward Arnold & Co.

A new edition of this has also been published (May, 1939) under the title *The 1939 Callendar Steam Tables*.

12. Unit of Heat. In the centigrade system, the thermal unit is the *calorie*. If the pound is taken as the unit of mass, then the heat unit is the pound caloric; if the gramme is the unit of mass, then the corresponding heat unit is the gramme calorie, and so forth.

The calorie is then defined as the heat required to raise unit mass of pure water from a temperature $t^{\circ}\text{C.}$ to $(t + 1)^{\circ}\text{C.}$

There are three such units, viz.—

- (1) *The 15° C. grm. calorie* ($t = 15^{\circ}\text{C.}$). This is equal to 4.184×10^7 ergs. [1 erg = 1 dyne cm.]
- (2) *The 20° C. grm. calorie* ($t = 20^{\circ}\text{C.}$). This equals 4.180×10^7 ergs.
- (3) *The mean calorie*, which is the $\frac{1}{100}$ th part of the heat required to raise the temperature of 1 grm. of water from 0°C. to 100°C. According to Barnes (2), this is equal to 4.184×10^7 ergs. Thus the mean calorie is equal to the 15°C. calorie.

Corresponding to the mean calorie, there is the *mean British Thermal Unit*, which is the $\frac{1}{180}$ th part of the heat required to raise the temperature of 1 lb. of water from 32°F. to 212°F. This unit will be used throughout the book, except where otherwise stated.

The International Steam Tables Conference has recently formulated a heat unit to be termed *the international steam tables calorie* (written I.T.Cal.) and defined as the $\frac{1}{860}$ th part of the international kilowatt-hour. One international electrical watt = 1.0003 absolute watts. Then one mean B.Th.U. = 251.996 I.T.Cals., and one I.T.Cal. per gramme = 1.8 B.Th.U. per lb.

13. Unit of Work. This is the foot-pound and is the work done in raising a mass of 1 lb. through a vertical distance of 1 ft. at sea-level in London. The acceleration of gravity, g , is there 32.19 ft. per sec. per sec.

14. Heat and Work. Heat may be partially converted into work and work into heat. In each case there is no loss of energy in the transformation and the rate of exchange of heat and work is such that

$$W = JH$$

where W = work done in ft.-lb.

H = heat utilized or heat generated, in B.Th.U.

J = Joule's equivalent = 777.8.

Note. In the recently published steam tables the value of J is given as 778.3.

15. Relationship between Temperature and Vapour Pressure. Consider a vessel, Fig. 8, partially filled with pure water and connected to a U-tube or a suitable pressure gauge, depending on the pressure to be measured. Assume that all gases are completely

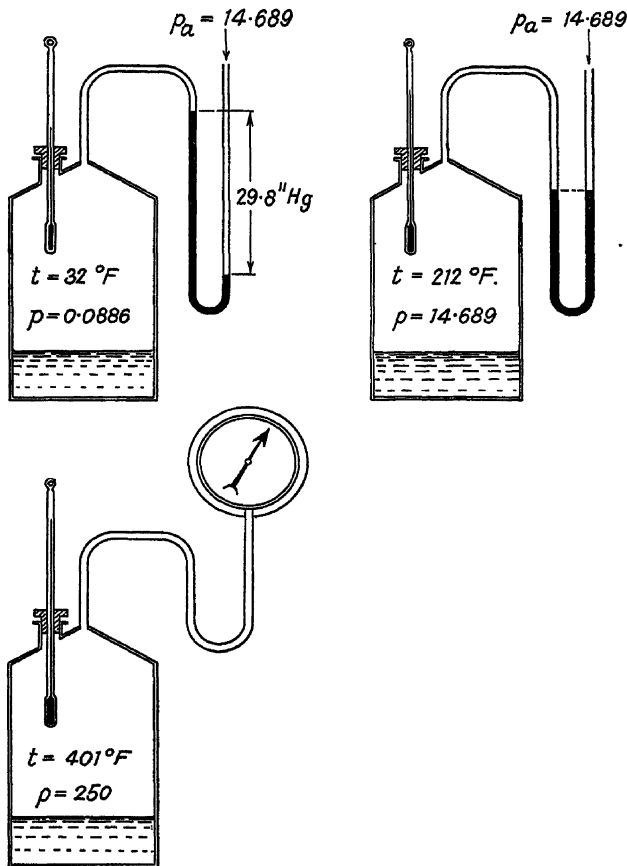


FIG. 8.—Diagram illustrating the Relationship between Temperature and Vapour Pressure

extracted from the space above the water. Then this space will be filled with water vapour which exerts a certain absolute pressure. It is found that this pressure depends on the temperature. For instance, assume first that the temperature is 32°F . and that the water is in the liquid state. Then it would be found that the pressure of the vapour would be 0.0886 lb. per sq. in. abs.

As the temperature is gradually raised, more and more of the water will be evaporated and the pressure exerted by the water

vapour will gradually increase. Thus at 212°F. and at sea-level, the liquid in the manometer tube would stand at the same level in both limbs and the steam pressure would be $14\cdot69$ lb. per sq. in. abs. At 401°F. , the vapour pressure would be found to be 250 lb. per

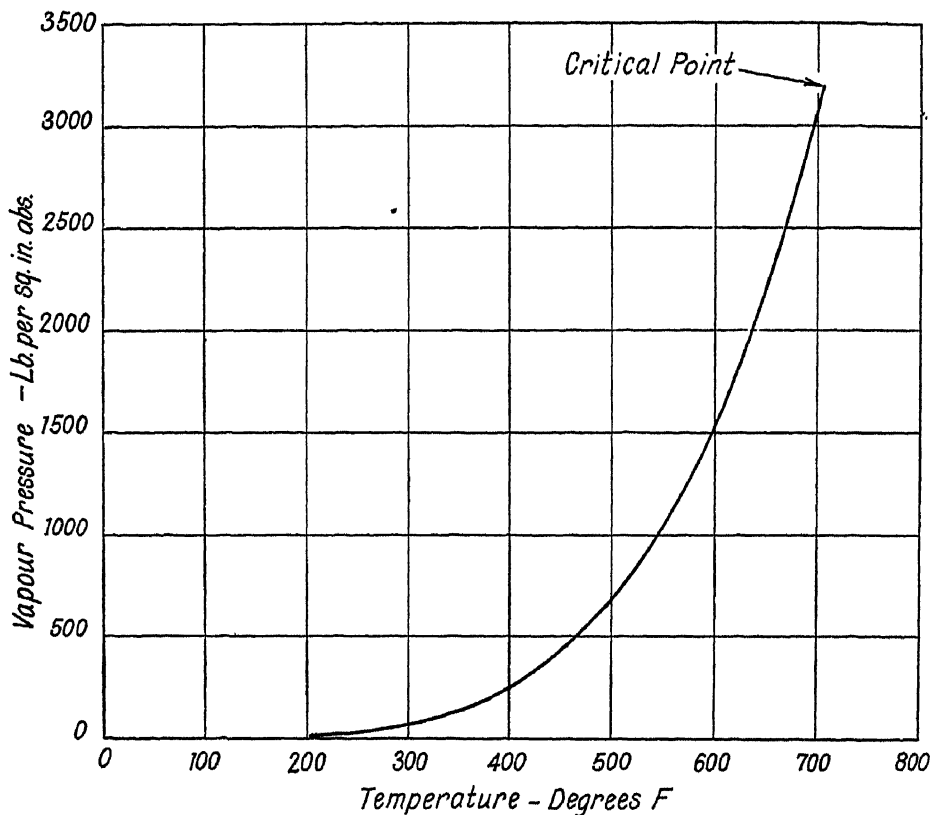


FIG. 9.—Vapour Pressure of Water

sq. in. abs., the pressure indicated by a Bourdon gauge being $250 - 14\cdot7$, or $235\cdot3$ lb. per sq. in.

The relationship between temperature and vapour pressure continues up to a temperature known as the *critical temperature*, which is defined in the following manner. At all temperatures above the critical, it is impossible to liquefy water vapour by pressure, no matter how great the pressure employed. The critical temperature of water is $374\cdot1^{\circ}\text{C.}$ or $705\cdot4^{\circ}\text{F.}$ As the temperature approaches the critical, the vapour pressure also approaches a definite pressure known as the *critical pressure*. This may be defined as the vapour pressure of pure water at a temperature under but infinitely close to the critical temperature. The critical pressure of steam is about

3,210 lb. per sq in. abs. Fig. 9 shows the variation of vapour pressure up to the critical point.

16. Sensible Heat of Water. This is defined as the amount of heat required to raise the temperature of unit mass of water to any given value. The liquid heat at the freezing point, 0°C. or 32°F. , is normally taken as the arbitrary zero. Then the sensible heat at $t^{\circ}\text{F.}$ is the quantity of heat which must be added to unit mass of water to raise the temperature from 32°F. to $t^{\circ}\text{F.}$

The specific heat C_w of water differs little from unity at low pressures, and for approximate calculations the sensible heat

$$h = t - 32 \text{ B Th.U. per lb.}$$

For higher pressures, the increase in C_w with increasing temperature must be taken into account. For a small increase in temperature δt , the additional heat supplied is

$$\delta h = C_w \cdot \delta t$$

and for any range t_1 to t_2 ,

$$h_2 - h_1 = \sum_{t_1}^{t_2} C_w \cdot \delta t$$

Similarly, for any temperature $t^{\circ}\text{F.}$, the sensible heat

$$h = \sum_{32} C_w \cdot \delta t.$$

Dieterici (3) gives C_w as a function of t as follows—

$$C_w = 1.00076 - 0.00009855t + 0.00000064t^2 \quad (1)$$

for temperatures between 95°F. and 572°F.

Callendar (4) gives

$$C_w = 0.98415 + \frac{0.9072}{t + 4} + 0.00002889t + 0.000000278t^2 \quad (2)$$

for temperatures between 32°F. and 392°F. Experimental values of C_w are shown in Fig. 10.

Actually, the heat which must be supplied to raise the temperature of unit mass of water varies, to a slight extent, according to the conditions under which the heat is supplied. This will be considered more fully in Art. 22.

17. Latent Heat. When, for example in a boiler, the temperature of the water has been raised to the saturation point corresponding to the pressure, the further addition of heat at constant pressure generates steam at constant temperature. The heat added during this process is termed latent heat because it is absorbed by the water without any change of temperature and so becomes "latent."

More specifically, the latent heat of vaporization L is defined as the heat required to convert 1 lb. of water at the saturation temperature into 1 lb. of dry and saturated steam.

The latent heat diminishes as the saturation temperature is raised,

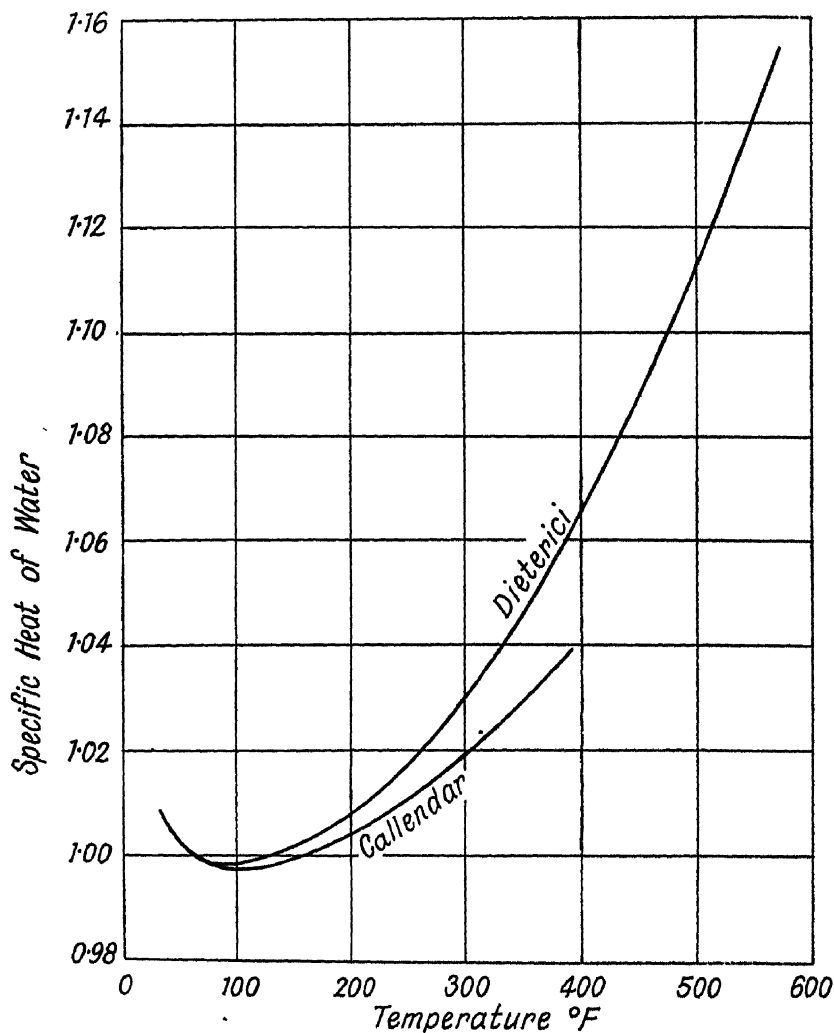


FIG. 10.—Specific Heat of Water

as is shown by Fig. 11. The results arrived at by various investigators are seen to be in good agreement. At low pressures, the latent heat is given approximately by the simple linear relation—

$$L = 1,100 - 0.63t \quad . \quad . \quad . \quad (3)$$

Up to a pressure of 100 lb. per sq. in. abs. the maximum deviation from the values given in the International Tables is under ± 0.5 per cent; at 250 lb. per sq in abs, the error reaches 2.7 per cent.

During the evaporation of water, the heat supplied is expended in two ways—

- (1) In overcoming the external resistance to change of volume.

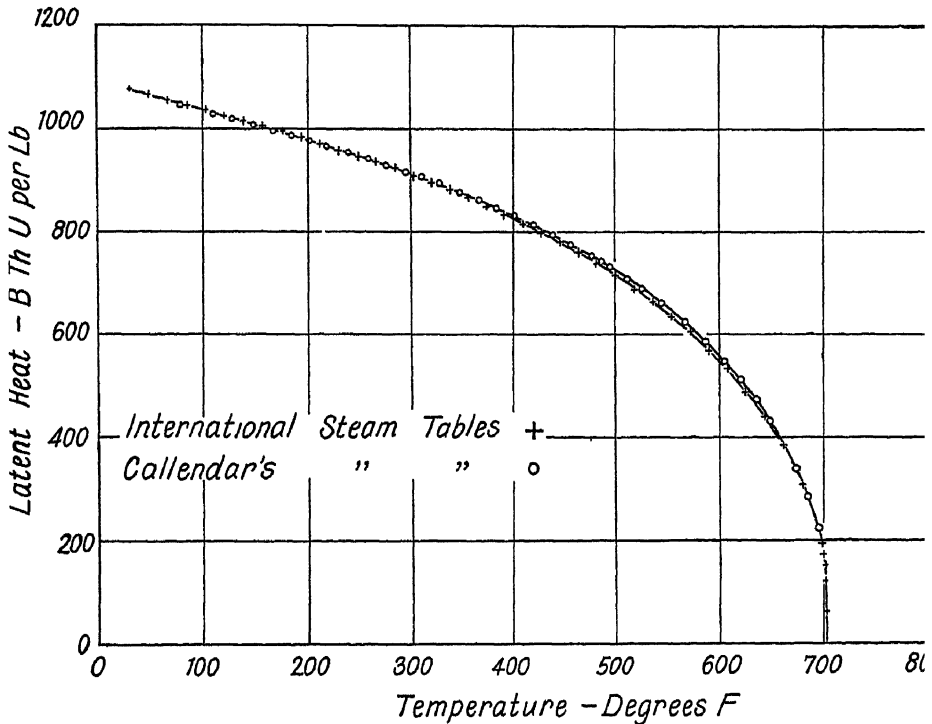


FIG. 11 —Latent Heat of Vaporization

Assume that Fig 12 (a) represents a cylinder fitted with a tight but frictionless piston, having an area of 1 sq ft. The space underneath the piston is filled with 1 lb of water at the saturation temperature corresponding to the steam pressure P lb per sq ft. Then the length of the cylinder occupied by the water will be v_w ft. When the water is heated and steam is being generated, the piston will rise; when the water has been entirely converted into dry and saturated steam, the volume occupied will be v_s . The external work done is clearly

$$\begin{aligned} & P(v_s - v_w) \text{ ft.-lb. per lb} \\ \text{or} \quad & \frac{P}{J}(v_s - v_w) \text{ B.Th.U. per lb.} \end{aligned} \quad (4)$$

(2) In overcoming the internal molecular resistance to change of state from saturated water to dry and saturated steam.

This is the change of internal energy E during evaporation, and equals $E_s - E_w$. Since, by the First Law of Thermodynamics,

Heat supplied = (Gain of internal energy)
+ (External work done)

$$L = (E_s - E_w) + \int P(v_s - v_w)$$

or
$$E_s - E_w = L - \int P(v_s - v_w) \text{ B.Th.U. per lb.} \quad (5)$$

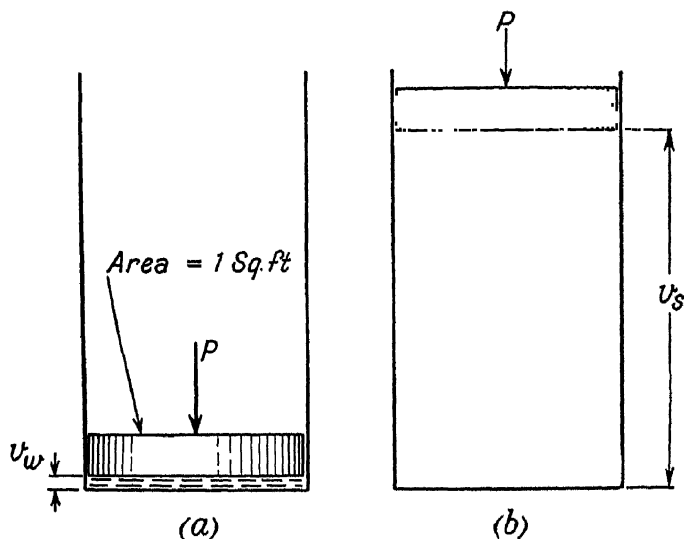


FIG. 12

EXAMPLE 1. Consider the generation of steam at 250 lb. per sq. in. abs. ($v_s = 1.852$; $v_w = 0.019$; $L = 830.7$).

$$\begin{aligned} E_s - E_w &= 830.7 - \frac{144 \times 250}{777.8}(1.852 - 0.019) \\ &= 830.7 - 84.9 \\ &= 745.8 \text{ B.Th.U. per lb.} \end{aligned}$$

This shows that the bulk of the heat supplied is utilized in changing the internal energy of the substance.

18. Heat of Generation H. The heat required to generate unit mass of steam from unit mass of water at 32° F. may be termed the heat of generation. It is frequently called the total heat of the steam, but as this term is sometimes (erroneously) used for another

quantity which differs slightly in amount from the heat of generation, confusion is apt to arise.

$$\begin{aligned} H &= \text{Sensible heat} + \text{Latent heat} \\ &= h + L \end{aligned}$$

19. Dryness Fraction. Steam which is taken from the steam drum of a boiler normally contains a small amount of water in the form of minute suspended drops. Then again, during the expansion of steam in the nozzles or blades of the turbine a certain amount of condensation takes place. This condensation is not due to the cooling of the steam by the metal parts (since under a steady load these are at a sensibly uniform temperature) but, as will be shown later, is a natural result of the expansion. The steam condenses partially and a fine mist is formed. Such steam is said to be wet, and the mixture of dry steam and water is often referred to as "stuff". The dryness fraction x of the steam is defined as the ratio—

$$x = \frac{\text{Weight of dry steam}}{\text{Weight of stuff}}$$

The wetness fraction $1 - x$ is, similarly, the ratio of the weight of moisture to the total weight of steam and moisture.

The sensible heat of water is clearly independent of x . The heat required for the partial evaporation of 1 lb. of water is x times that required for the complete evaporation, i.e. xL .

$$\therefore \text{Heat of generation} = h + xL$$

$$\text{Initial volume of 1 lb. of water} = v_w$$

$$\text{Final volume of 1 lb. of stuff} = xv_s + (1 - x)v_w$$

$$\therefore \text{Change of volume} = x(v_s - v_w)$$

$$\therefore \text{External work done during the partial evaporation}$$

$$= x \cdot \frac{P}{J}(v_s - v_w)$$

$$\therefore E_s - E_w = xL - \frac{Px}{J}(v_s - v_w)$$

$$= x \left\{ L - \frac{P}{J}(v_s - v_w) \right\} \quad \quad \quad (5a)$$

20. Superheated Steam. Imagine again a cylinder, Fig. 13, having a cross-sectional area of 1 sq. ft. and containing a tight yet frictionless piston sustaining a constant load of P lb. weight and hence subjecting the water or steam to a pressure P lb. per sq. ft. The external pressure on top of the piston is assumed to be zero, so that the pressure P is absolute.

Consider case (b) where the steam is just dry and saturated. If a certain amount of heat equal to $(1 - x)L$ be abstracted from the

steam while P is kept constant, some of the steam will condense and the volume will diminish (Fig. 13 (c)). If heat be again added, the suspended moisture will be evaporated at constant temperature, and once again the cylinder will contain 1 lb. of dry and saturated steam. If still more heat be added and the pressure be kept constant, the temperature of the steam will rise above the saturation temperature and the volume of the steam will increase to a value v_3' . Such steam is said to be "superheated." If the now superheated steam be gradually cooled, its temperature will fall steadily until the saturation temperature is again reached, when further cooling will cause condensation at constant temperature.

In practice, steam is superheated by heating it in a tubular superheater while the steam is actually flowing through the tubes

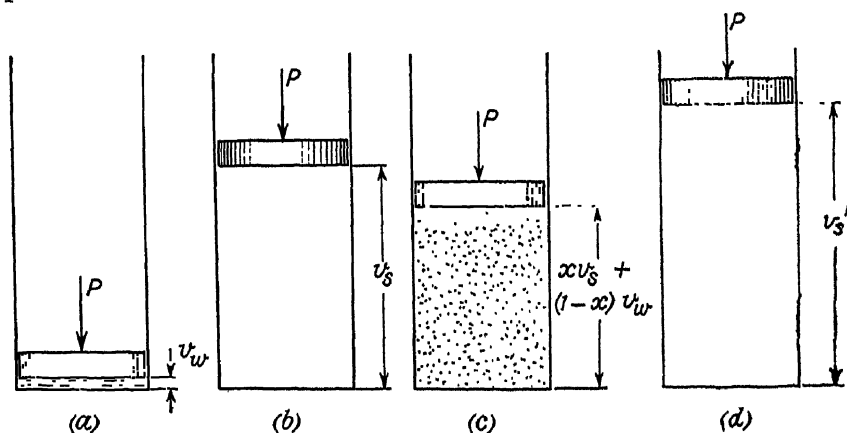


FIG. 13

and is not in close contact with the water in the boiler. In this case, the steam undergoes a slight pressure drop due to pipe friction, and the heating does not therefore take place at exactly constant pressure. But in many cases it is sufficiently accurate to assume that the steam is superheated at constant pressure.

The heat required to raise the temperature of 1 lb. of superheated steam by a small amount δt degrees when the steam is maintained at constant pressure is

$$\delta H = C_p \cdot \delta t$$

C_p being the instantaneous value of the specific heat of superheated steam at constant pressure for the given temperature and pressure.

The early measurements of C_p for steam at atmospheric pressure were made by Regnault (5) in 1862; he found that C_p was approximately equal to 0.48. It was many years after (1906) before the results of experiments at pressures different from atmospheric were published by Knoblauch and Jakob (6). Since then many

other researches have been carried out. It is beyond the scope of this book to do other than refer to the variation of C_p with pressure and temperature. Recent measurements made at Munich by Knoblauch and Koch (7) are shown in Fig. 14. These values are for pressures ranging from 30 to 120 atmospheres (450 to 1,800 lb per sq in. abs.) It will be observed that the value of C_p is very high at the saturation point and falls rapidly as the temperature is increased. Similar curves based on differences between the

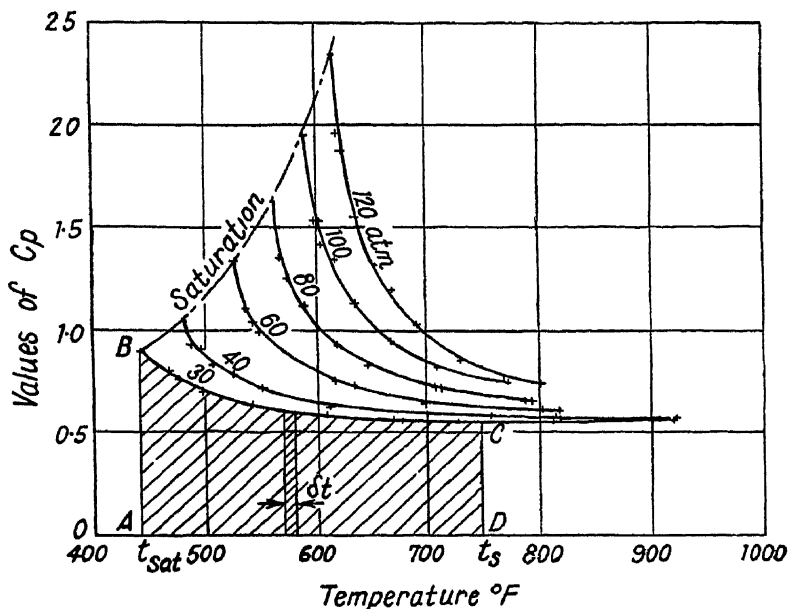


FIG. 14 — Specific Heat of Superheated Steam
(Knoblauch and Koch)

values of the enthalpy of superheated steam published in Callendar's Revised (1931) Steam Tables are shown in Fig. 15. These values are appreciably lower than the values published by Knoblauch and Koch, and other authorities, but as the enthalpy values for dry and saturated steam published by Callendar are somewhat higher than those published by other workers, the agreement between the values of the enthalpy of superheated steam at the higher pressures is relatively good.

As C_p is variable, the total amount of heat required to superheat steam from the saturation temperature $t_{sat.}$ to some temperature t_s equals

$$H = \sum_{t_{sat.}}^{t_s} C_p \delta t$$

and is represented by the area ABCD, Fig. 14

The mean specific heat for the range of temperature $t_{sat.}$ to t_s is accordingly the heat supplied divided by the temperature rise, i.e.—

$$C_{pm} = \frac{\sum_{t_{sat.}}^{t_s} C_p \cdot \delta t}{t_s - t_{sat.}}$$

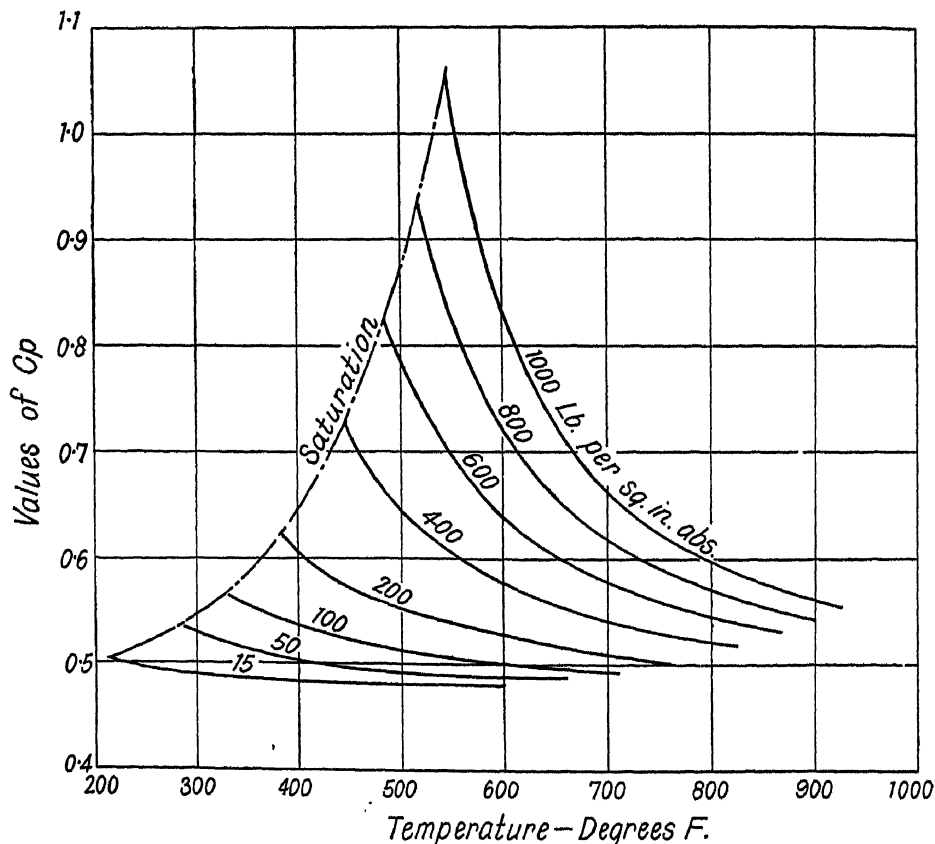


FIG. 15.—Specific Heat of Superheated Steam
(Callendar, 1931)

Values of C_{pm} calculated from Callendar's Tables are given in Fig. 16.

The external work done during superheating at constant pressure P lb. per sq. ft. is

$$\frac{P}{J}(v_s^1 - v_s) \text{ B.Th.U. per lb.}$$

Therefore, the change of internal energy during superheating

$$= C_{pm}(t_s - t_{sat.}) - \frac{P}{J}(v_s^1 - v_s)$$

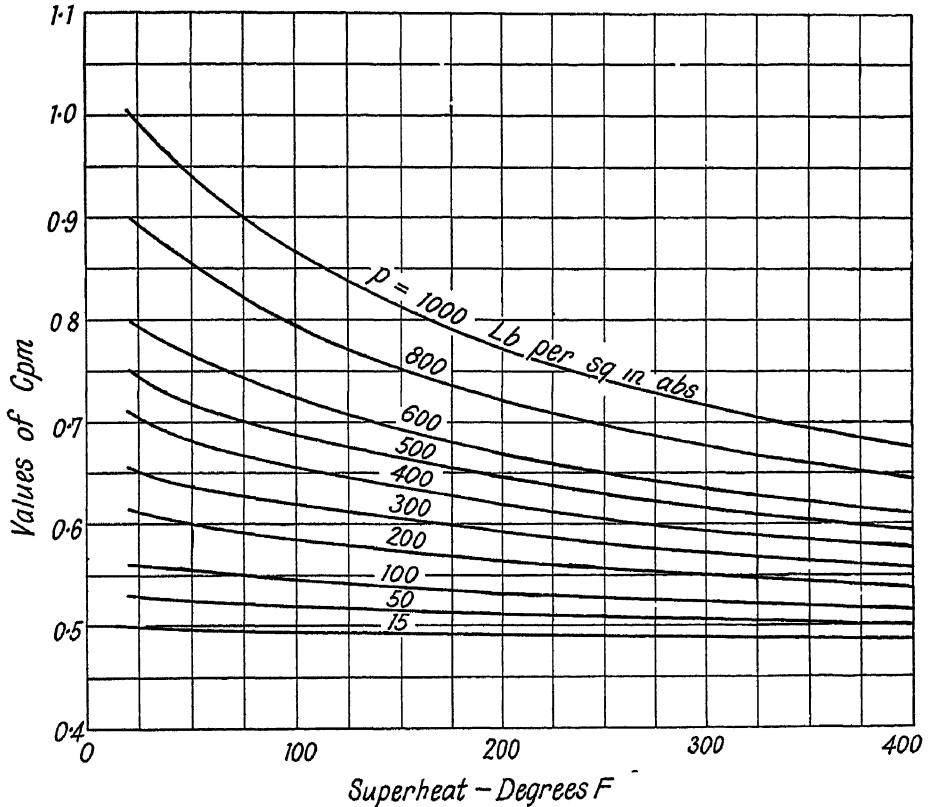


FIG 16 — Mean Specific Heat of Superheated Steam

21. The Enthalpy Function. The enthalpy of a substance is defined as the sum of the internal energy and the work done in forcing unit mass of the substance into an enclosure at the constant pressure under which the substance exists.

Thus, suppose the specific volume of steam in any given condition is v_s and the pressure P . Then the enthalpy I per unit mass is

$$I = E + \frac{P \cdot v_s}{J} \quad . \quad . \quad . \quad (6)$$

As will be seen later, this is a most important quantity in steam turbine theory. The name given to this function by Mollier is "total heat," and the use of this somewhat unfortunate term has become so general in engineering work that it is doubtful whether

any other term will ever be universally adopted. The term is apt to be confused with Regnault's term for the "total heat" of generation, and whilst the numerical difference between the two quantities is small at moderate pressures, there is a rather important distinction between them. The term "holotherm" has also been suggested for this function. The term "enthalpy," which was used in the first edition of this book, has been used in the recently published International Steam Tables (1). Callendar, in his work, used

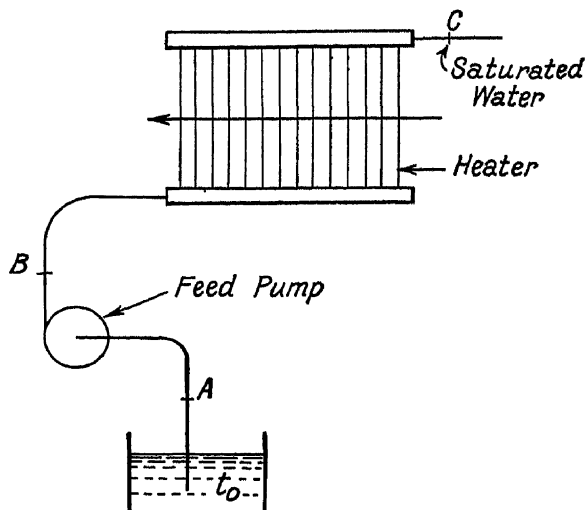


FIG. 17

the term "total heat," and represents this by the symbol H for steam and h for water.

22. The Enthalpy of Water. For the purpose of tabulating values of the enthalpy of water or steam, the enthalpy of water at 32°F . is taken as the arbitrary zero.

Let v_o = volume per lb. of water at 0°C . or 32°F . ($v_o = 0.01602$ cub. ft.).

v_w = volume per lb. of water at any pressure P lb. per sq. ft. abs.

Then

$$I_w = E_w + \frac{P \cdot v_w}{J} \quad (7)$$

At the freezing point

$$I_o = E_o + \frac{P_o v_o}{J}$$

and since I_o is arbitrarily taken to be zero, the internal energy E_o is negative and numerically equal to $\frac{P_o v_o}{J}$.

Since $P_o = 144 \times 0.0886$, $E_o = -0.00026$ B.Th U per lb. Actually, the internal energy of water at 32°F is, of course, a large positive quantity, the apparently negative value follows from the employment of an arbitrary zero for I_w .

Further reference will now be made to the sensible heat of water

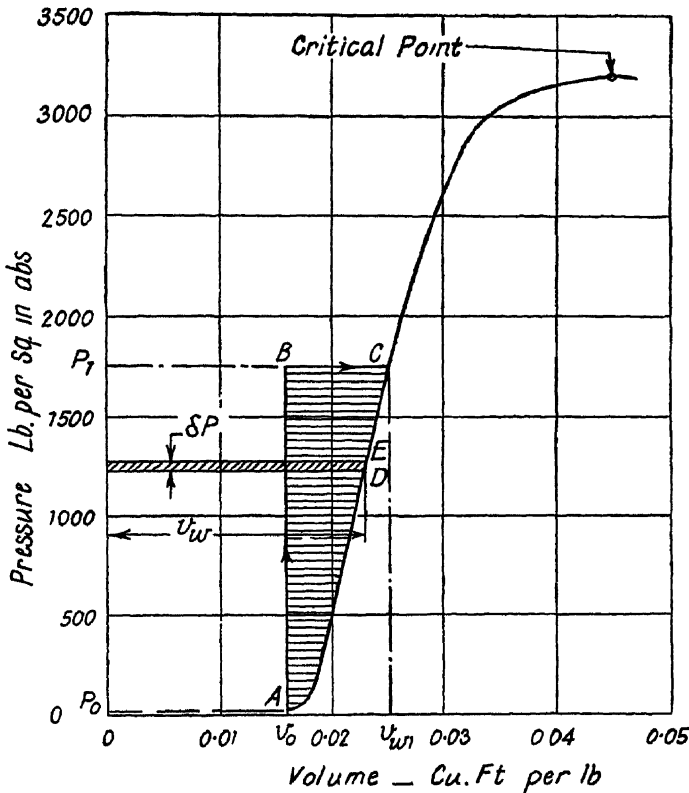


FIG. 18 — Specific Volume of Water

Consider the heating of unit mass of water from 32°F to any temperature $t_1^\circ \text{F}$, for which the vapour pressure is P_1 . This heating may be carried out in a number of ways; only two methods will be considered.

(1) The water is first compressed (as in a feed pump) adiabatically from P_o to P_1 and is then heated at constant pressure from t_b to t_o (Figs. 17 and 18).

$$\text{In any change of state, } dH = dE + \frac{P \cdot dv}{J}.$$

Also, since $I = E + \frac{Pv}{J}$

$$dI = dE + \frac{P \cdot dv}{J} + \frac{v \cdot dP}{J}$$

$$dI = dH + \frac{v \cdot dP}{J} \quad \dots \quad (8)$$

From A to B, $dH = 0$, and dP is positive.

$$\therefore I_B - I_A = \frac{v_o}{J}(P_1 - P_o)$$

assuming that the specific volume v_o of the water remains constant during the pressure rise.

Then $I_B = I_A + \frac{v_o}{J}(P_1 - P_o)$

From B to C, since $dP = 0$, $dH = dI$. Therefore the heat supplied is $I_C - I_B$, and since no heat is supplied between A and B, the whole of the sensible heat h supplied along the path ABC is

$$h_{ABC} = (I_C - I_A) - \frac{v_o}{J}(P_1 - P_o)$$

This type of heating process is the one normally used in practice. The pressure of the feed water is raised by the boiler feed pump to the boiler pressure P_1 . The water is then forced into the boiler (or economizer, etc.) and heat is supplied to raise the temperature to the saturation point. It should be noted that the heat supplied is less than the difference between the enthalpy values given in steam tables by the amount

$$\frac{v_o}{J}(P_1 - P_o)$$

For any exhaust pressure P_2 the values of P_2 and the corresponding specific volume v_{w2} may be substituted for P_o and v_o respectively.

(2) The water is kept in the saturated condition during the heating process. In other words, the pressure applied to the water at any instant is just equal to the vapour pressure corresponding to the water temperature. The path of the condition point is then along the saturated liquid line ADC. This would be a difficult process to carry out.

Since for any small change of state

$$dH = dI - \frac{v \cdot dP}{J}$$

$$h_{ADC} = (I_C - I_A) - \frac{1}{J} \int_{P_1}^{P_2} v_w \cdot dP$$

Then $h_{ABC} - h_{ADC} = \frac{1}{J} \int_{P_2}^{P_1} v_w \cdot dP - \frac{v_o}{J}(P_1 - P_o)$

i.e. the difference between the sensible heat along the path ABC and that along the path ADC is represented by the horizontally hatched area ABCDA. Values of $I_C - I_A$, h_{ABC} and h_{ADC} are given in the following table.

p lb per sq in	$I_C - I_A$ ($I_A = 0$)	$\frac{v_o}{J}(P_1 - P_o)$	h_{ABC}	$\frac{1}{J} \int_{P_o}^{P_1} v \, dP$	h_{ADC}	$h_{ABC} - h_{ADC}$
14.7	180.0	0.0434	179.9566	0.0446	179.9554	0.0012
200	355.5	0.594	354.906	0.654	354.846	0.060
400	424.6	1.188	423.412	1.357	423.243	0.169
600	472.5	1.782	470.718	2.096	470.404	0.314
800	511.0	2.376	508.624	2.862	508.138	0.486
1,000	544.1	2.97	541.13	3.66	540.44	0.69
1,500	614.4	4.35	610.05	5.68	608.72	1.33
2,000	676.4	5.94	670.46	8.12	668.28	2.18
2,500	739.0	7.43	731.57	10.71	728.29	3.28
3,000	816.2	8.9	807.3	13.6	802.6	4.7
3,200	862.3	9.5	852.8	14.9	847.4	5.4

23. The Enthalpy of Steam. If the steam is dry and saturated (specific volume v_s) then the enthalpy (see eq. 6)

$$I_s = E_s + \frac{P}{J} v_s$$

It has already been shown in Art. 17 that

$$\begin{aligned}
 E_s &= E_w + L - \frac{P}{J} v_s + \frac{P \cdot v_w}{J} \\
 \therefore I_s &= (E_w + L) + \frac{P}{J} v_w \\
 &= I_w + L \quad \dots \dots \dots (9)
 \end{aligned}$$

Consider the enthalpy function

$$I = E + \frac{Pv}{J}$$

Then
$$\frac{dI}{dv} = \frac{dE}{dv} + \frac{P}{J} + \frac{v}{J} \cdot \frac{dP}{dv}$$

That is, for a small change of state

$$dI = dE + \frac{1}{J}(Pdv + v \cdot dP)$$

and when P is constant

$$dI = dE + \frac{P}{J} \cdot dv \quad \dots \dots \dots (10)$$

But the heat supplied during a small change of state is

$$dH = dE + \frac{P}{J}dv$$

Therefore, when P is constant

$$dI = dH$$

and the gain of enthalpy is equal to the heat supplied. It follows, without further proof, that for wet steam

$$I_s = I_w + xL \quad . \quad . \quad . \quad . \quad (9a)$$

Similarly, if superheating takes place at constant pressure the enthalpy of the superheated steam is

$$I_s^1 = I_w + L + C_{pm}(t_s - t_{sat.}) \quad . \quad . \quad . \quad (11)$$

Thus, the enthalpy of steam in any condition is equal to that of water together with the heat supplied to convert 1 lb. of saturated water into steam with the given condition.

24. Specific Volume of Steam. If steam were a perfect gas, then the specific volume v_s , under all conditions of pressure and temperature, would be given by the well-known characteristic equation

$$v_s = \frac{RT}{P}$$

it being assumed that the steam is dry, i.e. either dry and saturated or superheated. The value of the "constant" R is given by the relation

$$R = \frac{J \cdot R_u}{m}$$

where R_u is the universal gas constant (1.985 heat units per degree) and m is the molecular weight of steam (18). Hence R for steam is 85.76 when T is expressed in degrees F. absolute.

The values of a quantity R^1 which is

$$R^1 = \frac{P \cdot v_s}{T}$$

have been calculated from Callendar's 1931 Tables for a few steam pressures and are shown in Fig. 19. This diagram shows clearly that steam behaves nearly as a perfect gas at low pressures, a fact which is borne out by the relative constancy of the specific heat at low pressures. But the curves also show clearly to what a remarkable extent the properties of steam deviate from those of a perfect gas, especially at and near the saturation point. It is apparent from Fig. 19 that the specific volume v_s is always less than the quantity $\frac{RT}{P}$ and by an amount which increases as P is increased,

but which, at any given pressure, diminishes as T is increased. Therefore, the "equation of state" connecting P , v_s and T is likely to be of the form

$$v_s = \frac{RT}{P} - f(P, T)$$

where $f(P, T)$ represents some function of P and T . Several such equations have been proposed from time to time. As far back as 1867, Zeuner proposed the equation

$$P \cdot v_s = 50.93T - 192.5 \sqrt[4]{P}$$

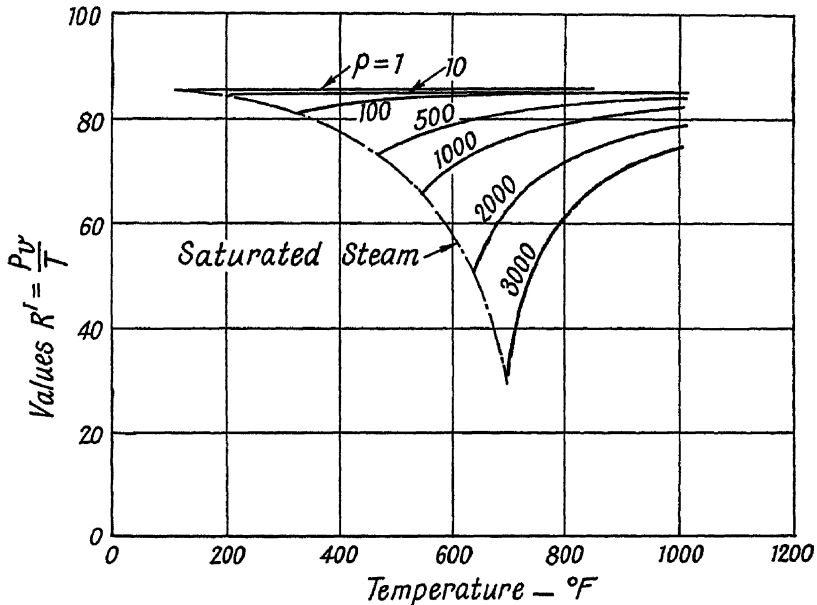


FIG. 19 —Values of $\frac{Pv}{T}$ for Dry Steam

where P is the pressure in kg per sq. m.
 T is the temperature in degrees C. abs.
 v_s is the volume in cub. m. per kg.

In British units (f.p. ° F.) this becomes

$$v_s = \frac{92.8T}{P} - \frac{939}{\sqrt[4]{P^3}} \quad (12)$$

This equation gives results which agree fairly well with modern data at moderate pressures. It is extremely simple, but its very simplicity involves the use of an R value which is 8 per cent in excess of the theoretical value.

Professor Callendar (8) proposed the formula

$$v_s = \frac{RT}{P} - c + b \quad (13)$$

in which, with v_s in cub. ft. per lb., P in lb. per sq. ft., and T in degrees F. absolute, b is the volume of 1 lb. of water at 0° C., i.e. $b = 0.01602$ cub. ft. The term c was called by Callendar the "co-aggregation volume" and represents the reduction in volume due to molecular association. The value of c is given as

$$c = 0.4213 \left(\frac{671.6}{T} \right)^{\frac{10}{3}} \quad (14)$$

$$\text{Hence } v_s = \frac{85.76T}{P} - 0.4213 \left(\frac{671.6}{T} \right)^{\frac{10}{3}} + 0.01602 \quad (15)$$

The specific volume of dry steam (superheated or supersaturated) is also given by the expression

$$v_s = 1.2464 \frac{(I_s - 835.2)}{p} + 0.0123$$

where I_s is the enthalpy of the dry steam, values being taken from the 1921 or 1930 Steam Tables or Diagrams.

Many years later, Professor Callendar carried out researches (9, 10, 11) on steam at pressures up to 4,000 lb. per sq. in. and at temperatures up to 950° F. He found that by slightly modifying the original equations they could be made to give the properties of steam for the full range of pressure.

The new form of the Callendar equation, used in the 1931 Callendar Steam Tables and the 1934 Abridged Callendar Steam Tables, is

$$v_s = \frac{85.76T}{P} - \frac{c}{1 - Z^2} + b \quad (16)$$

where c has the value given in (14) above.

$$Z = 4.0115 \cdot \frac{c}{T} \text{ and } b = -0.00280$$

Using values of I_s from the 1931 and 1934 Tables or from diagrams based thereon—

$$v_s = 1.2464 \frac{(I_s - 835.2)}{p} - 0.00212$$

In 1925, the late Dr. Mollier published his extended steam tables in which the equation of state is given as

$$v_s = \frac{85.84T}{P} - \frac{227.3}{\left(\frac{T}{100} \right)^{\frac{10}{3}}} - \frac{2.72 \times 10^6}{\left(\frac{T}{100} \right)^{\frac{14}{3}}} \left(\frac{P}{100} \right)^2$$

where T is in degrees F. absolute, and P is in lb. per sq. ft.

In most cases in practice, the specific volume will be found by direct reference to steam tables, or by interpolation from steam table data. In some cases, the required volumes may be read off the enthalpy-entropy diagram. But there are occasions when it is necessary to calculate v_g with some precision; the foregoing equations are then useful

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EXAMPLES II

Note. In these and all other examples in this book, except where otherwise stated—

- (1) Pressures are in lb. per sq. in. absolute.
- (2) Specific volumes are in cub. ft. per lb.
- (3) Data are taken from the 1934 edition of the *Abridged Callendar Steam Tables*.

1. Calculate the sensible heat of water at 400° F —

- (a) assuming the specific heat of water to be unity,

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(b) assuming the value of C_w given by Dieterici's formula

$$C_w = 1.00076 - 0.00009855t + 0.00000064t^2$$

(c) assuming the value of C_w given by Callendar's formula

$$C_w = 0.98415 + \frac{0.9072}{t + 4} + 0.00002889t + 0.000000278t^2$$

2. Water is heated from 32° F. ($v_w = 0.016$) to 327.9° F. ($p = 100$, $v_w = 0.0177$) at the constant pressure of 100 lb. per sq. in.

If the heat supplied during the process is 298.500 B.Th.U. per lb., calculate the change of internal energy.

3. Steam is generated at 100 lb. per sq. in. ($t_{sat.} = 327.9^\circ$ F.) from water at 327.9° F. The specific volume of water at this temperature is 0.0177.

Calculate the following—

- (a) the external work done;
- (b) the change of internal energy.

4. 20 lb. of wet steam at $p = 250$ occupy 32 cub. ft. If the specific volume of water at 401° F. and 250 lb. per sq. in. is 0.0187, what is the dryness fraction of the steam? ($v_g = 1.852$.)

5. The enthalpy of 1 lb. of dry and saturated steam at 500 lb. per sq. in. is 1213.2 B.Th.U. per lb. What is the internal energy per lb. of steam and also per lb. of water? The specific volume of saturated water at $p = 500$ is 0.0199.

6. The exhaust steam from a turbine enters a surface condenser with a dryness fraction of 86 per cent. The absolute pressure in the condenser is 1 lb. per sq. in. If the final temperature of the condensate is 90° F., how much heat is rejected per lb. of steam? If the inlet temperature of the circulating water is 45° F. and the outlet temperature is 75° F., how many lb. of circulating water must be circulated per lb. of steam condensed? Assume that the specific heat of the circulating water is unity.

7. Steam is condensed in a tank containing 300 lb. of water initially at 45° F. The increase in the weight of water in a certain time is 10.8 lb. and the final temperature of the water is 80° F. Calculate the dryness fraction of the entering steam if its pressure is 200 lb. per sq. in. and if the heat loss from the tank during the period of condensation amounts to 540 B.Th.U.

CHAPTER III

ENTROPY DIAGRAMS

25. The Entropy Function. Suppose that a small quantity of heat δH is added to unit mass of a substance during a small change of state. Suppose, moreover, that the said change of state is internally thermodynamically reversible. The absolute temperature T may remain constant, as, for example, during the evaporation of water at constant pressure, or the addition of the heat δH to the substance might produce a small increase in temperature δT , as, for example, during the heating of water below the saturation temperature. In the latter case, let δH be so small that δT is negligibly small compared with T . Then we may assume that T is sensibly constant. The function entropy is then defined by saying that the increase of entropy $\delta\phi$ is equal to

$$\delta\phi = \frac{\delta H}{T} \quad (17)$$

If δH is positive, i.e. if heat is added, then the entropy increases, if heat is abstracted then δH and $\delta\phi$ are both negative.

The above expression shows that entropy, absolute temperature, and heat added are related in a very simple manner. Consider a diagram, Fig. 20, in which values of entropy are plotted horizontally and of the absolute temperature are plotted vertically. Let A and B be points representing two states of a substance and let the temperature difference between A and B be small, so that T is sensibly constant. Then since

$$\delta\phi = \frac{\delta H}{T}$$

$$\delta H = T \cdot \delta\phi$$

and the area of the diagram which is cross-hatched therefore represents the heat supplied between the points A and B. It should be noted that the area under the curve AB must extend right down to the absolute zero of temperature.

Such a diagram is termed a temperature-entropy diagram, or T - ϕ chart. The T - ϕ chart for steam is a most useful instrument

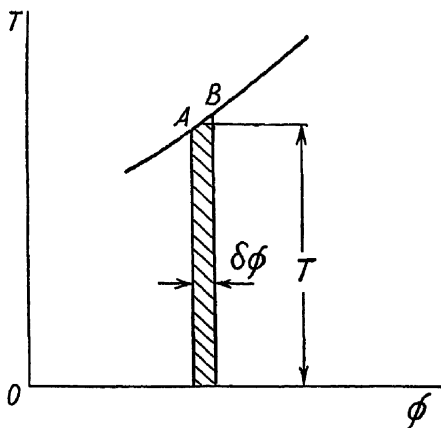


FIG 20

for the study of problems associated with the generation and expansion of steam and will now be discussed.

26. The Temperature-entropy Diagram for Steam. Consider first the changes of entropy which take place during the heating of water. For a small rise in temperature δt or δT , the heat supplied to the water is $C_w \cdot \delta T$, where C_w is the value of the specific heat of water at the existing temperature T .

Therefore, since
$$\delta\phi = \frac{\delta H}{T}$$

$$\delta\phi_w = C_w \cdot \frac{\delta T}{T}$$

and the total change of entropy in heating water from T_1 to T_2 would be

$$\phi_{w2} - \phi_{w1} = \sum_{T_1}^{T_2} \frac{C_w \cdot \delta T}{T} \quad . \quad . \quad . \quad (18)$$

It will be readily understood that the change of entropy calculated in this manner will be somewhat approximate, depending on the temperature increment δT which is chosen in making the calculation. The smaller the increments δT , the more accurate the result would be.

If the specific heat of water be assumed to be unity, as it may be for low pressures without incurring any appreciable error, then the change of entropy of water may be calculated as follows—

$$\delta\phi_w = \frac{\delta T}{T}$$

$$\therefore \phi_{w2} - \phi_{w1} = \int_{T_1}^{T_2} \frac{1}{T} \cdot dT = \log_e \frac{T_2}{T_1} \quad . \quad . \quad (19)$$

For the purpose of tabulating values of the entropy of water and steam in steam tables, it is customary to assume that the entropy of saturated water is zero at 32°F. , i.e. when $T = 492^\circ \text{F. abs.}$ Accordingly, at $T^\circ \text{F. abs.}$ —

$$\phi_w = \log_e \frac{T}{492} \text{ (approx.)} \quad . \quad . \quad . \quad (20)$$

If it is assumed that the specific heat of water may be represented by an equation of the form

$$C_w = a + bT + c \cdot T^2$$

where a , b , and c are constants, then

$$\delta\phi_w = \left(\frac{a}{T} + b + cT \right) \delta T$$

and is therefore the heat supplied during evaporation. The locus of such points as D is termed the "saturation curve" or "dry steam line."

If the evaporation is partial and x is the dryness fraction, then the increase of entropy is

$$\frac{xL}{T} = x \cdot \phi_e$$

Thus the point E represents the state of steam at T and having a dryness fraction of 0.7, the point E dividing BD such that $\frac{BE}{BD} = 0.7$. The locus of points such as E is a curve passing through the critical point C and representing a constant dryness fraction of 70 per cent.

The changes of entropy during superheating at constant pressure may be calculated in the following manner. Suppose that unit mass of steam is heated from temperature T to T + δT , and let C_p be the specific heat for the given pressure P and for the mean temperature T + $\frac{1}{2}\delta T$. Then the heat supplied is $C_p \cdot \delta T$ and the small increase in entropy

$$\delta\phi = C_p \cdot \frac{\delta T}{T}$$

The total change of entropy from the saturation temperature T_{sat} , degrees absolute to any superheat temperature T_s , could be found by the arithmetical integration.

$$\phi_s - \phi_{sat} = \sum_{T_{sat}}^{T_s} C_p \cdot \frac{\delta T}{T} \quad (22)$$

By plotting the total entropy of superheated steam against the superheat temperature, the constant-pressure line DF is obtained. The vertical height from D to F is clearly the degree of superheat at F, and the shaded area NDFR represents the heat supplied during superheating.

The entropy of superheated steam should always be obtained from reliable steam tables or steam diagrams. The change of entropy during superheating may be calculated *approximately* by assuming a mean specific heat C_{pm} for the given range of temperature. Then the change of entropy is

$$\begin{aligned} \phi_s - \phi_{sat} &= C_{pm} \int_{T_{sat}}^{T_s} \frac{1}{T} \cdot dT \\ &= C_{pm} \cdot \log_e \frac{T_s}{T_{sat}} \quad (23) \end{aligned}$$

It is assumed here that the *entropy-mean specific heat* is equal to the *temperature-mean specific heat*, which is not quite correct. The error made in such an approximation will be greater at higher pressures than at lower pressures, because of the greater variation in the specific heat at higher pressures. The error involved is not so serious as might at first be thought, as the following data show.

STEAM AT A PRESSURE OF 1,000 LB PER SQ IN

Degree of superheat (° F) .	40	100	200	400
Temp-mean specific heat, C_{pm} (Fig 16)	0.96	0.867	0.773	0.678
$C_{pm} \log_e \frac{T_s}{T_{sat}}$	0.0375	0.0823	0.1404	0.2273
Entropy change derived from <i>Callendar's Abridged</i> (1934) <i>Steam Tables</i>	0.0376	0.0825	0.1415	0.2311
Error	0.0001	0.0002	0.0011	0.0038
Percentage error	0.27	0.24	0.78	1.64

If in Fig. 21 the lines GH and JK are lines of constant pressure for pressures lower than that represented by DF, and if the points F, H, and K all represent a constant degree of superheat, e.g. 150° F., then the curve FHK is known as a constant superheat line.

Curves of constant volume may be drawn on the T - ϕ diagram in the following way. Suppose, for example, it is desired to draw a constant-volume line to represent 2 cub. ft. per lb. A reference to the Callendar Tables shows that for dry and saturated steam

$$v_s = 2.006 \text{ when } p = 230 \text{ and } T = 853.3$$

$$v_s = 1.965 \text{ when } p = 235 \text{ and } T = 855.2$$

A simple linear interpolation gives

$$v_s = 2.00 \text{ when } p = 230.73 \text{ and } T = 853.58$$

Then the point S on the saturation curve at $T = 853.58$ represents the condition of steam having a specific volume of 2.00 cub. ft. Consider a lower pressure, e.g. $p = 100$. Since the specific volume of dry and saturated steam is 4.429 cub. ft. per lb., it is clear that the steam must be wet to have a volume of only 2 cub. ft.

$$\text{Then } x \cdot v_s + (1 - x)v_w = 2.00$$

and since $v_w = 0.0177$

$$4.429x + 0.0177 - 0.0177x = 2.00$$

from which $x = 0.45$. This condition is represented by the state point T ($p = 100$, $T = 787.5$). Other points may be obtained in the same manner and a curve ST drawn.

If a pressure somewhat higher than 230.73 be considered, then, since the specific volume of dry and saturated steam is less than 2 cub. ft., it is clear that the steam must be more or less highly superheated. The precise temperature for any pressure may be

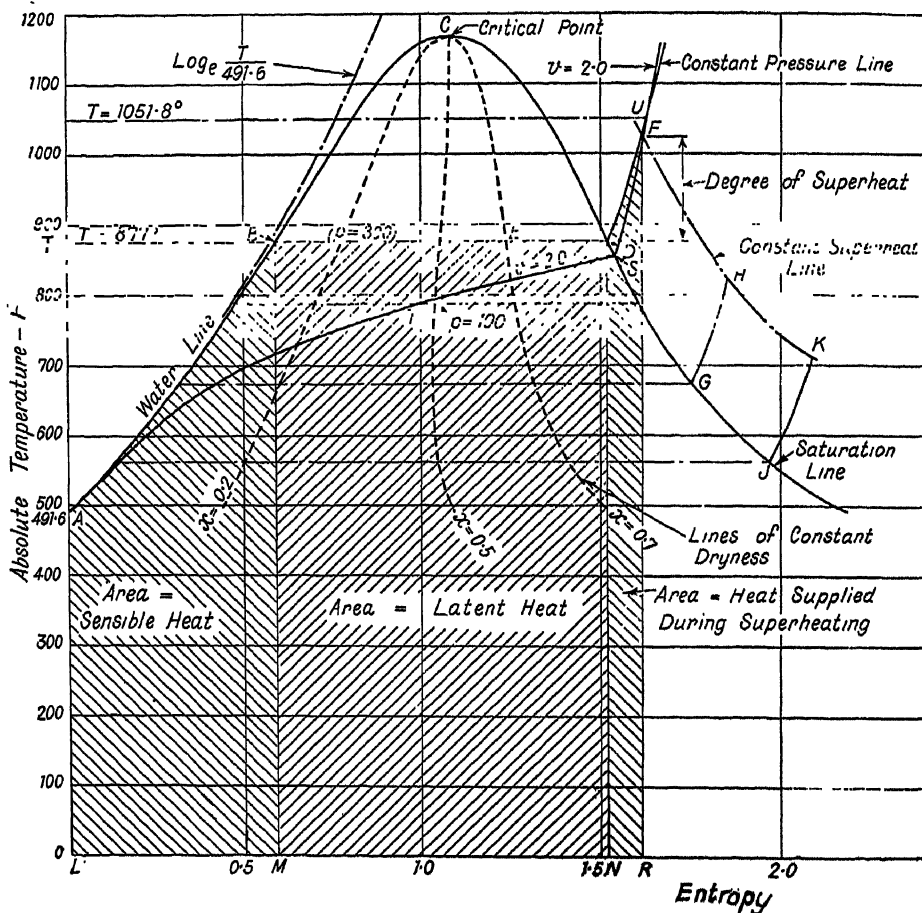


FIG. 21.—T- ϕ Diagram for Water and Steam

found by using the Callendar equation of state (15), or by interpolation from the Revised Callendar Tables. Thus, at the pressure of 300, the temperature corresponding to the volume of 2.00 cub. ft. per lb. is 592.2° F. or 1051.8° abs. This state is represented by the point U and the complete "constant-volume" curve for $v = 2.00$ is the discontinuous curve TSU.

27. The Enthalpy-entropy Diagram. In this diagram, proposed by the late Dr. Richard Mollier, of Dresden, and usually known as the Mollier diagram, the co-ordinates are entropy ϕ and enthalpy I .

A skeleton diagram for steam is shown in Fig. 22. The origin A represents zero entropy and zero enthalpy, corresponding to the state of water at 32° F. and under a pressure of 0.0883 lb. per sq. in. abs. Point B represents saturated water at 100 lb. per sq. in. abs. ($\phi_w = 0.4749$, $I_w = 298.5$). The locus of points such as B plotted up to the critical point C is the water line ABC. Point D represents dry and saturated steam at 100 lb. per sq. in. ($\phi_s = 1.6079$, $I_s = 1190.7$), and the locus of points such as D is the saturation line CDGJ. Since the value of I_s at the critical point is appreciably less than that at lower pressures (I_s is a maximum at about 520 lb. per sq. in., according to Callendar's Tables), the critical point is well removed from the peak of the curve, as is evident from Fig. 22.

Consider now the movement of the state point during evaporation at a pressure of 100 lb. per sq. in. abs. The initial state point is B. For any dryness fraction x , the increase in I is

$$I_s - I_w = xL$$

and the increase of entropy $\frac{xL}{T}$.

Therefore, since T is constant the increase in enthalpy is proportional to the increase in entropy, i.e. the evaporation line BD is straight. Moreover, the slope of the line is

$$\frac{I_s - I_w}{\phi_s - \phi_w} = xL \times \frac{T}{xL} = T$$

This relationship between the slope of the constant-pressure lines and temperature may be shown more generally. In Art. 23 it is shown that when P is constant

$$(dI)_p = dH = T \cdot d\phi$$

$$\therefore \left(\frac{dI}{d\phi} \right)_p = T \quad \dots \quad (24)$$

Thus all constant-pressure lines are straight in the wet region (P and T constant) and the slope of a high-pressure line is greater than that of a low-pressure line.

If lines such as BD are divided in the ratio of, say, 7 to 10, then a curve through the dividing points becomes a curve of constant dryness or quality. All such curves run to the critical point.

The constant-pressure lines may be produced into the superheat field. Since $\frac{dI}{d\phi} = T$, and T is increasing, the constant-pressure

lines are now curved. If points F, H, and K be taken on different constant-pressure lines for the same degree of superheat (400° F.), then the curve drawn through such points is a curve of constant superheat.

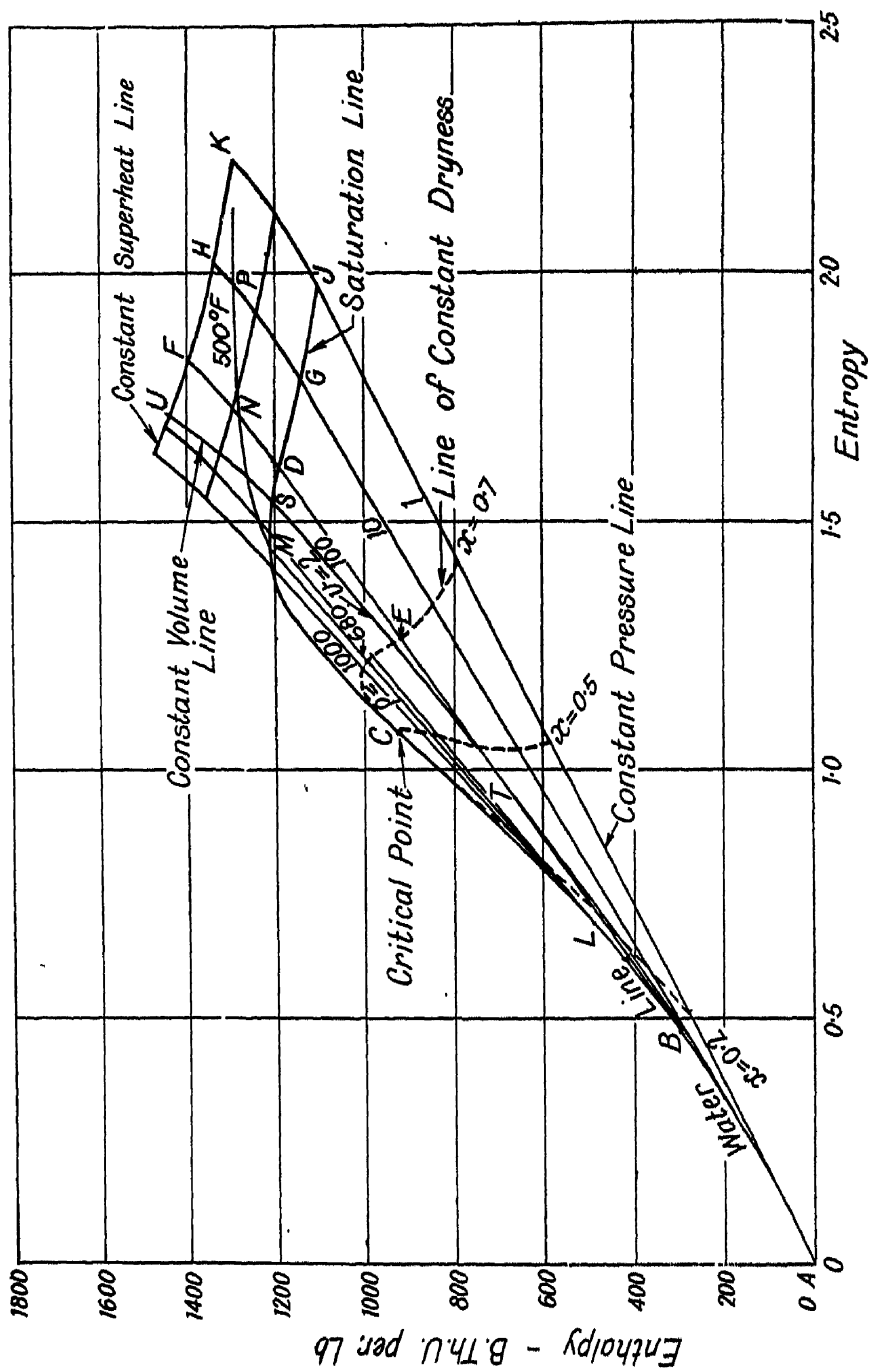


FIG. 22.—Mollier Diagram for Steam

Isothermals may be drawn in the following manner. Suppose, for example, it is desired to draw an isothermal for 500° F. The vapour pressure at 500° F is 680 lb per sq. in. abs. Therefore all points on the line LM (680 lb per sq. in.) in the wet steam region are on the required isothermal. Consider now the pressures 100 and 10 lb per sq. in.

Steam Pressure	Saturation Temperature	Superheat when — 500° F
100	327.9° F	172.1° F
10	193.2° F	306.8° F

Then if points N and P be taken on the lines for $p = 100$ and $p = 10$ respectively, to represent superheats of 172.1° F. and 306.8° F. respectively, the curve MNP will be part of the required isothermal. The discontinuous line LMNP represents the complete isothermal.

Constant-volume lines, such as TSU (drawn for 2.0 cub. ft. per lb.), may be set out in the manner described in Art. 26 for the constant-volume lines on the T - ϕ chart.

The ϕ - I diagram is, without a doubt, the most useful chart ever devised for the solution of problems connected with the theory and the design of steam turbines. For practical purposes, it is usual to use the upper part of the diagram only.

28. T - ϕ and ϕ - I Diagrams for Compressed Water. In steam power plants using high-pressure steam the feed water is compressed by the boiler feed-pump to a high pressure. This compression modifies, to a slight extent, the thermodynamic properties of the water. It has been already shown in Art. 22 that the compression of the feed water to a high pressure modifies the amount of heat which must be supplied.

It is beyond the scope of this book to explain how the changes in entropy and enthalpy during isothermal compression may be calculated, but the nature of the changes may be followed from Figs. 23 and 24, based on an article (1) by the author. Fig. 23 shows the T - ϕ diagram. The dotted line represents the boundary curve ABCD of Fig. 21. It should be noted that the curve ABC of Fig. 21 represents the relationship between the absolute temperature and the entropy of *saturated* water, i.e. water at the saturation temperature corresponding to the existing pressure. In Fig. 23, the line AB represents water at a pressure of 1,000 lb. per sq. in. abs. The point B coincides with the saturation line at an absolute temperature of 1004.3 degrees and again near the freezing point. The deviation from the saturation line is very small and has been purposely exaggerated in the figure.

Suppose now that water enters a feed pump at a temperature of 100° F. and at the corresponding pressure $p = 0.95$. Its condition is represented by the point K. The adiabatic compression of the water to $p = 1,000$ would be represented by the vertical line

KL, and during the subsequent heating to 544.7° F., the condition point would move along the curve LB. The figure given in the last column of the table in Art. 22 is represented by the small area between KLB and the curve KB. In Fig. 23, the full line AEF represents water and steam at the critical pressure, and AGH water and steam at 4,000 lb. per sq. in. abs. The parts of the curves

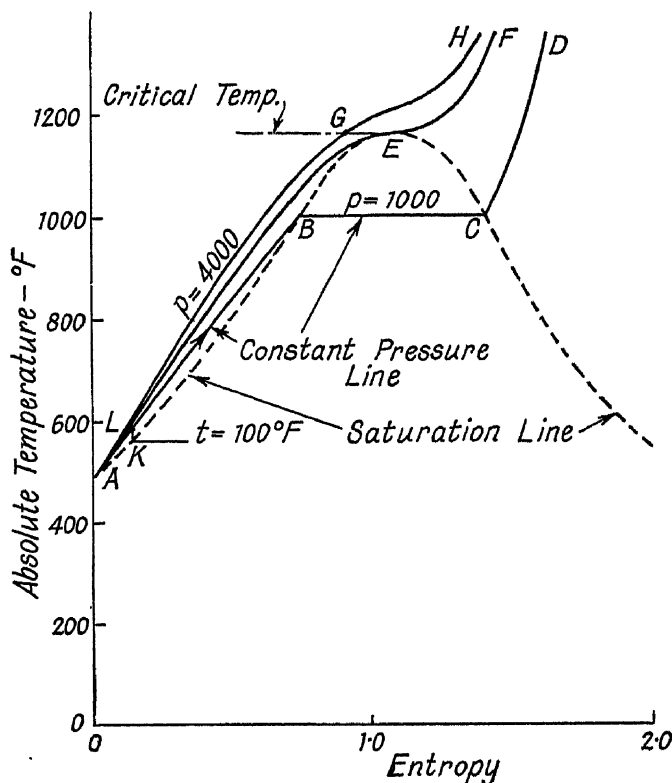


FIG. 23.—Sketch of T- ϕ Diagram for Compressed Water and Steam

lying below the critical temperature represent water in the liquid state, those parts above represent steam.

The ϕ -I diagram for compressed water is shown in Fig. 24. The dotted line OBE shows the relationship between I and ϕ for saturated water. Curve AB shows the same relationship for water at a pressure of 1,000, and it coincides with the saturated water line at the point $\phi = 0.7455$ and $I = 544.1$, but at no other point. Curve ABCD shows the complete constant-pressure line for $\phi = 1,000$. Similarly, A_1EF represents water and steam at the critical pressure, and A_2GH water and steam at $\phi = 4,000$.

Consider again the adiabatic compression of feed water from the

temperature of 100° F. to a pressure of 1,000 lb. per sq in. The initial condition is represented by the point K on the saturation line, the compression by the adiabatic line KL, and the subsequent heating to the saturation point B by the curve LB

It has been shown in Art 22 that the increase in enthalpy

$$dI = dH + \frac{v \cdot dP}{J}$$

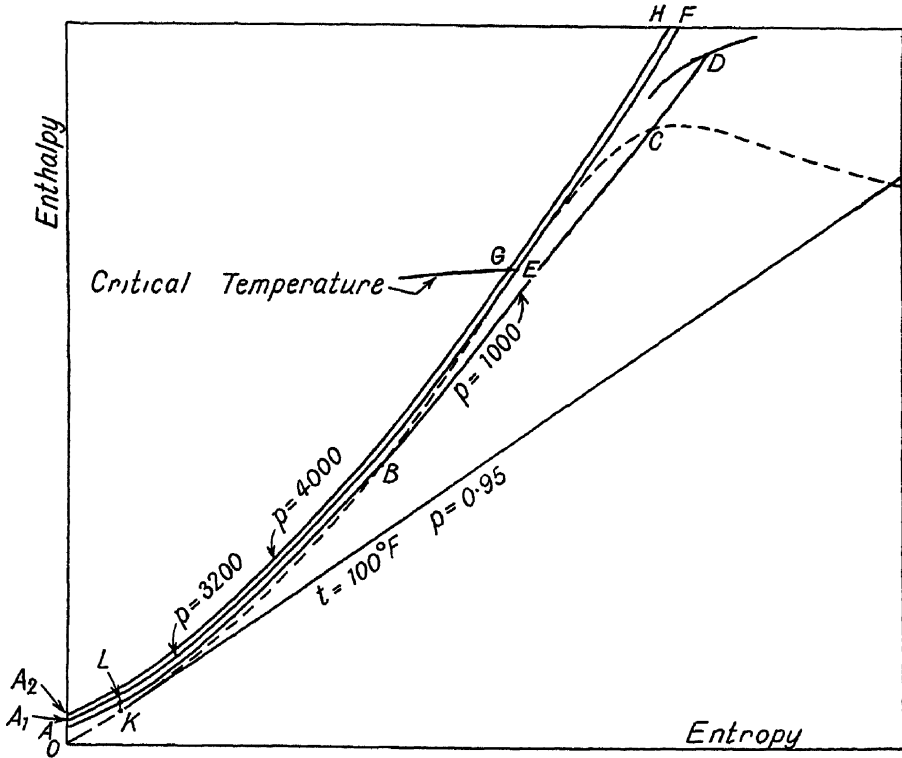


FIG 24 —Mollier Diagram for Compressed Water

In the adiabatic compression KL, Fig. 24, $dH = 0$, and therefore

$$I_L - I_K = \int \frac{v_w}{J} dP$$

and ignoring the extremely small change of volume during compression

$$I_L - I_K = \frac{v_w}{J} (P_L - P_K) \quad . \quad . \quad . \quad (25)$$

This simple relation would enable the change of enthalpy to be calculated for any increase of pressure at constant entropy.

Consider now the generation of steam at, say, 1,000 lb. per sq. in. from water at 100° F. and at its corresponding pressure to superheated steam at temperature t_D . During the adiabatic compression of the water to the pressure at which steam generation takes place there is, of course, no heat supplied or rejected. Since the pressure during the subsequent heating along the curve LB is constant, the heat supplied is exactly equal to the increase of enthalpy, i.e. to $I_D - I_L$.

$$\begin{aligned}\text{Heat supplied} &= I_D - I_L \\ &= (I_D - I_K) - \int_1^2 v_w (P_1 - P_2) \quad . \quad . \quad . \quad (26)\end{aligned}$$

This has already been shown in Art. 22, but is repeated here so as to make the matter clear with respect to the Mollier diagram.

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EXAMPLES III

1. Show that if the specific heat of a substance be expressed by the relation $C = a + bT + cT^2$, where a , b , and c are constants and T is the absolute temperature, the change of entropy due to the heating of the substance from T_1 to T_2 is

$$\phi_2 - \phi_1 = a \cdot \log_e \frac{T_2}{T_1} + b(T_2 - T_1) + \frac{c}{2}(T_2^2 - T_1^2)$$

2. Sixty units of heat are added to unit mass of a substance while its temperature is increasing from 400 to 490 degrees absolute. If the specific heat remains constant during the addition of heat, what is the change of entropy?

3. Assuming that the specific heat of water is unity and that the latent heat $L = 1,100 - 0.63t$, calculate for 1 lb. of steam at $p = 100$ ($t = 327.9^\circ$ F.) the following quantities—

(a) Change of entropy during the heating of water from 32° F. to 327.9° F.

(b) Change of entropy during evaporation.

(c) Approximate change of entropy during superheating, if the degree of superheat is 200° F. ($C_{pm} = 0.563$.)

4. Calculate the change of entropy of 1 lb. of water during the heating from 32° F. to 544.7° F. ($p = 1,000$), using as a basis the following modification of Dieterici's equation for the specific heat of water—

$$C_w = 1.18124 - 0.00068683T + 0.00000064T^2$$

T being the absolute temperature on the Fahrenheit scale. Compare this value with that based on the assumption of unit specific heat.

5 A boiler drum has a total capacity of 1,000 cub ft. It contains a certain amount of water and of dry and saturated steam, and the whole may be assumed to be at one temperature.

When the pressure is 200, the boiler is exactly half-full of water. Heat is added and the pressure is raised to 400. No steam is allowed to escape following—

- (a) The weight of water evaporated
- (b) The total amount of heat supplied
- (c) The total change of entropy of the whole mass of water and steam

Given the following information—

Specific volume of saturated water when $p = 200$, = 0.0184
 „ „ „ „ „ $p = 400$, = 0.0194

THEORETICAL STEAM TURBINE CYCLE

29. Definition of Cycle. The object of all heat engines is to convert heat into mechanical work. In all such engines we use some substance, termed the "working substance" or "working fluid," to which we supply heat, so causing the working substance to expand, and in overcoming some external resistance to perform a certain amount of mechanical work. After the expansion process, we have some cold "body" or receiver of heat to which the working substance rejects heat before taking in a new supply of heat from the hot body or source of heat.

The working substance is said to have undergone a cycle of changes of state when, after starting from some definite initial condition, receiving a supply of heat, expanding and performing mechanical work, and finally rejecting that quantity of heat which cannot be converted into work, it is brought back to its initial state or condition.

The cycle thus defined is a *closed* cycle when it is completed within the power plant itself, as, for example, in the case of a condensing steam turbine. When the cycle is virtually completed outside the power plant, as, for example, with an engine or turbine exhausting to atmosphere, the cycle is said to be an *open* one.

30. Actual Steam Turbine Cycle. Fig. 25 shows in diagrammatic form the essential parts of a steam turbine power plant. It comprises three fundamental elements—

(1) The boiler plant, in which the working substance, water, receives heat liberated by the combustion of the fuel.

(2) The steam turbine, in which the working substance undergoes changes in pressure, temperature, and heat content, thus performing mechanical work on the shaft.

(3) The condenser, in which the working fluid gives up to the cooling water that quantity of heat which, in virtue of the Second Law of Thermodynamics, cannot be converted into work and must be rejected in order to restore the working fluid to its initial condition.

The diagram shows in outline a Babcock & Wilcox boiler with a travelling-grate mechanical stoker. After leaving the boiler proper, the products of combustion flow over the tubes of an "economizer" or feed-water preheater, and then through the air preheater. The combustion air is supplied to the top of the air preheater by the forced-draught fan, and the hot air which leaves the heater then goes direct to the internal air compartments of the mechanical stoker.

The steam is normally superheated to a relatively high temperature, and the superheater tubes must be located in the path of the products in such a way that the temperature of the products is high enough to give the required steam temperature without an excessive amount of superheater surface. In this respect, Fig 25 is very diagrammatic

The steam generated by the boiler is led to a vessel termed a receiver into which other boilers may deliver steam. Steam pipes connect the receiver to the individual turbines and sometimes cross-connect the receiver to another receiver. With large turbine units it is customary to employ a single receiver per turbine. The steam pipes leading from the receiver to the turbine are equipped

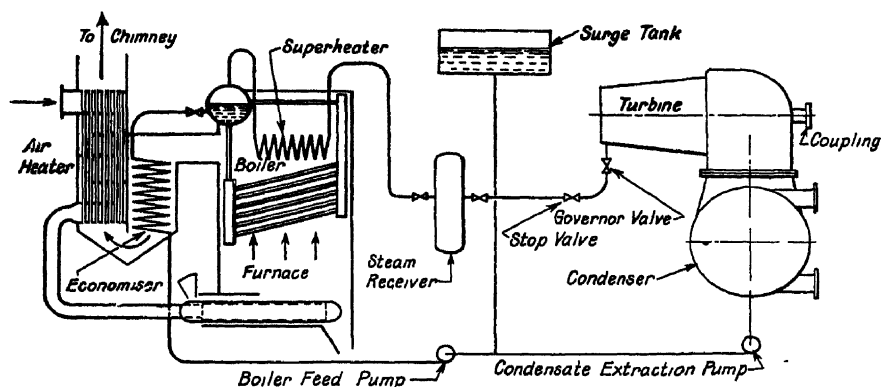


FIG 25 —Diagrammatic Arrangement of Steam Power Plant

with suitable valves, drain valves and means for permitting expansion. There are always at least two valves, or sets of valves. One, the stop valve, is used for starting and stopping the turbine, the other, the governor valve, is automatically controlled by the speed governor and maintains the speed of rotation at a sensibly constant value in spite of variations of load.

After doing work in the turbine, the steam is condensed. In the normal surface condenser, the steam is condensed on the outer surface of a very large number of metal tubes through which the cooling or "circulating" water flows. The condensed steam or "condensate" showers down over the tubes and is extracted from the bottom of the condenser by a pump, known as the condensate extraction pump. This pump delivers the condensate to the suction side of the boiler feed-pump and the latter returns the feed water to the boiler, via the economizer. In order to allow for the varying quantity of water in the boiler at different rates of steaming, it is necessary to provide some form of storage for the condensate between the condensate extraction pump and the boiler feed-pump.

This may take the form of an open "hot-well," a sealed "surge tank," as shown in Fig. 25, or other forms.

Owing to small leakages through joints and glands which are subjected to internal pressures lower than atmospheric pressure, a certain amount of air leaks into the turbine and condenser. Unless this air is continually extracted it accumulates in the condenser and raises the exhaust pressure, with adverse effects on the turbine performance. Air pumps, usually of the steam ejector type, are provided in all cases; but their effect on the thermal quantities involved is so small as to be negligible.

31. Rankine Cycle for Steam Initially Saturated. In order to compare the performance of actual engines and turbines, certain

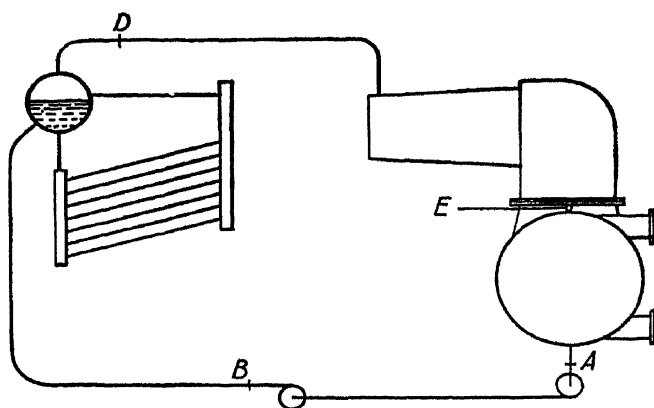


FIG. 26

theoretical or ideal cycles have been formulated. When the thermal properties of the working substance are sufficiently determined, it is possible by a study of these theoretical cycles to calculate the amount of heat which must be supplied to the working substance during one cycle, the amount of mechanical work performed by the substance under certain specified conditions, and the amount of heat which must be abstracted from the working substance in the condenser. We can thus calculate the theoretical thermal efficiency, which is the ratio of the theoretical work done to the heat supplied.

Consider, for example, a steam plant in which the steam is supplied to the turbine in the dry and saturated state. The plant is shown diagrammatically in Fig. 26, while the T - ϕ and ϕ - l diagrams for the cycle are shown in Figs. 27 and 28 respectively.

A convenient starting point in the cycle is A, at the extraction pump suction. Here we have unit mass of water in the saturated condition, at pressure p_2 and temperature T_2 . This water is next raised in pressure by the combined effects of the extraction pump

and boiler feed-pump to the higher limit of pressure p_1 , its condition being represented by the point B in Figs 26, 27, and 28, which are lettered to correspond. The compression AB is adiabatic.

The water now enters the economizer and/or boiler and is first raised to the evaporation temperature T_1 . During this process the state point moves along the curve BC. Moreover, as already explained (Arts 22 and 28), the heat supplied during the process is equal to $I_0 - I_B$, and is represented in Fig 27 by the area LBCM to some scale.

The water is now completely evaporated at pressure p_1 and temperature T_1 . The heat supplied is L_1 and equals $I_D - I_0$. In Fig 27 this is represented by the area of the rectangle MCDN.

The steam is now assumed to enter the turbine, where it undergoes a drop in pressure from p_1 to p_2 , usually in several stages. In the first place, the steam is assumed to pass through the turbine without any heat flowing into or out of the turbine cylinder. Furthermore, the expansion of the steam in the nozzles is assumed to take place without any surface friction losses and without the production of eddies, and the passage of steam through the rotor blades is likewise supposed to take place without friction or the dissipation of energy due to impact or eddies. If all these conditions could be satisfied then the steam would undergo the process of expansion without receiving heat in any way, either from external bodies or internally by the reception of dissipated mechanical energy in the form of heat. Such an expansion is said to be *adiabatic* and *frictionless*. Because the substance neither gains nor loses heat during such an expansion, the expansion takes place at constant entropy, i.e. it is *isentropic*.

An adiabatic or isentropic expansion of the swiftly moving steam in a steam turbine is utterly impossible in practice. The flow in a nozzle of good design must be attended by friction losses because of the molecular structure of, and molecular movements within, the flowing steam. Similarly, even though the rotor blades are so proportioned as to eliminate all shocks and eddies, there still remains the inevitable surface friction loss. In practice, therefore, the steam is continually receiving internally the heat equivalent of the mechanical losses incidental to the flow.

The adiabatic expansion of the steam is represented in Figs. 27 and 28 by the isentropic line DE. As soon as expansion begins, the steam (theoretically) commences to condense and at the turbine outlet has a dryness fraction x_D .

The steam is now condensed at p_2 and T_2 , the working substance thus returning to its initial condition at A. The heat rejected during condensation

$$= x_D \cdot L_2 = I_D - I_A$$

Now since all heat losses have been precluded, the work done

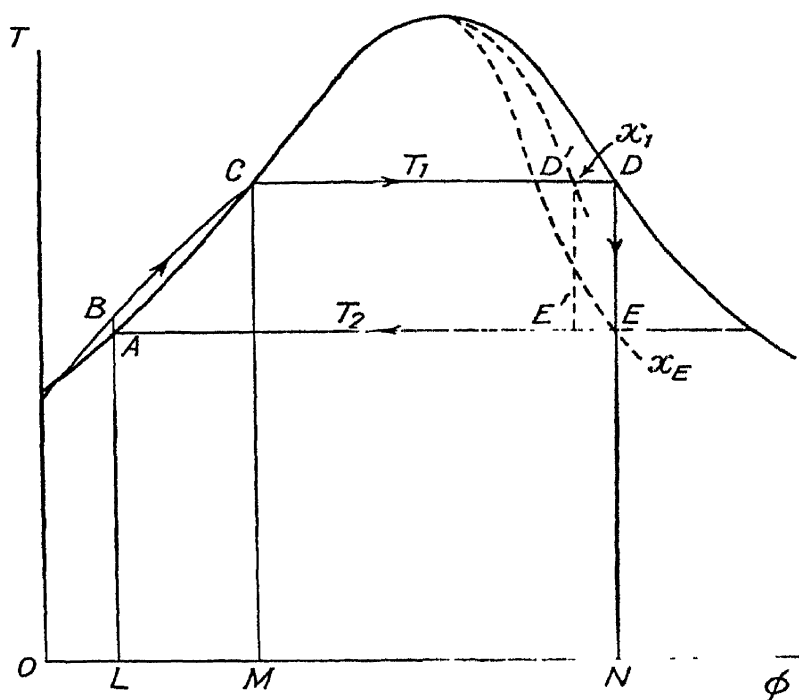


FIG. 27.— T - ϕ Diagram for the Rankine Cycle

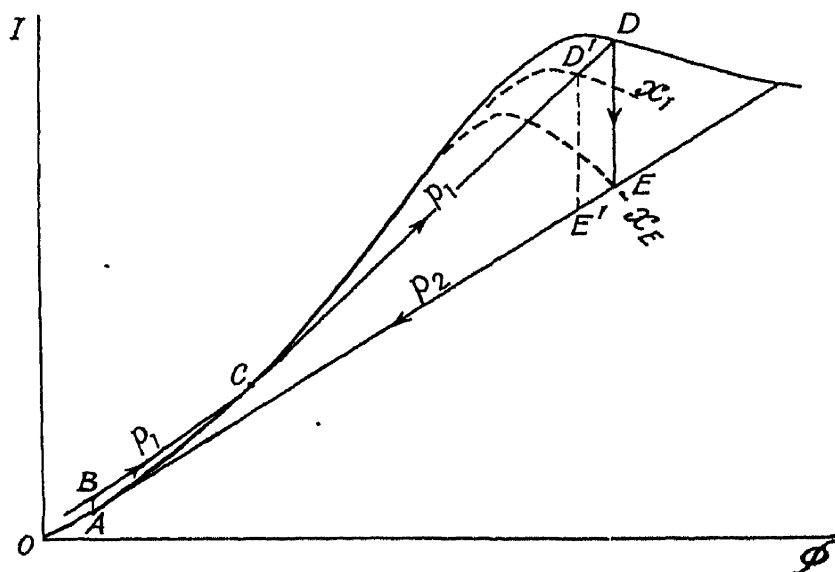


FIG. 28.— ϕ - I Diagram for the Rankine Cycle

is equal to the difference between the heat supplied and that rejected

$$\begin{aligned}\text{Heat supplied} &= (I_D - I_B) + (I_D - I_C) \\ &= I_D - I_B\end{aligned}$$

By eq. (26), Art. 28

$$I_B - I_A = \frac{v_{w2}}{J}(P_1 - P_2)$$

$\therefore$ Heat supplied

$$= (I_D - I_A) - \frac{v_{w2}}{J}(P_1 - P_2) \text{ B.Th U per lb.} \quad (27)$$

and heat rejected $= I_B - I_A$.

$\therefore$ Work done

$$= (I_D - I_B) - \frac{v_{w2}}{J}(P_1 - P_2) \text{ B Th U per lb.} \quad (28)$$

Therefore the theoretical thermal efficiency, or cycle efficiency η_c , is

$$\begin{aligned}\eta_c &= \left\{ (I_D - I_B) - \frac{v_{w2}}{J}(P_1 - P_2) \right\} \\ &\div \left\{ (I_D - I_A) - \frac{v_{w2}}{J}(P_1 - P_2) \right\} \quad (29)\end{aligned}$$

In the expression for the work done, the term $\frac{v_{w2}}{J}(P_1 - P_2)$ represents the work done by the feed-pump. Consequently, the gross work done by the turbine is given by $I_D - I_B$. Unless the pressure P_1 is relatively high the feed-pump work is small and may be neglected. The cycle efficiency is then equal to

$$\eta_c = \frac{I_D - I_B}{I_D - I_A} \quad (30)$$

If the steam has an initial dryness fraction x_1 , then the adiabatic expansion is represented by the isentropic D^1E^1 and the expressions for the work done, heat supplied, etc., are as above with D^1 and E^1 substituted for D and E respectively. An explanation of the methods of deriving the work done and the cycle efficiency will be given in the next article.

32. Rankine Cycle for Steam Initially Superheated. This is the cycle most frequently used in practice. The plant is shown schematically in Fig. 29 and differs from that dealt with in the previous article only in that a superheater is included.

It is unnecessary to discuss this cycle in detail and we shall proceed with an explanation of methods of deriving the values of the heat supplied, the work done, and the thermal efficiency. Three methods will be given.

EXAMPLE 2. Steam is supplied to a turbine at a pressure of 500 lb. per sq. in. and a temperature of 800° F. It is expanded adiabatically and without friction to a pressure of 0.5 lb. per sq. in. Calculate or derive the following—

- (a) External heat supplied per lb. of steam.
- (b) Work done by turbine per lb. of steam.
- (c) Net work done.
- (d) Theoretical thermal efficiency.

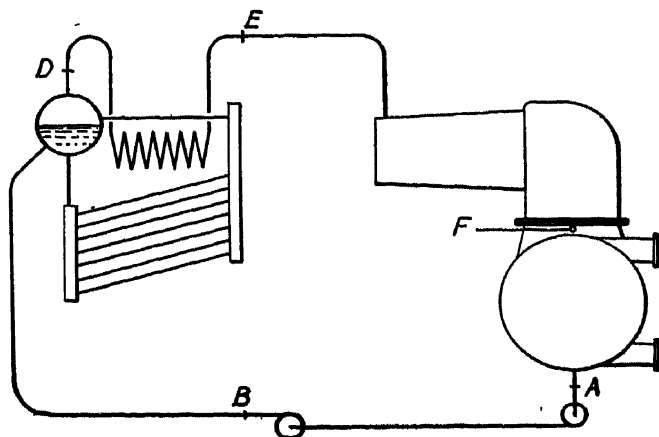


FIG. 29

Method 1. From first principles, and assuming mean values for the specific heats of water and superheated steam.

From Steam Tables—

Enthalpy of water

$$p = 500 (t = 467.1^\circ \text{ F.}) = 450.1$$

$$p = 0.5 (t = 79.5^\circ \text{ F.}) = 47.4$$

i.e. $I_0 - I_A$ (Fig. 30) = 402.7.

By Art. 22, heat supplied to the saturated water (path AC)

$$= (I_0 - I_A) - \frac{1}{J} \int_{P_2}^{P_1} v_w dP$$

$$= 402.7 - 1.7 = 401.0 \text{ B.Th.U. per lb.}$$

Note that v_w is the specific volume of saturated water and its variation with temperature has been taken into account.

Since the temperature rise is 387.6° F., the mean specific heat of the water is 1.035.

The initial superheat is $800 - 467.1$, or 332.9° F., and the mean

$$\begin{aligned}
 \therefore \text{Total heat supplied externally} \\
 &= 401.2 + 763.1 + (0.606 \times 332.9) \\
 &= \underline{1366.1 \text{ B.Th.U. per lb.}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Heat rejected} &= T_2 \times LR \\
 &= 539.1 \times 1.57 \\
 &= 846.5 \text{ B.Th.U. per lb.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Net work done in the power plant per lb. of steam passing} \\
 \text{through the turbine, i.e. the work done by the turbine less the work} \\
 \text{done in forcing the feed water into the boiler,} \\
 &= 1366.1 - 846.5 \\
 &= \underline{519.6 \text{ B.Th.U.}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Net thermal efficiency of cycle} \\
 &= \frac{519.6}{1366.1} \\
 &= \underline{38.05 \text{ per cent.}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Work done by turbine only} \\
 &= 519.6 + (\text{Feed pump work}) \\
 &= 519.6 + 1.5 \\
 &= \underline{521.1 \text{ B.Th.U. per lb.}}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Gross cycle efficiency} \\
 &= \frac{521.1}{1366.1} \\
 &= \underline{38.15 \text{ per cent.}}
 \end{aligned}$$

Method 2. By the use of Steam Tables, but without the use of the Mollier diagram.

$$\begin{aligned}
 \text{Heat supplied in cycle} \\
 &= I_B - I_A \\
 &= (I_B - I_A) - \frac{v_{w2}}{J}(P_1 - P_2) \\
 &= (1415.1 - 47.4) - 1.5 = \underline{1366.2 \text{ B.Th.U. per lb.}}
 \end{aligned}$$

$$\phi_F = \phi_B = 1.6619$$

$$\therefore \phi_F - \phi_A = 1.6619 - 0.0920 = 1.5699$$

$$\begin{aligned}
 \therefore \text{Heat rejected} &= T_2(\phi_F - \phi_A) \\
 &= 539.1 \times 1.5699 \\
 &= 846.5 \text{ B.Th.U. per lb.}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Net work done per lb. of steam} \\
 &= 1366.2 - 846.5 = 519.7 \text{ B.Th.U.}
 \end{aligned}$$

which is practically identical with the result obtained by the previous

method. The other results may be worked out by the methods given above.

Method 3. By the use of the Mollier chart

For this purpose the large Callendar diagram was used. This is shown in skeleton form in Fig. 31, which is lettered to correspond to Fig. 30.

In using the chart, the point E is first obtained from the given initial pressure and temperature. This immediately gives the value

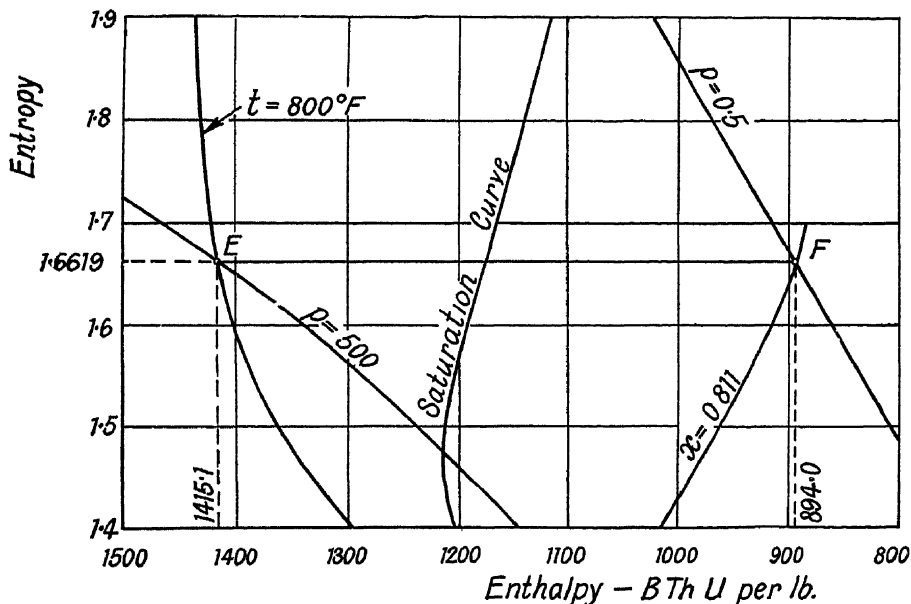


FIG 31

of the entropy, viz. 1.6619, and of the enthalpy, viz. 1415.1 B Th U per lb. The adiabatic and frictionless expansion is represented by the constant entropy line EF, which may be drawn on a sheet of tracing paper placed over the chart. Alternatively, the point F may be simply read off the chart for the pressure of 0.5 lb per sq. in. and the entropy 1.6619, without drawing any line at all. The enthalpy at F is 894.0 and the dryness fraction of the steam is 0.811.

The author recommends writing down the various quantities, given and derived, in the following tabular form—

p	Condition	ϕ	I
500	$t = 800^\circ\text{F.}$	1.6619	1415.1
0.5	$x = 0.811$	1.6619	894.0
			<u>521.1</u>

The steam, on adiabatic expansion, only becomes dry and saturated at the pressure p_3 corresponding to the temperature T_3 , which is found by the intersection of the adiabatic and isentropic expansion line EFH with the steam saturation line. If T_3 is lower than T_2 , the saturation temperature corresponding to the actual exhaust pressure, then the state point at the termination of the actual expansion will be F, the point of intersection of the isentropic EFH with the constant-pressure line GF. At the end of the adiabatic expansion the steam is superheated by the amount $T_F - T_G$. In order to restore the steam to its initial condition as saturated water at A, it is first necessary to de-superheat it, by cooling at constant pressure p_2 to the temperature T_2 and then to condense it.

The final steam temperature T_F may be calculated as follows. The total change of entropy LS may be found either by Method 1 of the last article, or from steam tables

$$\begin{aligned}\text{Then} \quad RS &= C_{pm} \log_e \frac{T_F}{T_G} \text{ (approx)} \\ &= LS - LR \\ &= LS - \frac{I_2}{T_2}\end{aligned}$$

$$\text{Heat rejected} = C_{pm}(T_F - T_G) + L_2$$

Since the heat rejection takes place at constant pressure throughout, the heat rejected also equals $I_F - I_A$. Therefore the net work done, i.e.

$$\begin{aligned}\text{Turbine work} - \text{feed-pump work} \\ &= (I_E - I_A) - \frac{v_{u2}}{J}(P_1 - P_2) - (I_F - I_A) \\ &= (I_E - I_F) - \frac{v_{u2}}{J}(P_1 - P_2) \text{ B Th U per lb}\end{aligned}$$

The work done by the turbine only is $I_E - I_F$. The treatment of this case by the Mollier diagram will now be illustrated briefly by an example

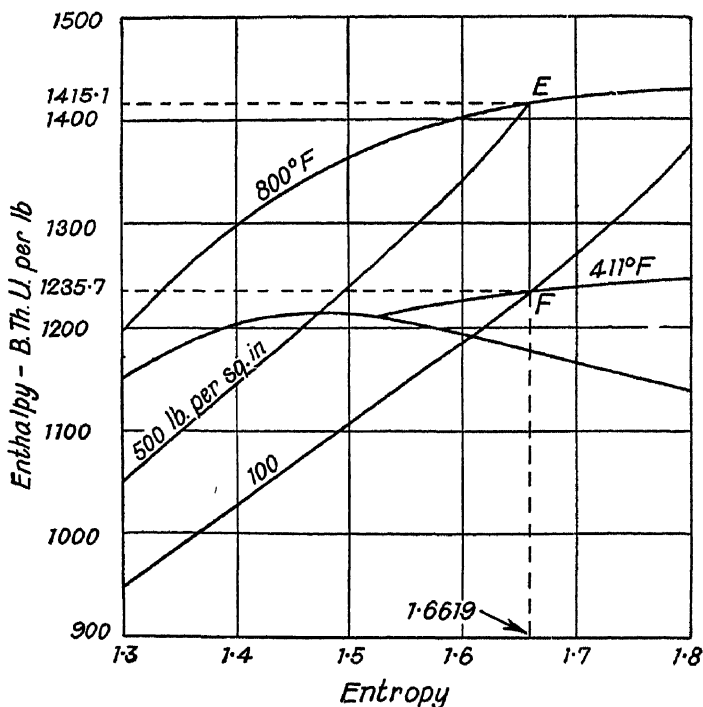
EXAMPLE 3. A turbine is supplied with steam at 500 lb per sq. in. and 800° F. The steam is expanded adiabatically to a pressure of 100 lb per sq. in.

Derive the following—

- (a) External heat supplied per lb. of steam.
- (b) Condition of the steam at the end of the expansion.
- (c) Work done by the turbine
- (d) Gross theoretical thermal efficiency.
- (e) Net theoretical thermal efficiency.

(a) Heat supplied in cycle

$$\begin{aligned}
 &= (I_E - I_A) - \frac{v_{w2}}{J} (P_1 - P_2) \text{ (Fig. 32)} \\
 &= (1415.1 - 298.5) - \left(\frac{0.0177 \times 144 \times 400}{777.8} \right) \\
 &= 1116.6 - 1.3 \\
 &= \underline{1115.3 \text{ B.Th.U. per lb.}}
 \end{aligned}$$

FIG. 33.— ϕ - I Diagram Illustrating Example 3

(b) For the expansion, the following table may be drawn up with reference to the Mollier diagram, Fig. 33.

p	Condition	ϕ	I
500	$t = 800^\circ \text{ F.}$	1.6619	1415.1
100	$t = 411^\circ \text{ F.}$	1.6619	1235.7
			179.4

At F the steam has a temperature of 411° F. and a superheat $= 411.0 - 327.9 = \underline{83.1^\circ \text{ F.}}$

(c) Work done in turbine per lb. of steam $= \underline{179.4 \text{ B.Th.U.}}$

H_w may be calculated from test data in the following manner.

Let W = steam consumption of turbine, lb. per hour.

h.p. = horse-power developed.

kW = power developed, kilowatts.

Now 1 h.p. = 33,000 ft.-lb. per minute.

= 42.43 B.Th.U. per minute.

= 2,546 B.Th.U. per hour.

But 1 h.p. = 746 watts.

∴ 1 kilowatt = 1.341 h.p.

= 56.89 B.Th.U. per minute

= 3,413 B.Th.U. per hour.

∴ Work done per lb. of steam

$$H_w = \frac{2546 \times \text{h.p.}}{W} \text{ B.Th.U.}$$

$$H_w = \frac{3413 \times \text{kW}}{W} \text{ B.Th.U.}$$

∴ Efficiency ratio

$$\eta_r = \frac{2546 \times \text{h.p.}}{W \cdot H_a} \quad . \quad . \quad . \quad . \quad (34a)$$

$$\eta_r = \frac{3413 \times \text{kW}}{W \cdot H_a} \quad . \quad . \quad . \quad . \quad (34b)$$

The absolute thermal efficiency

$$\eta_t = \frac{2546 \times \text{h.p.}}{W \cdot H} \quad . \quad . \quad . \quad . \quad (35a)$$

$$\text{or} \quad \eta_t = \frac{3413 \times \text{kW}}{W \cdot H} \quad . \quad . \quad . \quad . \quad (35b)$$

The steam consumption expressed in lb. of steam per unit power is

$$\frac{W}{\text{h.p.}} = \frac{2546}{H_w} = \frac{2546}{\eta_r \cdot H_a} = \frac{2546}{\eta_t \cdot H} \text{ lb. per h.p. hour}$$

$$\text{or} \quad \frac{W}{\text{kW}} = \frac{3413}{H_w} = \frac{3413}{\eta_r \cdot H_a} = \frac{3413}{\eta_t \cdot H} \text{ lb. per kW hour}$$

A more detailed study of losses and of efficiencies will be given in Chapters IX and X.

EXAMPLES IV

1. Dry and saturated steam at $p = 150$ and $t_s = 358.4^\circ \text{F.}$ is supplied to a turbine and expanded adiabatically to $p = 1.0$ ($t_s = 101.7^\circ \text{F.}$). Calculate the following—

(a) Heat supplied.

(b) Total change of entropy.

- (c) Heat rejected
- (d) Theoretical thermal efficiency.

Assume that the specific heat of water is unity and that the latent heats of vaporization at $p = 150$ and $p = 10$ are 867.9 and 1032.9 B Th U per lb respectively

2 Steam is supplied to a turbine at $p = 150$ ($t_s = 358.4^\circ \text{F}$) and with a dryness fraction of 80 per cent. The steam is expanded adiabatically to $p = 10$ ($t_s = 101.7^\circ \text{F}$). Calculate the following—

- (a) Heat supplied
- (b) Total change of entropy.
- (c) Heat rejected.
- (d) Theoretical thermal efficiency

Assume that the specific heat of water is unity and that L for $p = 150$ is 867.9 B Th U per lb

3 A steam turbine receives steam at $p = 250$ and superheated 200°F . The exhaust pressure is 10, and the expansion of the steam takes place adiabatically and without friction.

Using steam tables only, calculate the following—

- (a) Heat supplied, assuming that the feed-pump supplies water to the boiler at 250 lb per sq in
- (b) Heat rejected
- (c) Net work done
- (d) Work done by the turbine
- (e) Thermal efficiency.
- (f) Theoretical steam consumption

If the actual steam consumption is 11.2 lb per kWh, what is the efficiency ratio of the turbine? The specific volume of saturated water at $p = 10$ is 0.01614

4 A steam turbine plant is supplied with steam at $p = 250$, and superheated 180°F . The exhaust pressure is 0.85. The temperature of the condensate in the hot-well is actually 90°F ($v_w = 0.0161$). If the measured steam consumption is 10.9 lb per kWh and if the boiler efficiency is 82 per cent, what is the absolute thermal efficiency of the boiler and turbine plant?

5 Two steam turbines A and B operate with the same initial pressure, namely $p = 200$, and the same initial superheat of 160°F . In turbine A the exhaust pressure is 1.5 ($v_w = 0.01618$), whereas in turbine B the exhaust pressure is 0.5 ($v_w = 0.01607$). Calculate the following—

- (a) Theoretical steam consumption of turbines A and B, in lb per kWh.
- (b) Percentage reduction in steam consumption due to the lower exhaust pressure in turbine B
- (c) Theoretical thermal efficiency of turbines A and B
- (d) Percentage increase in thermal efficiency due to the lower exhaust pressure in turbine B

6. Steam expands adiabatically from an initial pressure p_1 (saturation temperature = T_1 abs.) and superheated to a temperature T_3 abs., to a final pressure p_2 (saturation temperature = T_2 abs.) Show that if the final pressure is lowered by an amount δp , the additional work done in the cycle per lb of steam is approximately

$$\frac{x_2 L_2}{T_2} \frac{dT}{dp} \delta p \text{ B Th U.}$$

where x_2 is the dryness fraction at T_2 . Assume that the water is maintained in the saturated condition during heating

Hence calculate the increase in work done when the final pressure is changed from 0.7 to 0.6 lb. per sq. in. abs., during the adiabatic expansion of steam from $p = 250$ and an initial superheat of 200° F.

7. The following are the means of the observations made during a test on a steam turbine—

Duration of test	2 hr.
Power developed (turbine coupling)	4,500 kW
Total weight of steam condensed	108,000 lb.
Steam pressure at stop valve	200
Steam temperature at stop valve	600° F.
Exhaust pressure	30.20 in. Hg.
Vacuum	28.57 in. Hg.

Calculate the thermal efficiency of the turbine and also its efficiency ratio. Assume that the condensate is returned to the boiler plant at the exhaust saturation temperature.

8. Two steam turbines operate between the same temperature limits, viz 800° F. and 79.5° F. In turbine A the initial pressure is 200, whereas in turbine B the initial pressure is 1,000. Using either the Revised Callendar Steam Tables or the ϕ -I Diagram, calculate the theoretical thermal efficiencies of turbines A and B and the values of the dryness fraction after adiabatic expansion.

9. Calculate the fuel cost per unit (kWh) generated in a steam power plant under the following conditions—

Steam pressure at turbine	300
Initial superheat	320° F.
Exhaust pressure	0.75
Turbine efficiency ratio	0.78
Boiler efficiency	0.82
Alternator efficiency	0.96
Calorific value of coal	12,800 B.Th.U. per lb.
Cost of coal	16s. per ton

The following assumptions may be made—

(a) That the feed temperature to the boiler is equal to the saturation temperature at 0.75 lb. per sq. in. abs.

(b) That 6 per cent of the steam generated is used to cover losses, and auxiliary power.

10. Two steam turbines A and B operate under the following conditions: initial pressure, 250; exhaust pressure, 0.7. In turbine A, the steam is initially dry and saturated, whereas in turbine B it is initially superheated by 200° F.

Using steam tables, calculate—

- Theoretical steam consumption, lb. per kWh.
- Theoretical thermal efficiency of the two turbines.
- Percentage reduction in steam consumption due to superheating.
- Percentage increase in thermal efficiency due to superheating.

11. To what temperature must steam at 500 lb. per sq. in. abs. be initially superheated so that, after adiabatic expansion to any given exhaust pressure, it may have the same condition (i.e. superheat or dryness fraction) as steam at 200 lb. per sq. in. abs. which has been initially superheated to a temperature of 520° F. and expanded adiabatically to the same exhaust pressure?

Assume that the thermal efficiency of superheated steam at constant pressure is constant and has the values 0.58 and 0.62 at $p = 200$ and $p = 500$ respectively.

12 Steam at 200 lb per sq in abs and superheated to a temperature of 600° F expands adiabatically to 1 lb per sq in abs. Using a Mollier diagram, find the pressure at which the steam is just dry and saturated, the final dryness fraction, and the adiabatic heat drop.

13 Assuming steam initially at a pressure of 200 and a 600° F, and also a barometric height of 30 in Hg, obtain diagram the adiabatic heat drops for the following vacua and state the percentage gain by increasing the vacuum above 25 in (a) 25, (b) 26, (c) 27, (d) 28, and (e) 29 in

THE FLOW OF STEAM THROUGH NOZZLES

35. Introduction. The object of the present chapter is to study the flow of steam as an elastic fluid flowing through passages of varying cross-sectional area. To the turbine engineer it is of the utmost importance to know what velocity will be acquired under any given conditions of expansion, and conversely what expansion conditions, such as pressure drop, must be provided in order that the steam will reach a certain velocity under any given conditions. In addition it is equally important to predict the mass flow of steam through a nozzle or similar channel of given shape under given conditions of pressure and temperature, and conversely to work out the proportions of a nozzle or blade channel to accommodate a certain rate of mass flow.

Not only is it necessary to understand the factors governing the flow under ideal conditions when the expansion is adiabatic and frictionless; it is equally important to be able to allow, at least approximately, for the effects of friction and other losses on the velocity acquired and the area required under any given conditions.

The theory is based on certain fundamental equations which will now be considered.

36. Equation of Continuity. Consider the flow of steam through a nozzle and let w be the rate of steady flow in pounds of steam per second. Consider a certain section of the nozzle where the cross-sectional area normal to the direction of steam flow is A sq. ft., and at which the volume per lb. of steam is v cub. ft. Then if the velocity across the section is uniform and equal to V ft. per sec., the volume of steam passing the given section is

$$AV \text{ cub. ft. per sec.}$$

But the volume of steam flowing past the given section is also

$$wv \text{ cub. ft. per sec.}$$

Hence

$$wv = AV \quad . \quad . \quad . \quad . \quad . \quad (36)$$

This simple equation is known as the "equation of continuity" and, provided the steam fills the nozzle at every section, it must be satisfied at all points in the steam path.

37. Steady Flow Equation for Turbine, Nozzle, Blades, etc. Consider the steady flow of 1 lb. of steam through a turbine, nozzle, or a system of blades. Assume that the conditions at inlet and outlet are as represented in Fig. 34 by the suffixes 1 and 2 respectively. The turbine, nozzle, or apparatus in question is represented diagrammatically by a rectangle.

Let the steam flow towards the apparatus through a pipe of area A_1 sq. ft., and leave it through a pipe of area A_2 . V_1 and V_2 are the velocities of flow through the inlet and outlet pipes respectively.

Let W = mechanical work (if any) done by the apparatus, in B Th U per lb of steam, e.g. the internal work in a steam turbine or engine. The mechanical work does not include the gain of kinetic energy in a nozzle.

H = heat supplied (if any) to the apparatus, in B.Th.U per lb of steam. In the case of a heat loss, H is negative.

W_f = work done in opposing the forces of friction within the apparatus

Imagine two pistons, one in the inlet pipe and the other in the outlet pipe. If, further, we suppose that these can move without

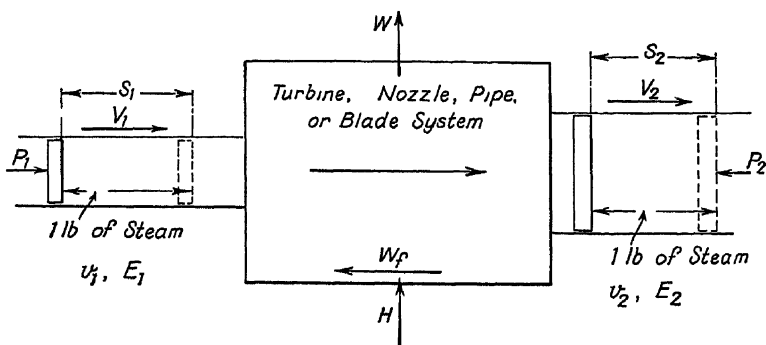


FIG 34

friction, then they will do so with velocities equal to those of the steam in the pipes. The distance moved forward by the piston in the inlet pipe during the flow of 1 lb. of steam through the apparatus is clearly $s_1 = \frac{v_1}{A_1}$. The force on the piston is $P_1 A_1$, and therefore the work done by the steam in forcing into the apparatus the steam which lies directly in front of it is equal to

$$\frac{P_1 A_1}{J} \times s_1 = \frac{P_1 v_1}{J} \text{ B Th U per lb.}$$

Similarly, the work done by the steam which is leaving the apparatus in forcing out the steam which is moving immediately in front of it is

$$\frac{P_2 v_2}{J} \text{ B.Th.U. per lb}$$

The difference $\frac{1}{J}(P_1 v_1 - P_2 v_2)$ represents the net work done in

introducing the steam into the system and expelling it from the system, and represents a supply of energy. E_1 and E_2 represent the initial and final values of the internal energy respectively.

Then, by the Principle of the Conservation of Energy, we may write—

$$\begin{aligned} & \text{(Work done by steam on inlet side of apparatus in forcing 1 lb.} \\ & \text{of steam into the apparatus) } + \text{(Initial internal energy) } + \\ & \text{(Initial kinetic energy) } + \text{(Heat supplied)} \\ = & \text{(Work done by steam in expelling 1 lb. of steam from the} \\ & \text{apparatus) } + \text{(Final internal energy) } + \text{(Final kinetic energy)} \\ & + \text{(External mechanical work performed).} \end{aligned}$$

$$\text{i.e.} \quad \frac{P_1 v_1}{J} + E_1 + \frac{V_1^2}{2gJ} + H = \frac{P_2 v_2}{J} + E_2 + \frac{V_2^2}{2gJ} + W$$

$$\text{and since} \quad I = \frac{Pv}{J} + E$$

$$I_1 + \frac{V_1^2}{2gJ} + H = I_2 + \frac{V_2^2}{2gJ} + W \quad . \quad . \quad (37)$$

This is the “steady flow equation” and is quite general and rigorously exact. It may be applied in many ways. Reverting for the moment to the ideal cycle considered in the previous chapter, if the initial and final kinetic energies of the steam are so small as to be negligible, and if the expansion is adiathermal ($H = 0$), then the work done

$$W = I_1 - I_2$$

this being the case whether the expansion is strictly adiabatic or not.

In the special case of a nozzle, the mechanical work W is obviously zero, and the general equation for a nozzle is therefore

$$I_1 + \frac{V_1^2}{2gJ} + H = I_2 + \frac{V_2^2}{2gJ} \quad . \quad . \quad (38)$$

If the flow through the nozzle is adiathermal, then $H = 0$, and the equation becomes

$$I_1 + \frac{V_1^2}{2gJ} = I_2 + \frac{V_2^2}{2gJ}$$

$$\text{Then} \quad \frac{V_2^2}{2gJ} = (I_1 - I_2) + \frac{V_1^2}{2gJ}$$

$$\text{or} \quad V_2 = \sqrt{2gJ(I_1 - I_2) + V_1^2} \quad . \quad . \quad (39)$$

The gain in kinetic energy between any two sections of a nozzle is equal to the loss of enthalpy.

If the steam expands from a state of rest ($V_1 = 0$) then the velocity V_2 reached in the expansion is

$$V_2 = \sqrt{2gJ} \cdot \sqrt{(I_1 - I_2)} \quad . \quad (40)$$

Substituting $g = 32.19$ ft per sec.², and $J = 777.8$,

$$V_2 = 223.8 \sqrt{(I_1 - I_2)} \quad . \quad (40a)$$

If the heat drop is expressed in C.H U's per lb of steam, then $J = 1,400$ and

$$V_2 = 300.2 \sqrt{(I_1 - I_2)} \quad . \quad (40b)$$

In each case, V_2 is in ft. per sec

These expressions are quite general for adiathermal flow whether the flow is accompanied by friction losses or not. It will be observed that the term W_f does not enter into the energy equation at all, although the existence of friction naturally affects the final condition of the steam at exit from the apparatus.

In the special case of frictionless flow, the expansion is adiabatic and takes place at constant entropy. Hence the heat drop $I_1 - I_2$ is the adiabatic heat drop and may be readily obtained from a Mollier diagram, or by calculation.

38. Calculation of Nozzle Area in General Case of Adiabatic and Frictionless Flow. Assume, for example, that steam initially dry and saturated at pressure p_1 expands to a pressure p_2 . It is required to calculate the area required for sections of the nozzle at which the steam pressure has certain values between p_1 and p_2 , and when the nozzle is required to discharge a quantity w lb. per sec. Let p be any intermediate pressure. The isentropic expansion line ab , Fig. 35, is drawn on the Mollier diagram, intersecting the p line in c , and so giving the adiabatic heat drop $I_1 - I$ between p_1 and p .

Let the steam have an initial velocity V_1 . Then by eq. (39) the velocity

$$V = \sqrt{2gJ(I_1 - I) + V_1^2}$$

If v_s is the volume per lb of dry and saturated steam at p , and x is the dryness fraction as given by the chart, Fig. 35, then the actual volume per lb of steam is very nearly equal to $x \cdot v_s$, and by the equation of continuity (36)

$$A = \frac{w}{V} \frac{x}{v_s} \text{ sq ft.}$$

It should be noted that while the expression enables us to calculate the area of the cross-section corresponding to any value of the steam pressure, it does not tell us anything about the shape of the nozzle. The latter depends on the rate of pressure drop along the axis of the nozzle, and conversely, if the shape is chosen then the rate of pressure drop will be governed by the shape. A valuable insight into the process of steam flow through a nozzle may be

obtained by assuming that the rate of pressure fall along the axis of the nozzle is uniform. This will be done in the following example.

EXAMPLE 4. To determine the longitudinal section of a round nozzle to discharge 1 lb. of steam per sec. The initial pressure is 200 lb. per sq. in. abs., the steam is initially dry and saturated,

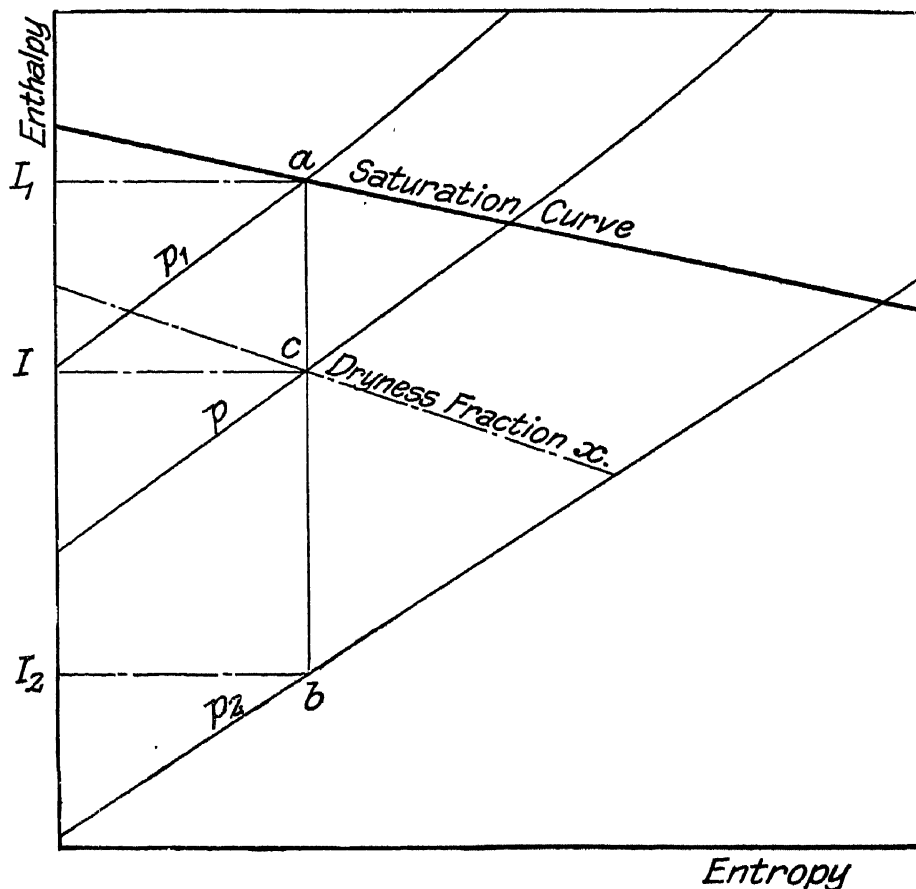


FIG. 35

and the final pressure is 1 lb. per sq. in. Assume that the flow is adiabatic and frictionless and that the pressure drop per unit length of nozzle is constant.

The calculation is shown in Table I. Column 2 gives the adiabatic heat drop, while 3 gives the theoretical steam velocity. In columns 4, 5, and 6 are given, respectively, the specific volume of dry and saturated steam, the dryness fraction, and the actual specific volume of the wet steam. Column 7 gives the required area of the

TABLE I

CALCULATION OF CROSS-SECTIONAL AREAS OF NOZZLE TO PASS 1 LB OF STEAM PER SEC, ASSUMING FRICTIONLESS ADIABATIC FLOW STEAM INITIALLY DRY AND SATURATED AT A PRESSURE OF 200 LB PER SQ IN ABSOLUTE

(Note Values based on Callendar's Steam Tables and ϕ -I Diagram)

1	2	3	4	5	6	7	8	9	10
Pressure p	$I_1 - I$	v ft. per sec.	v_g cu ft per lb.	v Dry- ness frac- tion	w_g Actual Sp Vol	$\frac{A}{V}$ $= \frac{av_g}{V}$ sq ft	Diam of Sec- tion in ft	Diam of Sec- tion in in	Mass Flow in lb per sq ft per sec
180	8.9	668	2.536	0.992	2.52	0.00377	0.0693	0.832	266
160	18.7	968	2.836	0.983	2.78	0.00288	0.0605	0.726	348
140	29.8	1,220	3.221	0.973	3.13	0.00257	0.0572	0.686	389
120	42.1	1,451	3.727	0.963	3.59	0.00247	0.0561	0.673	405
100	56.4	1,680	4.429	0.951	4.21	0.00251	0.0565	0.679	399
80	73.6	1,920	5.466	0.938	5.13	0.00267	0.0584	0.700	375
60	94.9	2,180	7.162	0.922	6.60	0.00303	0.0621	0.745	331
40	124.2	2,496	10.49	0.901	9.45	0.00379	0.0695	0.834	264
20	170.8	2,922	20.08	0.871	17.48	0.00599	0.0873	1.048	167
15	189.1	3,080	26.27	0.859	22.56	0.00733	0.0966	1.159	137
10	213.9	3,278	38.39	0.844	32.4	0.00988	0.112	1.344	101
5	253.9	3,562	73.44	0.820	60.1	0.0169	0.147	1.76	59.2
2	302.8	3,898	173.5	0.792	137.2	0.0353	0.212	2.54	28.4
1	336.8	4,106	333.1	0.772	257.1	0.0626	0.282	3.38	16.0

nozzle, while 8 and 9 give the corresponding diameters of the circular sections. Since the nozzle is designed for a mass flow of 1 lb per sec, the inverse of the area given in column 7 will give the rate of mass flow in lb per sq ft of nozzle area per sec at any given pressure. This is given in column 10.

The calculated results are plotted on a base of back pressure (uniform scale) in Fig. 36. A longitudinal section of the nozzle is also shown.

Writing $v = x \cdot v_g$, then since $w = 1$, $A = \frac{v}{V}$. Differentiating with respect to p , we have

$$V^2 \cdot \frac{dA}{dp} = V \cdot \frac{dv}{dp} - v \frac{dV}{dp}$$

From the initial pressure down to a pressure of about 120 lb. per sq. in., the second term in the above expression is numerically greater than the first, and the area A diminishes as the pressure falls. At pressures below about 120 lb. per sq. in. the first term is numerically greater than the second, and the area of the nozzle increases as the pressure is diminished. At a certain pressure, the required cross-sectional area is a minimum, thus giving the nozzle

a "throat." This is also brought out by the discharge curve. Suppose, for example, that the back pressure was 180. Then the nozzle, if correctly designed for the given conditions, should terminate at the section A-A and, if the area of this minimum section were 1 sq. ft., the mass of steam discharged per sec. would be 266 lb. Now suppose the back pressure is reduced to 160 lb. per sq. in. and that the nozzle be made to end at section B-B. Then, if the minimum area of this nozzle be also 1 sq. ft., the mass flow per second would be increased from 266 to 348 lb. Inspection of column 10, Table I, and of the discharge curve, Fig. 36, shows that by reducing the back pressure the mass flow per unit outlet area may be increased up to a certain limit. The limit is obviously that corresponding to the minimum area of the nozzle and occurs when the back pressure is reduced to about 116 lb. per sq. in. abs., i.e. about 0.58 of the initial pressure.

The pressure at which the area is a minimum and the discharge per unit area a maximum is termed the "critical pressure." The common use of this term is rather unfortunate, but it is not likely that it would ever be confused with the true critical pressure of a substance. The critical pressure in nozzle flow will be considered more fully in later paragraphs.

39. Adiabatic Equation for Steam. It may be readily shown that when a perfect gas expands adiabatically the pressure p and the volume v are related by the simple expression $pv^\gamma = C$, where C is a constant and γ is the ratio of the specific heat at constant pressure to that at constant volume. It should be emphasized that this relation applies strictly to an ideal gas obeying the laws of Boyle and Charles and having constant specific heats. Steam, even when superheated to an appreciable extent, deviates considerably from the laws of perfect gases, and there are no real grounds for expecting that, during an adiabatic expansion of steam, the p - v relationship should be of the form

$$pv^n = \text{constant} \quad . \quad . \quad . \quad (41)$$

where n is a constant. The test is, however, easily made. Corresponding values of p and v may be obtained from a Mollier diagram and the logs. of their values plotted. Then if

$$pv^n = C \\ \log. p + n \log. v = \log. C$$

and we have a straight-line relationship. When the steam is assumed to be initially dry and saturated, then plots made for different initial pressures deviate very little from straight lines. Mean values of n , where

$$n = \frac{\log. p_1 - \log. p_2}{\log. v_2 - \log. v_1}$$

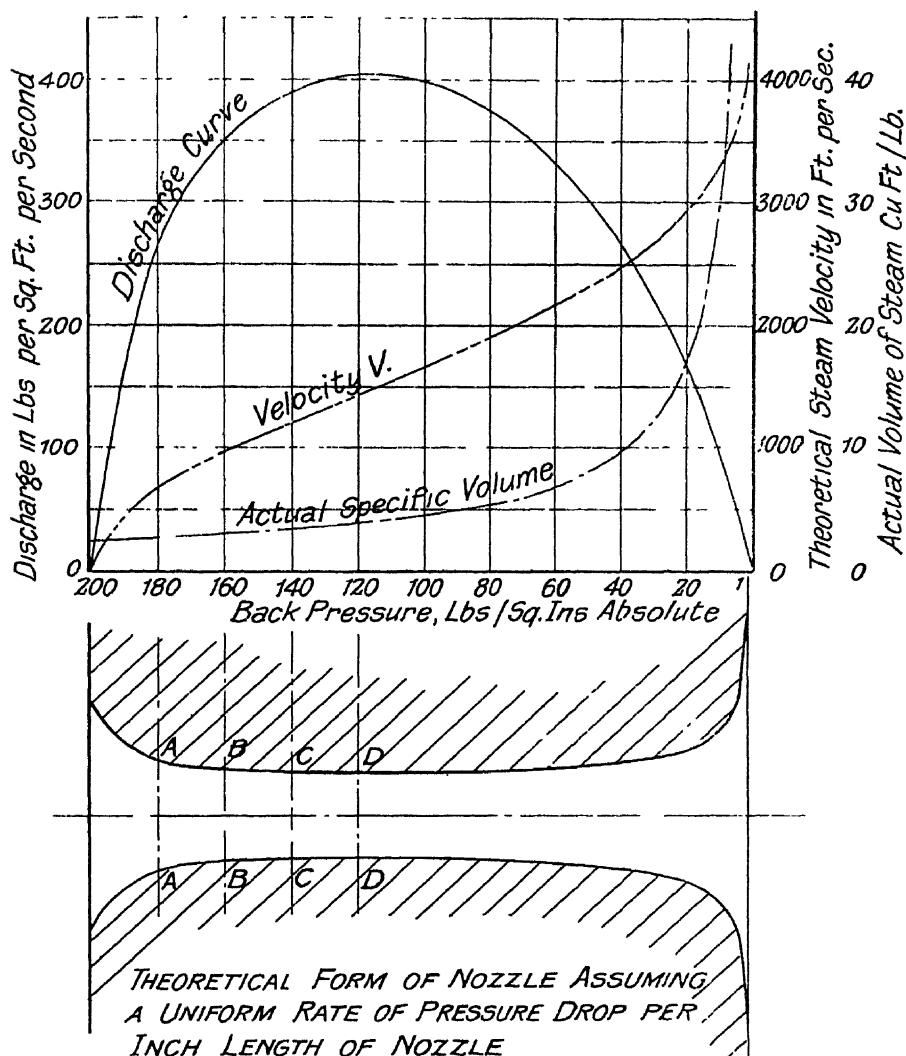


FIG. 36

calculated for various ratios of $\frac{p_2}{p_1}$ and based on Callendar's Enlarged Mollier Diagram, first increase very slightly as $\frac{p_2}{p_1}$ is diminished and then diminish slightly as $\frac{p_2}{p_1}$ is further reduced. The mean value of n for the ratio $\frac{p_2}{p_1}$ is shown in Fig. 37.

Rankine (1) states that for adiabatic expansion "it has been deduced by trial that for such pressures as usually occur in the working of steam engines, $pv^{1.11} = \text{constant}$." Zeuner (2) gives the

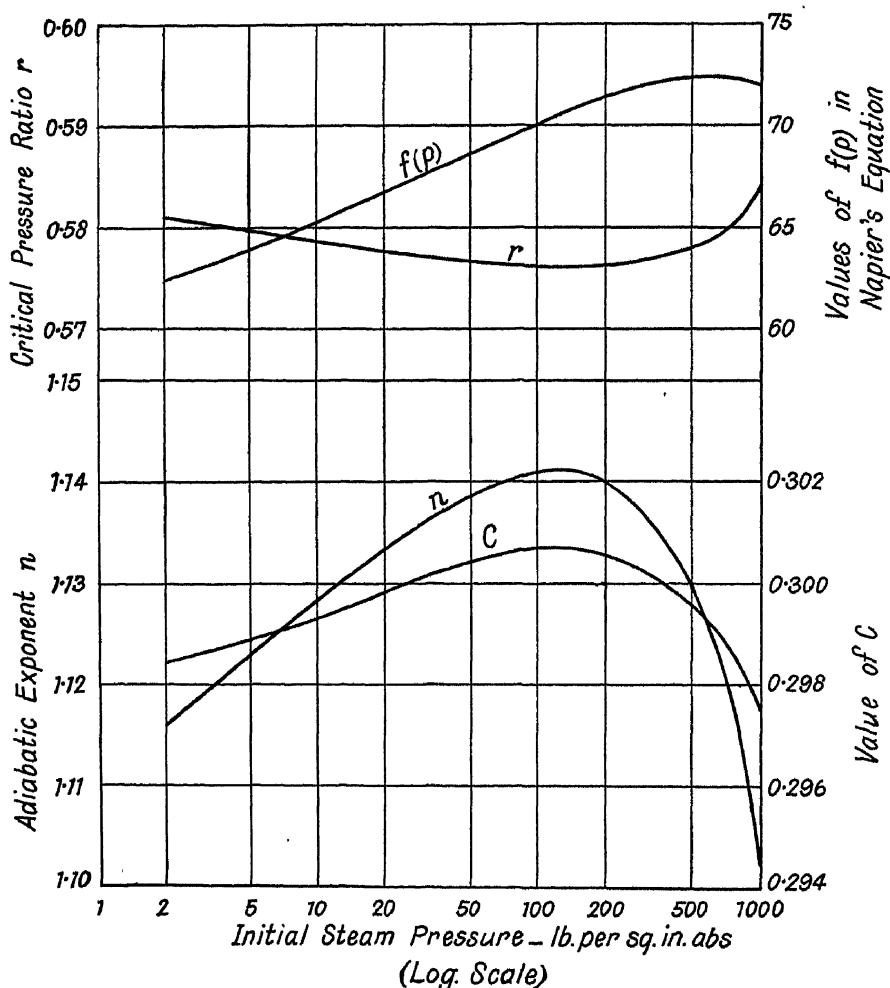


FIG. 37.—Nozzle Characteristics for Adiabatic Flow with Steam Initially Dry and Saturated

well-known equation $n = 1.035 + 0.1x_1$, where x_1 is the initial dryness fraction of the steam. This makes $n = 1.135$ for steam initially dry and saturated. Callendar found experimentally that when steam expands in the superheated condition, the relationship between p and v is given by the expression

$$p(v - b)^{1.3} = \text{constant} \quad . \quad . \quad . \quad (42)$$

where b is -0.0028 cub. ft per lb according to the Revised Calendar Tables (1931) Normally, b is so small compared with v that the expression may be written

$$pv^{1.3} = \text{constant} \quad . \quad (42a)$$

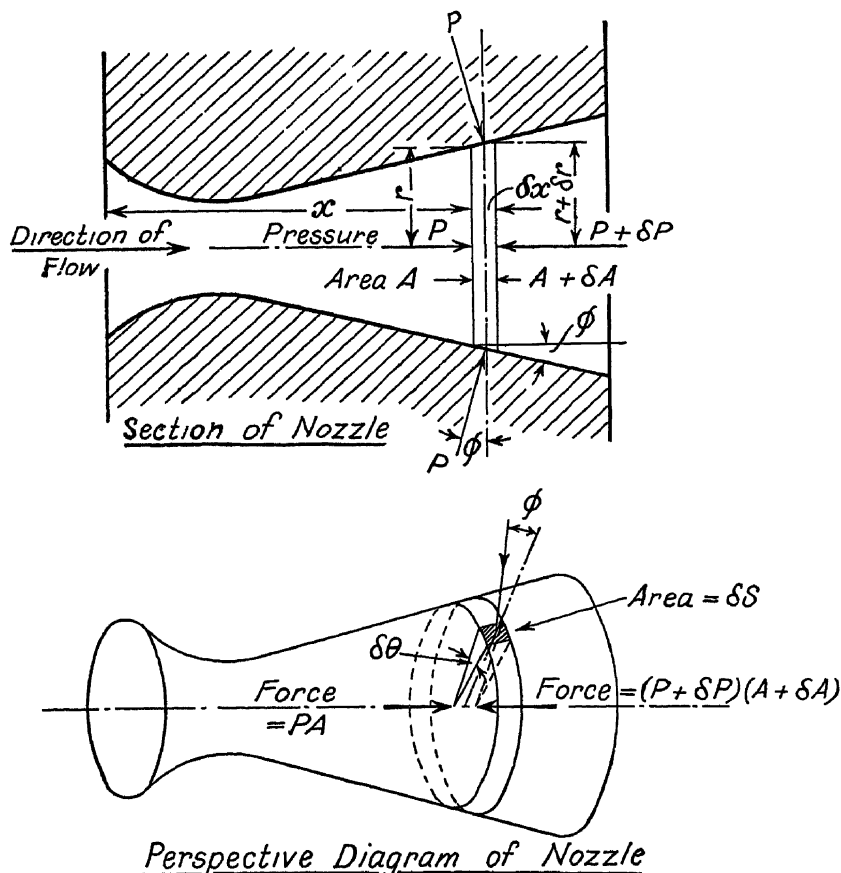


FIG. 38

40. Alternative Method of Calculating Velocity of Expanding Steam. Consider the flow of steam through the nozzle shown in Fig. 38, and let the surfaces of constant pressure be plane surfaces transverse to the axis of the nozzle. Consider a small mass of steam enclosed between two transverse sections of the nozzle δx apart. Let the pressure on the upstream side be P and that on the downstream side $P + \delta P$, δP being negative. Moreover, let it be assumed that the steam velocity across each section is uniform.

In actual fact, this is not true; the steam velocity near the walls of the nozzle varies across a relatively thin layer of fluid (the boundary layer) from zero at the surface of the wall to a value which is somewhat less than the velocity at the centre of the nozzle.

Let V = steam velocity at given section, ft. per sec.

v = specific volume of steam, cub. ft. per lb.

The mass of steam enclosed between the two transverse sections is being urged forward by the resultant axial force due to steam pressure. This resultant force is utilized to accelerate the mass of steam enclosed between the two sections considered. The mass enclosed is $\frac{A \cdot \delta x}{v}$ lb.

The forces on the enclosed mass are the following—

(1) The force on the upstream side = $P \cdot A$.

(2) The force on the downstream side

$$= (P + \delta P)(A + \delta A) = P \cdot A + P \cdot \delta A + A \cdot \delta P$$

the product $\delta P \cdot \delta A$ being neglected.

(3) The resultant of the force exerted by the boundary walls of the nozzle. The steam exerts a pressure on the nozzle wall, and equally the nozzle wall exerts a pressure on the steam. If the section of the nozzle considered lies in the diverging part of the nozzle, as shown in Fig. 38, then the force exerted by the nozzle wall on the steam is an accelerating force.

Consider the small element of area

$$\delta S = r \cdot \delta \theta \cdot \delta x \cdot \sec \phi$$

The normal force on the surface is $P \cdot r \cdot \delta \theta \cdot \delta x \cdot \sec \phi$, and the axial component of this force is

$$\begin{aligned} & P \cdot r \cdot \delta \theta \cdot \delta x \cdot \sec \phi \cdot \sin \phi \\ &= P \cdot r \cdot \delta \theta \cdot \delta x \cdot \tan \phi \end{aligned}$$

Then the total axial force exerted by the nozzle wall on the steam is

$$\begin{aligned} & 2\pi \cdot P \cdot r \cdot \delta x \cdot \tan \phi \\ &= 2\pi \cdot P \cdot r \cdot \delta r \\ &= P \cdot \delta A \end{aligned}$$

Therefore the resultant accelerating force

$$\begin{aligned} &= P \cdot A - P \cdot A - P \cdot \delta A - A \cdot \delta P + P \cdot \delta A \\ &= -A \cdot \delta P \end{aligned}$$

Since mass-acceleration = accelerating force

$$\frac{A \cdot \delta x}{v} \times \frac{d^2x}{dt^2} = -g \cdot A \cdot \delta P$$

or
$$\frac{\delta x}{\delta t} \frac{d^2x}{dt^2} = -g \cdot v \frac{\delta P}{\delta t}$$

Integrating, we have

$$\begin{aligned} \frac{1}{2} \left(\frac{dx}{dt} \right)^2 &= -g \int v \cdot \frac{dP}{dt} \cdot dt + \text{constant} \\ &= -g \int v \cdot dP + \text{constant} \end{aligned}$$

But
$$\frac{dx}{dt} = V$$

$\therefore \frac{V^2}{2g} = - \int v \cdot dP + \text{constant} \quad . \quad (43)$

It must be remembered that this equation is true only for frictionless flow. In deriving the expression we have not taken account of the retarding effect of friction.

The foregoing result may also be obtained as follows—

Since
$$I = E + \frac{Pv}{J}$$

$$dI = dH + \frac{v}{J} dP \quad (\text{See Art. 22})$$

For anadiabatic and frictionless flow, $dH = 0$, and therefore
$$dI = \frac{v}{J} dP$$

But it has already been shown (Art. 37) that

$$\begin{aligned} \frac{V_2^2 - V_1^2}{2gJ} &= I_1 - I_2 \\ &= \int_{I_2}^{I_1} dI \\ &= \frac{1}{J} \int_{P_1}^{P_2} v \cdot dP \end{aligned}$$

The value of the integral may be easily calculated if we assume that $P \cdot v^n$ is constant during the adiabatic expansion. The value is represented by the cross-hatched area in Fig. 39.

Since $Pv^n = C, v = \left(\frac{C}{P}\right)^{\frac{1}{n}}$

and
$$\int_{P_2}^{P_1} v \cdot dP = C^{\frac{1}{n}} \int_{P_2}^{P_1} P^{-\frac{1}{n}} \cdot dP$$

$$= \frac{n \cdot C^{\frac{1}{n}}}{n-1} \left(P_1^{\frac{n-1}{n}} - P_2^{\frac{n-1}{n}} \right)$$

$$= \frac{n}{n-1} (P_1 v_1 - P_2 v_2)$$

i.e.
$$\frac{V_2^2 - V_1^2}{2g} = \frac{n}{n-1} (P_1 v_1 - P_2 v_2) \quad (44)$$

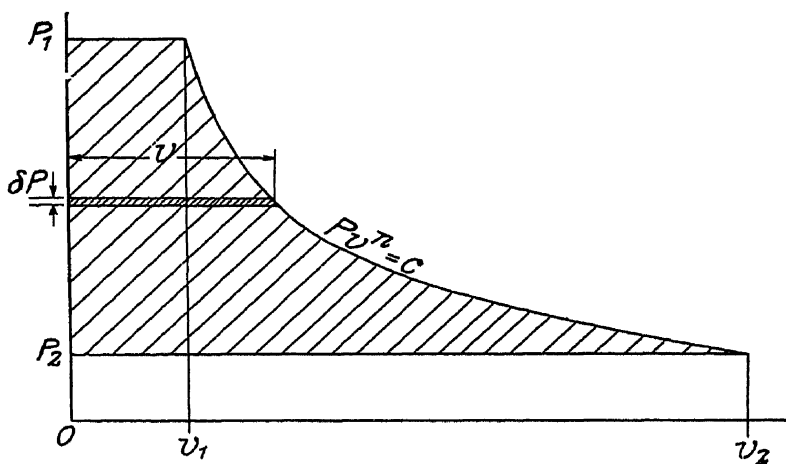


FIG. 39

Assuming expansion from rest ($V_1 = 0$)

$$V_2 = \sqrt{\frac{2gn}{n-1} (P_1 v_1 - P_2 v_2)}$$

$$= \sqrt{\frac{2g \cdot n}{n-1} \cdot P_1 v_1 \left\{ 1 - \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \right\}} \quad (45)$$

41. Proof of the Existence of a Critical Pressure in Nozzle Flow. Consider the expansion of steam from an initial condition P_1, v_1 , and from a state of rest. Assume further that the expansion is

adiathermal and frictionless, and that the relationship between pressure and volume may be represented by the expression $Pv^n = C$, where C is a constant

Then at any pressure P , the velocity V acquired by the steam is

$$V = \sqrt{\frac{2g}{n-1} \cdot P_1 v_1 \left\{ 1 - \left(\frac{P}{P_1} \right)^{\frac{n-1}{n}} \right\}}$$

Let the mass rate of flow be w lb. per sec and let the area be A at the section where the pressure is P Then, by the equation of continuity

$$\frac{w}{A} = \frac{V}{v}$$

and since

$$v = v_1 \left(\frac{P_1}{P} \right)^{\frac{1}{n}}$$

$$\begin{aligned} \frac{w}{A} &= \frac{1}{v_1} \left(\frac{P}{P_1} \right)^{\frac{1}{n}} \sqrt{\frac{2gn}{n-1} \cdot P_1 v_1 \left\{ 1 - \left(\frac{P}{P_1} \right)^{\frac{n-1}{n}} \right\}} \\ &= \sqrt{\frac{2gn}{n-1} \cdot \frac{P_1}{v_1} \left\{ \left(\frac{P}{P_1} \right)^{\frac{2}{n}} - \left(\frac{P}{P_1} \right)^{\frac{n+1}{n}} \right\}} \end{aligned}$$

g , n , P_1 , and v_1 are constants Hence the rate of flow per unit area is a function of the ratio of P to P_1 Write r for $\frac{P}{P_1}$

Then $\frac{w}{A}$ is a maximum when

$$r^{\frac{2}{n}} - r^{\frac{n+1}{n}} \text{ is a maximum}$$

i e. when
$$\frac{d}{dr} \left(r^{\frac{2}{n}} - r^{\frac{n+1}{n}} \right) = 0$$

or when
$$r = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}} \quad . \quad . \quad . \quad (46)$$

This is known as the critical pressure ratio and depends on the value of n .

42. Physical Explanation of Critical Pressure. The implications of the existence of a critical pressure in nozzle flow may be expressed in another way. Suppose we have two vessels A and B, A containing steam at a steady pressure P_1 Suppose that the pressure in B may be varied at will. A and B are separated by a diaphragm containing a convergent nozzle, as shown in Fig 40 (a).

Assume at first that P is equal to P_1 ; then there is no flow of steam through the nozzle. Now let P be gradually reduced. The discharge w through the nozzle will increase as shown by the curve, Fig. 40 (*b*), this curve being the left-hand branch of the discharge curve of Fig. 36. As the pressure P approaches the critical, the discharge rate gradually approaches its maximum value, and when P is reduced below the critical value, the discharge rate does not increase but remains at the same value as that at the critical pressure. The extraordinary result that P can be reduced well below the

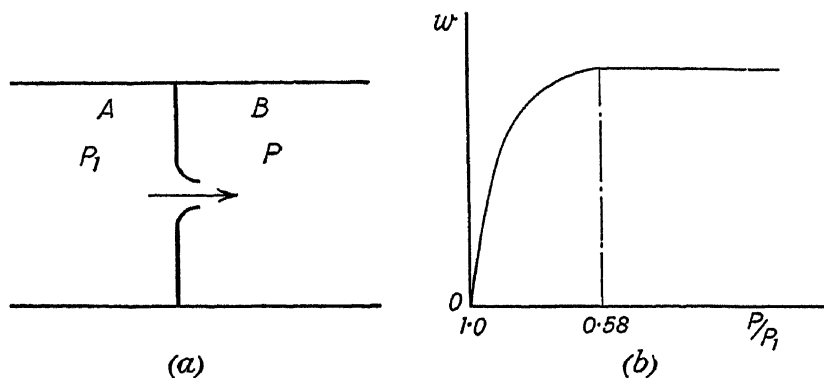


FIG. 40

critical pressure without influencing the mass flow was first discovered by R. D. Napier (3).

Again assuming flow from rest, the velocity reached at any pressure P is

$$V = \sqrt{\frac{2gn}{n-1} (P_1 v_1 - P v)}$$

$$= \sqrt{\frac{2g \cdot n}{n-1} \cdot P v \left\{ \left(\frac{P}{P_1} \right)^{\frac{1-n}{n}} - 1 \right\}}$$

If, now, we make the ratio

$$\frac{P}{P_1} = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}}$$

which makes P the critical pressure, then

$$V = \sqrt{\frac{2gn}{n-1} \cdot P_v \cdot \frac{n-1}{2}}$$

$$\text{or} \quad V = \sqrt{g \cdot n \cdot P \cdot v} \quad . \quad . \quad . \quad (47)$$

Since n is the adiabatic index, eq. (47) represents the velocity of sound in steam at pressure P and density $\rho = 1/v$. It appears, therefore, that the velocity of steam in adiabatic and frictionless flow reaches a critical value, i.e. at the nozzle throat, equal to the velocity of sound in steam at that point. In other words the critical velocity is equal to the local acoustic velocity. In fact, if the curves of steam velocity and acoustic velocity be drawn for a convergent-divergent nozzle, Fig 41, it will be found that the curves intersect at the critical pressure

The explanation of this, first expounded by Professor Osborne Reynolds (4), is as follows. Consider steam flowing through a tube or nozzle with a velocity V , and let S be the sonic velocity in steam at the given point, S being a function of the pressure and density of the steam. Then the velocity with which a disturbance such as, for example, a sudden change of pressure, will be transmitted back through the flowing steam will be $S - V$. Referring to Fig 41, let the final pressure P at the outlet of a converging nozzle be, say, 0.8 of P_1 . Here $S - V$ is clearly positive and any change in the pressure P would produce a change in the rate of flow. When the pressure has been reduced to the critical value, $S - V$ becomes zero and any further reduction of pressure in the space surrounding the nozzle outlet has no effect on the pressure P at the outlet section or on the rate of flow.

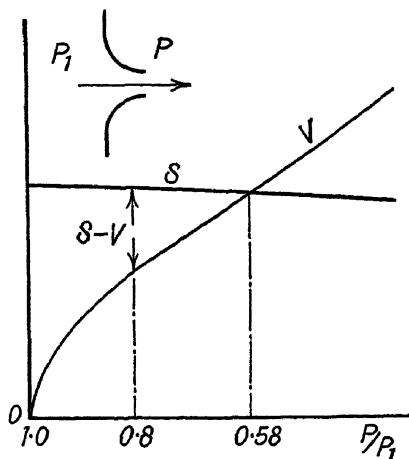


FIG 41

43. Maximum Discharge of Saturated Steam. The value of the critical pressure ratio given by eq. (46) has been calculated for steam initially dry and saturated from the values of n given in Fig 37 and plotted in the same figure. It will be observed that the variation in the value of r is quite small. If the value of r be substituted in the equation

$$\frac{w}{A} = \sqrt{\frac{2g \cdot n}{n-1} \cdot \frac{P_1}{v_1} \left\{ 1 - r^{\frac{2}{n}} - r^{\frac{n+1}{n}} \right\}}$$

together with the appropriate value of n , the flow rate

$$w \text{ lb per sec.} = C \cdot A_t \sqrt{\frac{P_1}{v_1}} \quad (48)$$

where C is a factor

A_t = throat area of nozzle, sq. in.

p_1 = initial pressure, lb. per sq. in. abs.

v_1 = initial specific volume, cub. ft. per lb.

Values of C are plotted in Fig. 37 as a function of the pressure. The maximum variation over the range from 2 to 1,000 lb. pressure is only about 1 per cent. A mean value of 0.3 may be taken, so that

$$w = 0.3 A_t \sqrt{\frac{p_1}{v_1}} \quad . \quad . \quad . \quad . \quad (49)$$

An approximate relationship between p_1 and v_1 for dry and saturated steam is

$$p_1 v_1^{\frac{16}{15}} = 490 \quad . \quad . \quad . \quad . \quad (50)$$

Substituting for v_1 in eq. (49), we have

$$w = 0.0164 p_1^{\frac{31}{32}} \cdot A_t \quad . \quad . \quad . \quad . \quad (51)$$

This expression shows that w is nearly proportional to p_1 . The expression for w may be written

$$w = \frac{p_1 \cdot A_t}{f(p_1)} \quad . \quad . \quad . \quad . \quad (52)$$

where $f(p_1)$ is a quantity which is a function of p_1 . Values of $f(p_1)$ have been calculated and are plotted in Fig. 37. Equation (52) is another form of the so-called Napier's equation, for which $f(p_1)$ is normally taken as a constant and given the value 70, which corresponds to the theoretical value for $p = 100$.

Equations (49), (51), and (52) give the maximum theoretical discharge with adiabatic and frictionless flow of steam which is initially dry and saturated and when the pressure in the space surrounding the nozzle outlet (provided the nozzle is of the convergent type) is equal to or less than the critical pressure. If the pressure to which the steam expands is greater than the critical, then the nozzle area or mass flow must be found by the method described in Art. 38.

44. Mouth Area of Convergent-divergent Nozzles. Consider the flow of steam through a convergent-divergent nozzle. In the following expressions, let

Suffix 1 refer to inlet conditions

„ t refer to throat conditions

„ m refer to mouth conditions

and let all pressures P be in lb. per sq. ft., volumes v in cub. ft. per lb., and areas in sq. ft.

Then $V_m = \sqrt{\frac{2g \cdot n}{n-1} \cdot P_1 v_1 \left\{ 1 - \left(\frac{P_m}{P_1} \right)^{\frac{n-1}{n}} \right\}}$

$$w = \frac{A_m \cdot V_m}{v_m} \quad \text{But } v_m = v_1 \left(\frac{P_1}{P_m} \right)^{\frac{1}{n}}$$

$$\begin{aligned} \therefore w &= \frac{A_m}{v_1} \left(\frac{P_m}{P_1} \right)^{\frac{1}{n}} \sqrt{\frac{2g \cdot n}{n-1} \cdot P_1 v_1 \left\{ 1 - \left(\frac{P_m}{P_1} \right)^{\frac{n-1}{n}} \right\}} \\ &= A_m \sqrt{\frac{2gn}{n-1} \frac{P_1}{v_1} \left\{ \left(\frac{P_m}{P_1} \right)^{\frac{2}{n}} - \left(\frac{P_m}{P_1} \right)^{\frac{n+1}{n}} \right\}} \end{aligned}$$

But, by eq. (49) $w = (0.3 \times 144) A_t \sqrt{\frac{P_1}{144 v_1}}$

i.e. $w = 3.6 A_t \sqrt{\frac{P_1}{v_1}}$

If there is steady flow, w is the same for all sections of the nozzle at any time.

$$\text{Hence } \frac{A_m}{A_t} = \frac{3.6 \sqrt{\frac{P_1}{v_1}}}{\sqrt{\frac{2gn}{n-1} \frac{P_1}{v_1} \left\{ \left(\frac{P_m}{P_1} \right)^{\frac{2}{n}} - \left(\frac{P_m}{P_1} \right)^{\frac{n+1}{n}} \right\}}}$$

Substituting 32.2 for g and 1.135 for n

$$\frac{A_m}{A_t} = \frac{0.155}{\sqrt{\left(\frac{P_m}{P_1} \right)^{\frac{2}{n}} - \left(\frac{P_m}{P_1} \right)^{\frac{n+1}{n}}}} \quad (53)$$

Values of the ratio $\frac{A_m}{A_t}$ have been calculated for values of $\frac{P_m}{P_1}$; the results are shown graphically in Fig 42

45. Maximum Discharge of Steam Initially Superheated. Provided that steam is supplied with an initial superheat of not less than 72° F. it will remain superheated until the throat pressure is

reached. In this case $n = 1.3$ and the corresponding value of r is 0.5457. Substituting these values in the expression

$$\frac{w}{A} = \sqrt{\frac{2gn}{n-1} \cdot \frac{P_1}{v_1} \left\{ r^{\frac{2}{n}} - r^{\frac{n+1}{n}} \right\}}$$

$$w = 0.3155 A_t \sqrt{\frac{p_1}{v_1}} \quad (54)$$

where w , A_t , p_1 and v_1 have the same meanings as in Art. 43.

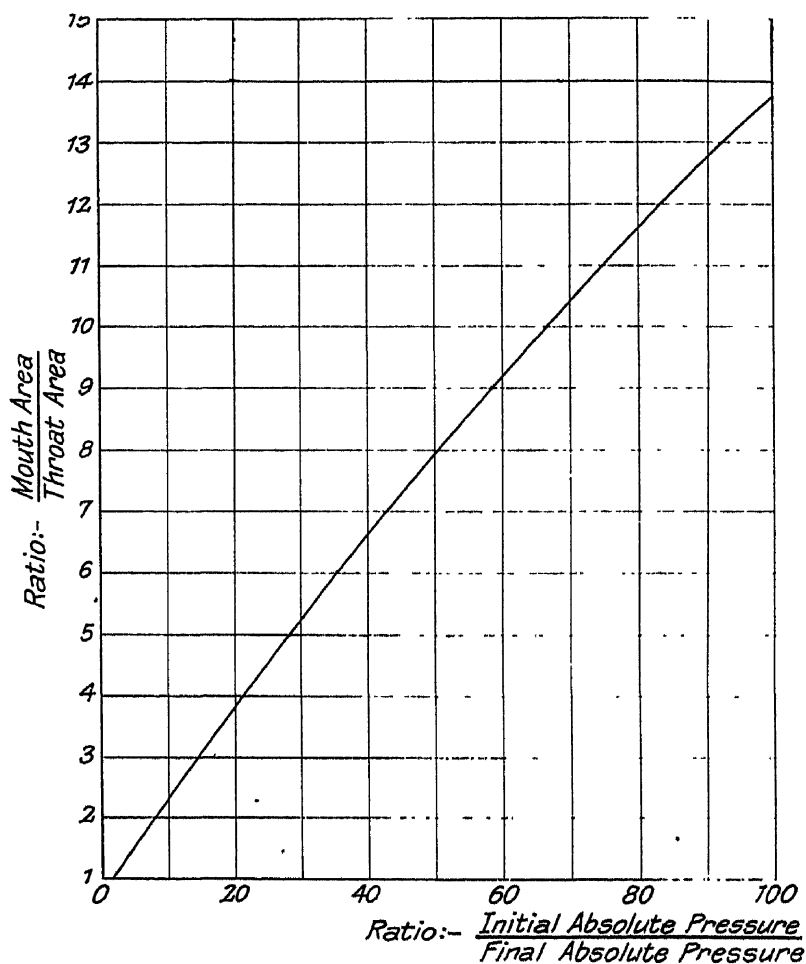


FIG. 42

46. Effect of Friction in Nozzle Flow. In the foregoing sections of this chapter we have considered the flow of steam through nozzles when the flow takes place without friction. The analysis is simplified by the assumption of frictionless flow and leads to important conclusions regarding flow phenomena. In nature, however, the flow is never frictionless and adiabatic, and we must know how to take friction into account

It is hardly necessary to state at the outset that with a given nozzle operating with steam expanding between given limits of pressure and temperature, the effects of friction are to reduce the efflux velocity and also the mass flow below the adiabatic values

Let I_1 = initial enthalpy at pressure p_1

V_1 = initial velocity of steam

I_2, x_2, V_2 be the enthalpy, dryness fraction, and velocity, respectively, at pressure p_2 , after adiabatic and frictionless expansion

I_2', x_2', V_2' be the values after expansion to p_2 with friction

Then, assuming adiathermal flow ($H = 0$) and applying the steady flow eq (38), the adiabatic heat drop is

$$I_1 - I_2 = \frac{1}{2gJ} (V_2^2 - V_1^2)$$

The change of enthalpy during the actual flow is

$$I_1 - I_2' = \frac{1}{2gJ} \{(V_2')^2 - V_1^2\}$$

If we define the ratio of the actual gain in kinetic energy to the theoretical gain in kinetic energy as the nozzle efficiency η_n , then it follows that

$$\eta_n = \frac{I_1 - I_2'}{I_1 - I_2} \quad (55)$$

In Fig 43, AB represents the adiabatic and frictionless expansion from p_1 to p_2 and AC the frictionally resisted flow. The path AC of the state point depends on the nature of the losses in the nozzle. With a well-shaped inlet and the losses occurring chiefly near the outlet, the path would be represented by a curve such as ADC. If the losses occurred mainly near the inlet, then the locus would be a curve such as AEC. Friction causes internal heating of the steam with a consequent increase of entropy

It will be seen that the final dryness fraction is increased by friction from a value x_2 to some value x_2^1 . If v_{s2} is the specific volume

of steam at pressure p_2 , and A_2 , A_2' are the outlet areas for the adiabatic and actual flows respectively, then

$$A_2 = \frac{w \cdot x_2 v_{s2}}{V_2} \quad . \quad . \quad . \quad (56)$$

$$A_2' = \frac{w \cdot x_2' \cdot v_{s2}}{V_2'} \quad . \quad . \quad . \quad (56a)$$

Since $V_2' < V_2$ and $x_2' > x_2$, $A_2' > A_2$, but chiefly on account of the reduction in velocity.

When steam expands wholly in the superheated state, friction

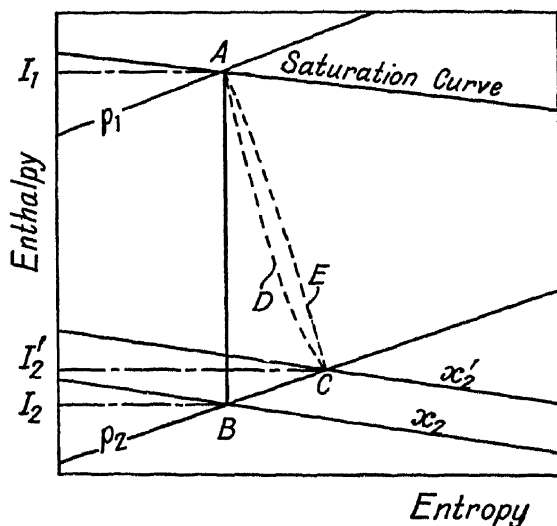


FIG. 43

increases the temperature of the steam at the nozzle outlet and hence increases the specific volume.

Friction alters slightly the value of the index n in the expression $Pv^n = \text{constant}$, but the effect on the critical pressure ratio is very slight and may be neglected.

In an expansion from rest, the nozzle efficiency

$$\eta_n = \frac{(V_2')^2}{V_2^2}$$

Writing η_v = velocity coefficient for nozzle (not a true efficiency)

$$\text{i.e.} \quad \eta_v = \frac{V_2'}{V_2}$$

$$\text{then} \quad \eta_n = \eta_v^2$$

EXAMPLE 5. Dry and saturated steam enters a nozzle at a pressure $p = 110$ and expands from rest to the critical pressure $p = 63.5$. Using the Mollier diagram, obtain the following—

- (a) Outlet velocity.
- (b) Final dryness fraction.
- (c) Nozzle outlet area for a mass flow of 0.05 lb per sec
- (d) Nozzle diameter for a round nozzle

Assume in Case I that the flow is frictionless, and in Case II that the nozzle efficiency is 0.9

Case I. Frictionless Expansion The following data are obtained from the Mollier diagram—

p	x	ϕ	I
110	1.0000	1.6004	1192.6
63.5	0.9643	1.6004	1148.8
			43.8

Then, since $V_1 = 0$, $V_2 = 223.8 \sqrt{43.8} = 1,481$ ft/sec By interpolation from steam tables, $v_{s2} = 6.791$

$$\begin{aligned} \text{Hence} \quad A_2 &= (0.05 \times 0.9643 \times 6.791) - 1,481 \\ &= 0.0002212 \text{ sq ft} \end{aligned}$$

from which the diameter required is 0.2014 in

Case II. Flow with Friction. Since the nozzle efficiency is 0.9, the actual heat drop is $0.9 \times 43.8 = 39.42$. Hence $I_2' = 1192.6 - 39.42 = 1153.18$. Thus the conditions at the nozzle outlet (point C, Fig. 43) are as follows—

p	x	ϕ	I
63.5	0.9694	1.6062	1153.18

$$\text{Then} \quad V_2' = 223.8 \sqrt{39.42} = 1,405 \text{ ft per sec}$$

$$\begin{aligned} \text{Hence} \quad A_2' &= (0.05 \times 0.9694 \times 6.791) - 1,405 \\ &= 0.0002344 \text{ sq ft.} \end{aligned}$$

from which the diameter required is 0.2073 in. The increase in nozzle area is 6.0 per cent; in other words, the discharge would, theoretically, be about 6.0 per cent less than with adiabatic and frictionless expansion

The effect of inlet velocity is not so great as it might seem on first considerations. This will be made clear by the following example

EXAMPLE 6. Using the data given in Example 5, but assuming an inlet velocity of 400 ft. per sec, calculate the final velocities and nozzle diameters for both frictionless and frictionally resisted expansions.

Case I. Adiabatic Expansion. Since the adiabatic heat drop is 43.8 B.Th.U. per lb.

$$\begin{aligned}\frac{V_2^2 - V_1^2}{2gJ} &= 43.8 \\ \therefore \frac{V_2^2}{2gJ} &= 43.8 + \frac{(400)^2}{2gJ} \\ &= 47.0\end{aligned}$$

from which $V_2 = 1,534$. That is, an increase of 400 ft. per sec. in the initial velocity causes an increase of only 53 ft. per sec. in the outlet velocity. The quality of the steam, and hence the specific volume, are unaffected by the initial velocity. Therefore, the nozzle area is 0.0002135 sq. ft. and the diameter 0.1978 in.

Case II. Frictionally Resisted Flow.

$$\begin{aligned}\text{Here } \frac{(V_2')^2 - V_1^2}{2gJ} &= I_1 - I_2' \therefore 39.42 \\ \therefore \frac{(V_2')^2}{2gJ} &= 39.42 + \frac{(400)^2}{2gJ} \\ &= 42.62\end{aligned}$$

from which $V_2' = 1,461$. With the definition of nozzle efficiency given in eq. (55), the condition of the steam and, therefore, its specific volume are the same as in the previous example. Substituting the appropriate values in eq. (56a), we have the nozzle area equals 0.0002254 sq. ft. and the outlet diameter 0.2032 in.

47. Experimental Measurements of Steam Discharge through Nozzles. Experiments on this subject date from about 1870 and the majority have been made on nozzles of simple form, e.g. converging nozzles with rectilinear axes and outlet sections perpendicular to those axes. In most cases the mass flow has been measured directly by condensing the steam discharged by the nozzle and weighing the condensate, but in some cases indirect methods of measurement have been employed.

Experiments with Initially Dry and Saturated Steam. R. D. Napier, in a number of letters to *The Engineer* (3), discusses the subject at considerable length and gives a number of experimental results from which the following figures have been taken. These refer to tests in which the final pressure was lower than the critical. The figures given in the second column of the table on p. 91 have been calculated by the author using eq. (52) and values of $f(p_1)$ taken from Fig. 37. It will be observed that the measured discharge is greater than the calculated discharge in every case, notwithstanding the fact that the experimental flow would be accompanied by losses. In some experiments made by H. Parenty (6) on the flow of initially dry and saturated steam through orifices it was also observed that

p_1 lb per sq in abs	Discharge of Steam per sq in of Nozzle Outlet Area per sec		Ratio $\frac{\text{Experimental}}{\text{Calculated}}$
	Calculated	Experimental	
30	0 445	0 451	1 013
40	0 587	0 610	1 039
45	0 659	0 682	1 035
50	0 729	0 753	1 033

the discharge was either practically equal to, or slightly greater than, the theoretical discharge

Probably the most careful and searching experiments on the subject were made by Professor Rateau (7) in the year 1895. The nozzles used are shown in Fig. 44. A and C were convergent nozzles with a curved profile, while B was a conical convergent nozzle. The experimental results are also shown in Fig. 44, the quantity

$\frac{w}{p_1 A_t}$ being plotted against p_1 plotted to a log scale. The theoretical curve for saturated steam is based on eq. (52) and the values of $f(p_1)$ plotted in Fig. 37. Here again it is clear that, with few exceptions, the experimental discharge is in excess of the theoretical. These results will be further discussed in Art. 49.

Experiments with Superheated Steam Particulars of the mass flow with superheated steam are given in a paper by Professors Mellanby and Kerr (8). The flow through several different types of nozzles was investigated by means of a search tube and the mass flow observed by condensing the steam and weighing the condensate. We shall refer only to the results of experiments made on a simple convergent nozzle having a diameter of 0.2507 in. and an axial length of 0.25 in. Two series of tests were made on this nozzle, one series with a relatively high superheat and the other with a superheat which, though relatively low, is sufficiently high to maintain the steam in the superheated condition down to the critical pressure. The results are given in the table on p. 93. The coefficients C_t and C_e are, respectively, the theoretical and experimental values of the coefficient C in the expression

$$w = C \cdot A_t \cdot \sqrt{\frac{p_1}{v_1}}$$

Hence the ratio of C_e to C_t may be described as the coefficient of discharge. It will be seen that this is always lower than unity, i.e. the observed flow is always less than that calculated theoretically. In the case of the other nozzles tested, the measured flow was still lower than the theoretical flow.

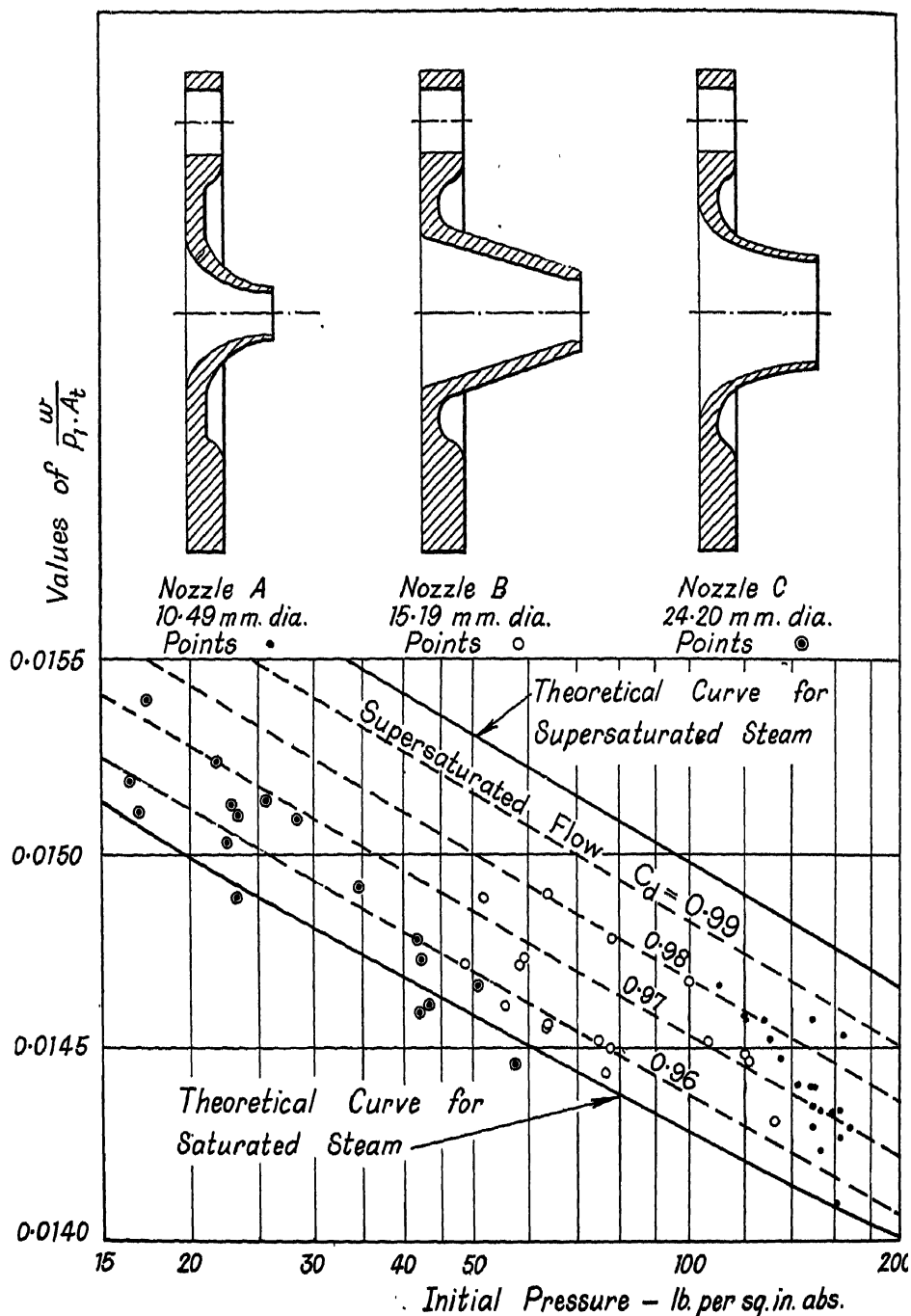


FIG. 44.—Rateau's Experiments on the Flow of Steam through Nozzles

p_1	t_1 °F	Initial Superheat °F	Ratio $\frac{p_2}{p_1}$	C_t	C_e	Ratio $\frac{C_e}{C_t}$
76.0	559	250.6	0.580	0.315	0.306	0.971
76.0	565	256.6	0.574	0.315	0.306	0.971
76.0	568	259.6	0.593	0.314	0.304	0.969
76.0	569	260.6	0.710	0.294	0.285	0.970
76.0	567	258.6	0.843	0.239	0.230	0.962
75.7	385	76.6	0.574	0.315	0.309	0.980
75.6	395	86.6	0.570	0.315	0.308	0.976
75.7	402	93.6	0.589	0.314	0.307	0.979
75.7	388	79.6	0.692	0.300	0.290	0.967
75.7	399	90.6	0.837	0.245	0.230	0.942

48. **Supersaturation in Steam Turbine Nozzles.** Attention has been drawn in the preceding article to the astonishing fact that when steam, initially dry and saturated, is caused to expand in a nozzle the actual measured steam flow in lb. per sec is in excess of that given by a theoretical analysis and expressed by eqs (49), (51), or (52). In view of the friction losses which occur in nozzles, it would be expected that the actual discharge would be less than the theoretical value, the ratio being approximately equal to the velocity coefficient η_v . Thus we would expect the actual mass flow to be about 3 to 5 per cent less than the theoretical, whereas experiments have shown it to be from 1 to 3 per cent greater. On the other hand, the actual discharge of superheated steam is in good agreement with the theoretical value when allowance is made for friction losses. This remarkable difference between the behaviour of dry and saturated steam and that of superheated steam is due to the fact that, during the rapid expansion of initially dry and saturated steam in a nozzle, the state or condition of the steam is not that visualized in the simple theory based on adiathermal expansion in thermal equilibrium. This theory presupposes that at every point in the expansion, condensation proceeds at the correct rate and that the heat given up by the steam due to this condensation has all been liberated. By condensation we do not mean condensation which might take place on the walls of nozzles or blades, but true condensation within the flowing steam itself, i.e. the sudden coalescence of steam molecules to form minute droplets of water.

Consider the vapour pressure in the neighbourhood of such a minute water droplet. The vapour pressure of water at any given temperature is that corresponding to a flat water surface. The attractive force exerted by a liquid molecule on the surrounding molecules is only effective over a very small distance, say a , Fig. 45. Hence a molecule A in the surface of the water is held by the

attraction of the molecules lying within a hemisphere of radius a . Consider now a molecule B lying in the surface of a droplet of radius r . It will be clear that this molecule is held by a smaller force since it loses the attractive forces of the molecules lying within the sphere of action and between the outer surface of the droplet and the tangent plane at B, i.e. in the space indicated by the cross-hatching. In consequence, *at any given temperature*, the forces which tend to maintain a molecule within a liquid droplet are reduced as the droplet radius r is made smaller. This means that a greater number of molecules will escape into the space surrounding the droplet than would be the case with a flat liquid surface. Now the vapour pressure, being proportional to the number of vapour molecules per unit volume at the given temperature, must be greater in the space surrounding the droplet when there is equilibrium than it would be at a flat liquid surface. In consequence, the molecules of

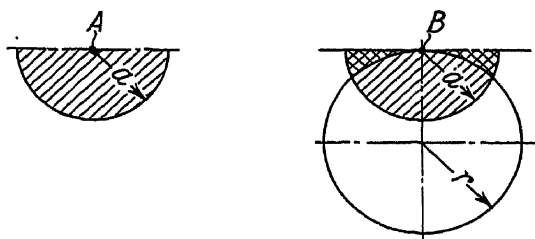


FIG. 45

a very small water droplet will go on evaporating into a space that is saturated, and the space will therefore become *super-saturated* with water vapour. This condition is commonly described by saying that the steam is supersaturated.

It follows from the foregoing statements that during rapid expansion pure steam will not condense of itself, for if a very minute droplet were accidentally formed it would immediately evaporate. Consequently, it is quite feasible to expect that when pure steam, initially dry and saturated, is expanded rapidly it will remain dry. Under these conditions the steam expands more or less like a perfect gas and the frictionless adiabatic expansion causes its temperature to fall well below the saturation temperature corresponding to the pressure. Hence such steam is sometimes described as being *undercooled*.

Where there is a large number of dust particles in the flowing steam these may act as nuclei on which the steam could condense. In turbine practice there is little doubt that the steam does contain such nuclei, but, to judge from the numerous experiments made with ordinary boiler steam, the number is much too small to effect condensation and the steam appears to behave as though it were dust-free.

There is, however, a limit to the expansion as a supersaturated vapour. Mr. C. T. R. Wilson (9) made experiments on the sudden expansion of air and water vapour and found that when dust-free air saturated with water vapour was rapidly expanded, no condensation took place until a certain ratio of expansion $\frac{v_2}{v_1} = 1.25$

had been reached, when a rain-like condensation occurred. When the ratio of expansion had been further increased to 1.375 a very dense cloud of mist particles was formed.

Professor Stodola (10) made some interesting experiments in which steam was allowed to flow through a glass nozzle illuminated by an arc lamp. The nozzle was of the convergent-divergent type and it was found that a mist was first formed at a point just beyond the throat. With an initial pressure $p_1 = 71$, the pressure at the point of condensation was about 38.5 i.e. about 0.542 times the initial pressure.

49. Equations for Supersaturated Steam. Undercooled steam behaves like superheated steam and the equation for the adiabatic expansion of supersaturated vapour is

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \quad (57)$$

where p_1 and p_2 are the initial and final pressures, and T_1 and T_2 are the initial and final absolute temperatures respectively. Since $n = 1.3$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{3}{13}} \quad (57a)$$

In Stodola's experiment quoted above, $p_1 = 71$, $T_1 = 763.4^\circ \text{F abs}$, and $p_2 = 38.5$. Hence $T_2 = 663.3$, or $t_2 = 203.7^\circ \text{F}$. The saturation temperature corresponding to $p = 38.5$ is 264.9°F . Thus the steam at the point of condensation in the nozzle was undercooled by 61.2°F .

The saturation pressure p_s , corresponding to the temperature 203.7°F , is 12.4, whereas the actual pressure p_{ss} of the supersaturated steam was observed to be 38.5

Let ρ_s = density of saturated vapour

ρ_{ss} = density of supersaturated vapour

Then the degree of supersaturation S is defined as the ratio

$$S = \frac{\rho_{ss}}{\rho_s} \quad (58)$$

Since, at the same temperature, $\frac{\rho_{ss}}{\rho_s} = \frac{p_{ss}}{p_s}$, it follows that

$$S = \frac{p_{ss}}{p_s} \quad (59)$$

In Stodola's experiment the degree of supersaturation reached was therefore 3.1.

More recently, Mr. J. I. Yellott, junr. (11), has carried out experiments on the flow of steam through a glass-sided nozzle. The observed degree of supersaturation varied from 3.05 to 5.64, according to the initial steam pressure. Mr. A. M. Binnie and Dr. M. W. Woods (12), experimenting with a brass nozzle, found that the limiting degree of supersaturation varied from 4.1 to 5.0.

Callendar's adiabatic equation for dry steam, which applies to supersaturated steam as well as to superheated steam, is

$$P(v - b)^{1.3} = \text{Constant}$$

As pointed out in Art. 39, b is normally so small compared with v that it may be neglected. Hence $Pv^{1.3} = \text{constant}$. Then the critical pressure with adiabatic expansion of supersaturated steam is 0.5457 times the initial pressure, and the maximum discharge of steam is given by eq. (54), i.e.

$$w = 0.3155 A_t \sqrt{\frac{p_1}{v_1}}$$

If the steam is initially dry and saturated, we can substitute for v_1 from the equation

$$p_1 v_1^{\frac{16}{15}} = 490$$

from which

$$w = 0.01730 p_1^{\frac{31}{32}} \cdot A_t \quad . \quad . \quad . \quad (60)$$

It will be seen that the discharge given by this formula is about 5 per cent greater than that given by eq. (51); and if 3 or 4 per cent be deducted to allow for the effect of friction, the actual discharge will still be 1 or 2 per cent in excess of that based on adiabatic expansion in the saturated condition, which result agrees with those of the experimenters already mentioned.

Returning for a moment to Rateau's experiments described in Art. 47, the theoretical curve for supersaturated steam, based on eq. (60), has been drawn in Fig. 44, together with curves for varying coefficients of discharge. It will be seen that Rateau's experimental results are in agreement with the assumption of supersaturated flow and the reasonable value of C_d varying from about 0.96 to 0.97.

Consider now the adiabatic heat drop with supersaturated flow from an initial pressure P_1 to a final pressure P_2 . Since by eq. (8)

$$dI = dH + \frac{v}{J} \cdot dP$$

and

$$dH = 0. \quad dI = \frac{v}{J} \cdot dP$$

Work done per second

$$= \frac{w_1 \cdot V_1^2}{gJ} (2\rho \cos \alpha - \rho^2)$$

Total adiabatic heat drop per second

$$\begin{aligned} &= w(h_n + h_b) \\ &= \frac{w \cdot V_1^2}{g \cdot J} \frac{1}{\eta_n} \{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)\} \end{aligned}$$

Since the leakage loss is practically the only loss affecting the efficiency of a reaction turbine stage -- apart from blade friction and shock losses which have already been considered and whose effects are included in the above expressions—we may write—

Stage efficiency

$$\eta_s = \frac{w_1}{w} \frac{\eta_n(2\rho \cos \alpha - \rho^2)}{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)} \quad . \quad . \quad (100)$$

If ordinary unshrouded blades are used then, since

$$\begin{aligned} \frac{w_1}{w} &= 1 - \frac{3c}{l} \\ \eta_s &= \left(1 - \frac{3c}{l}\right) \frac{\eta_n(2\rho \cos \alpha - \rho^2)}{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)} \quad . \quad (101) \end{aligned}$$

EXAMPLES VIII

1 The following particulars refer to a stage of a Parsons steam turbine comprising one ring of fixed blades and one ring of moving blades—

Mean diameter of blade ring	26 in
R p m	3,000
Steam velocity at exit from blades	525 ft per sec
Blade outlet angle	20°
Steam flow through blades	15.5 lb per sec

Draw the velocity diagram and derive the following—

- (a) Blade inlet angle
- (b) Tangential force on ring of moving blades
- (c) Horse-power developed in stage.

2 In a stage of an impulse-reaction turbine operating with half-degree reaction, the fixed and moving blades are of identical section and the outlet angle is 18°. The absolute velocity of discharge from the moving blades is 130 ft per sec. in a direction 110° to the direction of motion of the blades, and the change of velocity produced by the moving blades is parallel to that direction.

Draw the velocity diagram and derive—

- (a) Mean velocity of the blades.
- (b) Ratio of blade velocity to the velocity of efflux from the fixed blades.
- (c) Work done per lb. of steam

$$\begin{aligned}
 I_1 - I_2 &= 0.8024 p_1 v_1 \left\{ 1 - \left(\frac{p_2}{p_1} \right)^{\frac{3}{13}} \right\} \\
 &= 0.8024 \times 20 \times 20.08 (1 - 0.8521) \\
 &= \underline{47.66 \text{ B.Th.U. per lb.}}
 \end{aligned}$$

50. **The Wilson Line.** Constant-pressure lines for supersaturated steam on the ϕ - I chart are continuations of the superheat lines, as

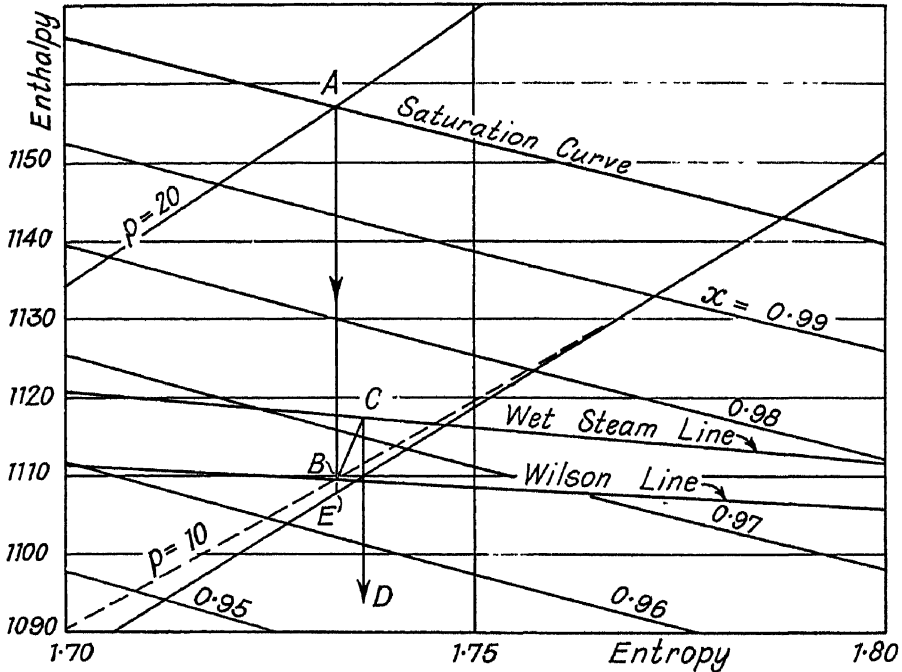


FIG. 46

shown by the dotted line in Fig. 46. The expansions worked out in the previous example are shown, AE representing the saturated expansion and AB the supersaturated expansion. In a series of articles (13) on this subject, the late Mr. H. M. Martin suggested that the locus of the points on the Mollier diagram representing the limit of supersaturated flow should be termed the Wilson line (after C. T. R. Wilson).

It may be shown that if steam is flowing through a convergent-divergent nozzle suited for the terminal pressures then there must be a rise in pressure when condensation takes place. This rise in pressure was clearly observed by Binnie and Woods, and is shown in Fig. 46 by the sloping line BC. The subsequent adiabatic expansion of the steam (now saturated) is shown by the isotropic CD.

The expansion of steam in a turbine takes place with such rapidity that it is doubtful whether, after condensation has actually occurred, it goes on at such a rate as to maintain thermal equilibrium at every point in the expansion

51. Jet Deflection in Turbine Nozzles. In the preceding articles of this chapter it has been shown that there exists a certain critical pressure in nozzle flow. With steam initially dry and saturated this pressure is about 0.58 of the initial pressure, whereas with superheated steam it is about 0.55 of the initial pressure. The practical significance of the critical pressure lies in the fact that if expansion takes place in a nozzle to a pressure which is equal to or greater than the corresponding critical pressure, then the nozzle must be convergent, the mass flow being controlled by the outlet area. On

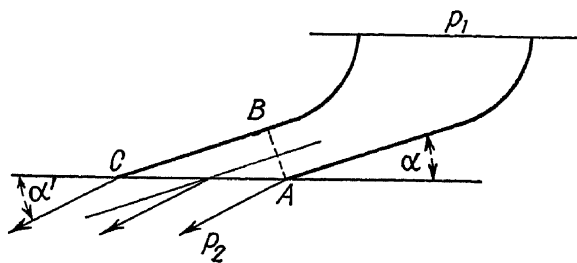


FIG 47

the other hand, if the expansion is to extend to a pressure lower than the critical pressure, then the nozzle must be of the convergent-divergent type and, provided the pressure on the outlet side is not too high, the mass flow will be controlled by the throat area.

In view of the fact that the nozzle must be inclined to the plane of the wheel, the turbine nozzle must be provided with an oblique outlet. Fig 47 shows diagrammatically such a nozzle of the convergent type. Provided the steam pressure p_2 on the outlet side of the nozzle is not less than the critical pressure, then the steam jet will issue at an angle equal to the vane angle α . In practice it is found that with thick guide vanes the jet angle may be less than the vane angle. If, however, the pressure p_2 is much lower than the critical pressure, then there will be an expansion in the wedge-shaped space ABC and the jet will issue at an angle α' greater than α .

Consider now a convergent-divergent nozzle designed for an initial pressure p_1 and an outlet pressure p_2 . If the cross-sectional area at the section AB, Fig 48 (a), is made equal to the calculated outlet area A_2 , and if the wall BC is made parallel to the nozzle axis, then, provided the actual pressure p_2' on the outlet side is equal to p_2 , the steam will flow through the wedge-shaped portion ABC at sensibly constant pressure and parallel to the nozzle axis

If, however, p_2' is lower than p_2 there will be expansion in the wedge ABC and the jet will undergo a certain deflection $\alpha' - \alpha$.

When the nozzle is made divergent for the full length, as shown in Fig. 48 (b), then if the section at AB is made equal to the required

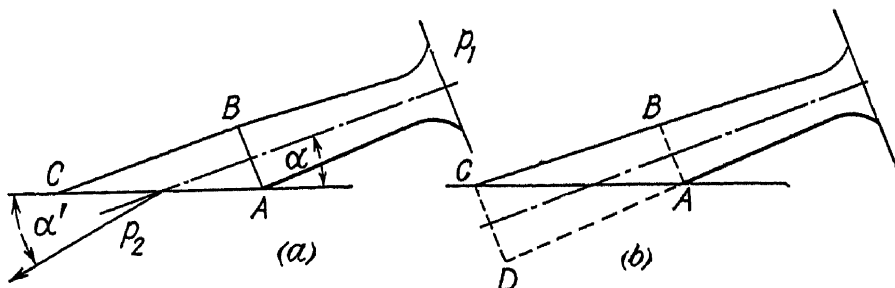


FIG. 48

outlet area A_2 the remaining part BC is too divergent, whereas if the "section" at CD is made equal to A_2 , part of the expansion will be completed in the wedge ABC. In either case a certain amount of jet deflection seems inevitable. This type of nozzle is not suitable for large ratios of expansion.

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EXAMPLES V

1 The heat available during the adiabatic expansion of 6 lb of steam is 240 B Th U Calculate the final velocity of the steam if it was initially at rest

2 If the efficiency of the nozzle in Question 1 is 90 per cent, calculate the exit velocity.

3 Dry and saturated steam enters a nozzle at a pressure $p = 150$ and with an initial velocity of 300 ft per second The outlet pressure is 90 lb per sq in abs and the outlet velocity 1,450 ft per sec If the heat loss from the nozzle amounts to 2 B Th U per lb of steam flowing through the nozzle, what must be the dryness fraction of the steam as it leaves the nozzle?

4 Steam expands adiabatically from an initial pressure of 200 lb per sq in abs to a pressure of 120, the relationship between pressure and volume during expansion being

$$p v^{1.135} = \text{constant}$$

If the initial specific volume of the steam is 2.29 cu ft per lb and the initial velocity is 170 ft per sec, calculate—

(a) Velocity at $p = 120$

(b) Area required at $p = 120$ if the weight of steam discharged per sec is 1.4 lb

5 Steam at a pressure of 1,000 lb per sq in abs and a temperature of 1,000° F ($I = 1505.6$ B Th U per lb) expands adiabatically to a pressure of 600 Assuming that the relationship between p and T is

$$p = \text{constant} \times T^{\frac{1.3}{8}}$$

calculate the degree of superheat of the steam at $p = 600$, and obtain the enthalpy value by interpolation from the steam tables. Hence find the adiabatic heat drop and the velocity from rest

6 Assuming that the ratio of the throat pressure to the initial pressure with adiabatic expansion from a state of rest is

$$r = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}}$$

where n is the exponent of v in the expression $pv^n = \text{constant}$, prove that the throat velocity for steam initially dry and saturated is 1,280 $p_1^{\frac{1}{3.5}}$, where p_1 is the initial pressure in lb per sq in abs Assume $n = 1.135$ and $p v^{\frac{1.6}{3.5}} = 490$ for dry and saturated steam

7 Calculate the throat area of a nozzle to pass 4,200 lb. per hour, assuming that the final pressure is lower than the critical, that the steam is superheated during the whole of the expansion, and that expansion takes place without losses Given that—

Initial pressure $p_1 = 250$

„ superheat = 250° F

„ specific volume = 2.56

8. Steam expands in a nozzle under the following conditions—

Initial pressure $p_1 = 240$

„ temperature = 500° F

Final pressure $p_2 = 60$

Mass flow = 1.2 lb. per sec

Calculate the required throat and outlet areas, using the Mollier diagram—

(a) When the expansion is frictionless.

(b) When the friction loss at any pressure amounts to 10 per cent of the total heat drop down to that pressure.

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9. The effective heat drop in a nozzle is 60 B.Th.U. per lb. of steam and the final pressure is 20. What is the gain of entropy during an adiabatic expansion if the nozzle efficiency is 85 per cent and the final condition of the steam is wet?

10. Steam which is initially dry and saturated expands in a nozzle from $p_1 = 200$ to $p_2 = 80$. The expansion is frictionless throughout. During the expansion from 200 to 100 lb. per sq. in., the steam remains in the dry state. What is the maximum degree of supersaturation and of undercooling?

If the steam were to revert instantaneously to the saturated state at constant enthalpy and if the further expansion takes place in thermal equilibrium, calculate the change of entropy, the loss due to undercooling, and the percentage loss.

STEAM NOZZLE RESEARCH

52. Introduction. The objects of steam nozzle research may be enumerated briefly as follows—

(a) To measure the mass flow under certain conditions of pressure and temperature and to compare it with values calculated from theory. This part of steam nozzle research has been briefly dealt with in the preceding chapter

(b) To measure the velocity acquired by the steam under given conditions and compare it with the theoretical velocity.

(c) To determine the direction of the issuing steam jet. This has an important bearing on the design of the blades.

(d) To study special problems such as the distribution of pressure, effect of roughness, effect of nozzle angle, effect of entrance conditions, and so forth

53. Reaction Method of Measuring Velocity. When a jet of fluid issues from a nozzle at a certain velocity it exerts a backward reaction on the nozzle. The simultaneous measurement of the steam flow in lb. per sec and the force of reaction enables the velocity of efflux to be deduced

One arrangement of apparatus is shown diagrammatically in Fig 49. High-pressure steam passing along the pipe A enters the inside of the trunnion B and passes down the pipe C into the steam chest D to which the nozzle E is attached. The pendulum pipe C is free to swing in bearings attached to the casing F. The latter surrounds the pendulum pipe and leads to a surface condenser. The trunnion B is supported on ball bearings and packed by labyrinth packing so that there is very little friction in the apparatus. By using suitably designed packing and a suitable arrangement of leak-off connections, the errors due to steam leakage may be kept within very small limits. An arm G is attached to the trunnion so that the turning moment on the latter may be measured, e.g. by a small weighing machine. The pressure of the steam in the supply pipe may be read off the gauge H_1 , while the superheat or dryness may be obtained by a thermometer or thermocouple, or by a calorimeter, respectively. A small-bore pipe K is coupled to the steam chest and led through the trunnion (not shown in diagram) to a gauge H_2 , which will, therefore, give the steam pressure at the nozzle inlet. H_3 (mercury column) gives the back pressure. Suitable balance weights are added to the swinging parts so as to put them in neutral equilibrium.

A measurement of the turning moment on the trunnion enables

the reaction R to be calculated. A simultaneous reading of the mass flow w lb. per sec. must be obtained.

Then, since
$$R = \frac{w \cdot V}{g} \text{ lb. weight}$$

$$V = \frac{gR}{w} \text{ ft. per sec.}$$

With reference to the testing of convergent nozzles, it may be shown that the velocity given by the above formula is strictly only the

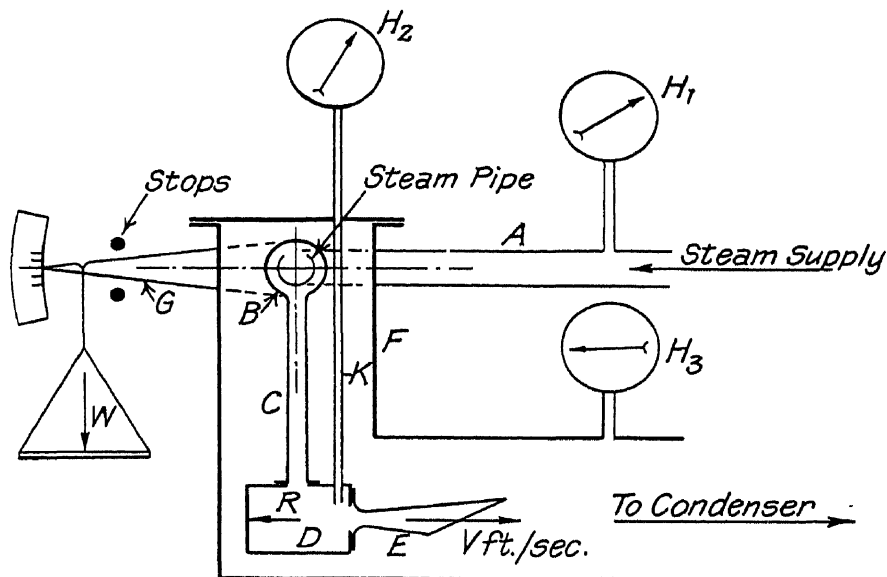


FIG. 49.—Reaction Method of Measuring Steam Velocity

velocity of efflux from the nozzle when the pressure at the nozzle outlet is equal to the pressure in the exhaust space. When steam is expanded through a convergent nozzle, the velocity of efflux increases as the external pressure is reduced until the critical pressure is reached. When the critical velocity is reached, the velocity of efflux cannot increase further, but it is found that the reaction still continues to increase owing to the fact that the steam continues to expand outside the nozzle.

54. Impulse Method of Measuring Velocity. In this method the steam jet, on emerging from the nozzle, is allowed to impinge on a flat plate as shown in Fig. 50. The jet spreads radially in all directions from the surface of the plate and its initial momentum is destroyed.

Let P = force on plate, lb weight
 w = steam flow, lb. per sec.
 V = jet velocity, ft per sec.

Then
$$V = \frac{P \cdot g}{w}$$

Actually the velocity is not uniform throughout the jet, it is almost uniform over a large central area, but diminishes rapidly near the nozzle walls. If V be the velocity of a small element of steam mass δm , then the total momentum of the steam is $\Sigma V \delta m$

Hence

$$\Sigma V \cdot \delta m = P \cdot g \quad (62)$$

If we divide the total momentum by the mass flow per second, we have

$$\frac{\Sigma V \cdot \delta m}{\Sigma \delta m} = V_1 \quad (63)$$

which is a quantity having the dimensions of a velocity and is in fact the mass-mean velocity of efflux.

The ratio $\frac{V_1}{V_t}$ (where V_t is the theoretical velocity of efflux based on the adiabatic heat drop) is the velocity coefficient η_v for the nozzle.

The simple apparatus shown in Fig 50 is not satisfactory in use owing to the circulation currents which are set up under the plate by frictional entrainment. These appear to augment the thrust, resulting in apparent jet velocities higher than the true velocity.

In the apparatus used by the Institution of Mechanical Engineers Steam Nozzles Research Committee and described in Art. 59, these objections were satisfactorily overcome.

55. Search-tube Method of Determining Pressure Distribution in a Nozzle. A small-bore tube A, Fig. 51, closed at the exhaust end and communicating with a pressure gauge B at the other end, passes

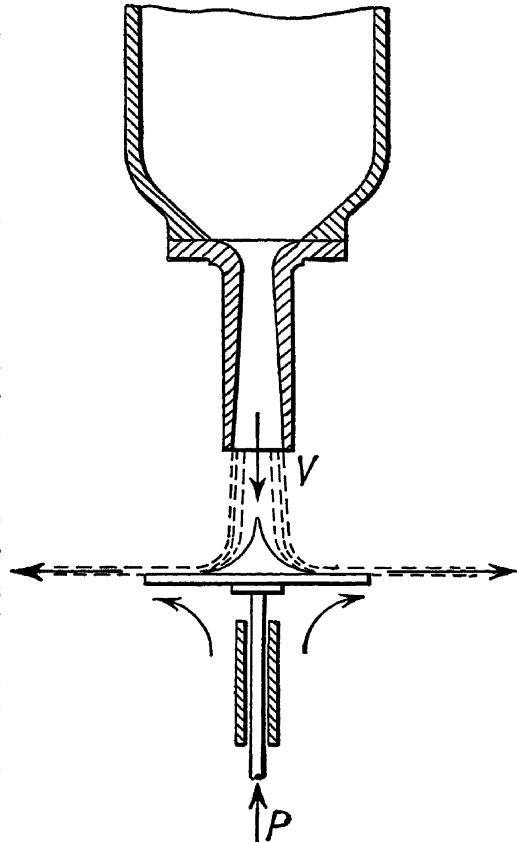


FIG 50 —Impulse Method of Measuring Steam Velocity

along the axis of the nozzle. Small holes are drilled in the tube in a transverse plane for the purpose of determining the pressure at the core of the steam at the particular section where the searching holes happen to be. The axial position of the searching holes can be varied by means of the micrometer wheel C. Professor Stodola (1) made some experiments in which he measured the pressures at the rim of the stream simultaneously with those at the core. He found

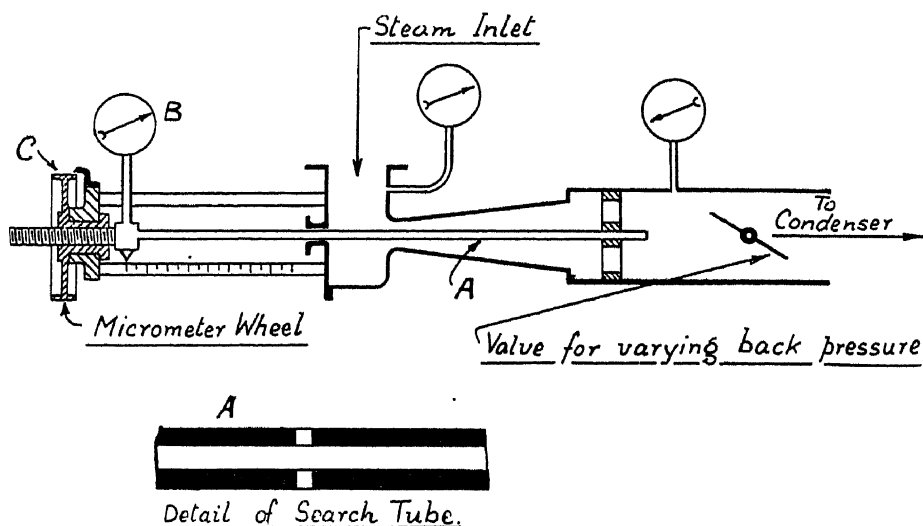


FIG. 51.—Diagrammatic Arrangement of Search-tube Apparatus

that the pressure at the axis of the stream was slightly higher than that at the rim, but the difference was very small.

Besides giving information regarding any possible disturbances in the nozzle, the results of search-tube experiments, combined with simultaneous mass-flow measurements, may be utilized for investigating the distribution of loss inside a nozzle, provided the following assumptions are made—

- (a) That there is no heat loss by conduction through the nozzle.
- (b) That the steam completely fills the nozzle at all sections and that the velocities of all steam particles across any section are identical.

Let I_1 = enthalpy of steam at the beginning of the expansion.

w = mass flow, lb. per sec.

V_1 = initial velocity of steam, which may be calculated from w and the initial condition of the steam.

p_2 = observed pressure at any section.

A_2 = nozzle area at any section, sq. ft.

I_2 = enthalpy at p_2 .

Then by assumption (a)

$$\frac{V_2^2}{2gJ} + I_2 = \frac{V_1^2}{2gJ} + I_1$$

If the steam is superheated at p_2 and the specific volume is v_2 , then by assumption (b)

$$V_2 = \frac{w \cdot v_2}{A_2}$$

Then

$$\frac{1}{2gJ} \left(\frac{w}{A_2} \right)^2 \cdot v_2^2 + I_2 = \frac{V_1^2}{2gJ} + I_1 \quad . \quad . \quad (64)$$

The condition of the steam at p_2 to satisfy this equation may easily be found by trial and error. This enables the locus of the state point to be drawn on the Mollier chart and the loss down to any pressure to be easily determined

If the steam is wet at p_2 and x_2 is the dryness fraction, then very nearly

$$\frac{V_2^2}{2gJ} + h_2 + x_2 L_2 = \frac{V_1^2}{2gJ} + I_1$$

and since

$$w = \frac{A_2 V_2}{x_2 \cdot v_{s2}}$$

$$\frac{V_2^2}{2gJ} + h_2 + \frac{A_2 V_2 L_2}{w \cdot v_{s2}} = \frac{V_1^2}{2gJ} + I_1 \quad . \quad (65)$$

This equation is a quadratic in V_2 , all other terms being known from the observed pressure and mass flow. When V_2 is known we can obtain the dryness fraction x_2 from the equation of continuity. By means of the Mollier chart the dryness fraction x_a , after adiabatic expansion, may be found. The energy loss in the nozzle down to the pressure p_2 will then be

$$(x_2 - x_a) L_2 \text{ B Th U. per lb}$$

56. Stodola's Search-tube Experiments. Stodola (*loc cit*, pp. 95 and 106) made many illuminating experiments with the aid of search tubes. Fig. 52 shows a convergent-divergent nozzle with normal inlet and outlet sections. The initial steam conditions were practically constant throughout the experiments, but the back pressure was gradually increased by the closure of a valve placed between the nozzle outlet and the condenser. Curve 1 shows the distribution of pressure in the nozzle when the outlet pressure was that for which the nozzle was designed. The other curves (2, 3, 4, etc.) show the pressure distribution as the back pressure was gradually increased.

It will be noticed that in such cases the pressure follows the correct expansion line down to a pressure which is well below the final value and then rises in sudden leaps. It appears that the

steam fills the nozzle for a certain distance and expands at a rate which is controlled entirely by the area of the nozzle. Then, at a certain point, these conditions break down; the rapidly flowing

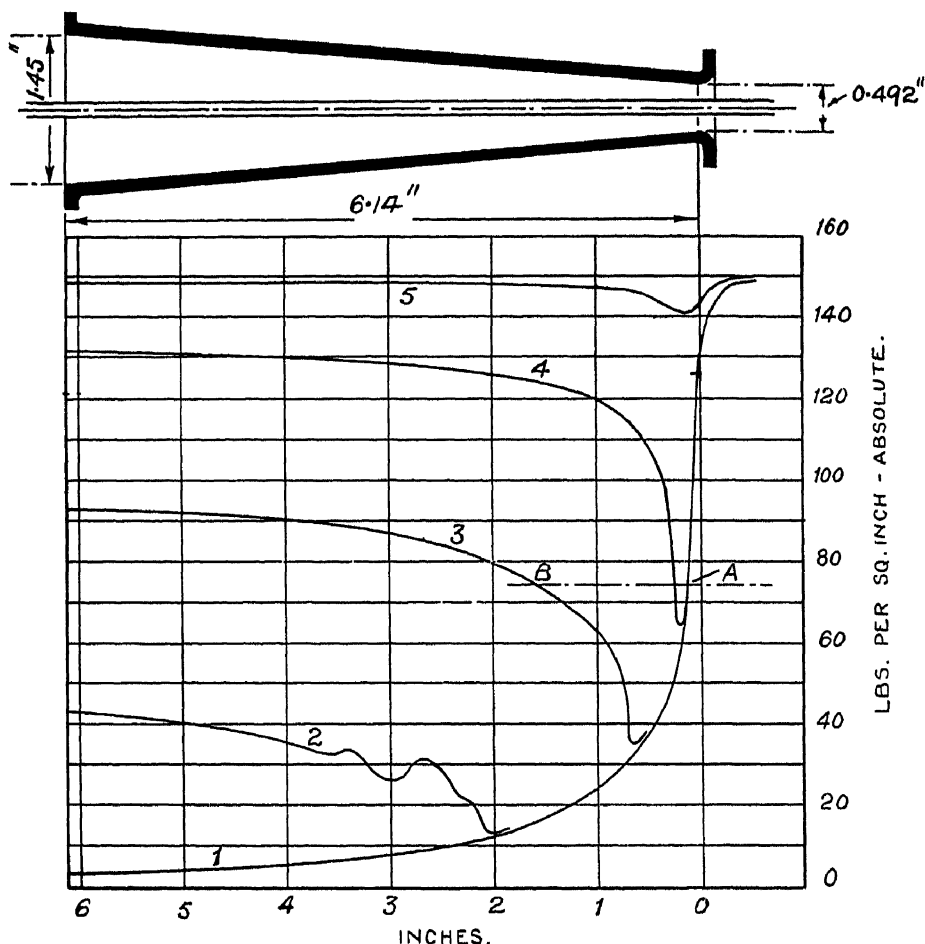


FIG. 52.—(Stodola)

steam enters a region where the steam velocity is lower, with the result that there is an impact and a sudden rise of pressure. By taking two points A and B at the intersection of a line of constant pressure AB with any one of the curves, and comparing the kinetic energy of the steam at B with that at A, it is possible to estimate the loss of energy due to shock. Stodola points out that, except for small pressure drops, the loss of energy is great in all cases.

In some experiments with a divergent nozzle the outlet pressure was varied and the pressure before and after the nozzle outlet

section was measured by means of a search tube. Some of the curves are shown in Fig. 53. When the pressure in the outlet space was lower than that corresponding to the design of the nozzle the

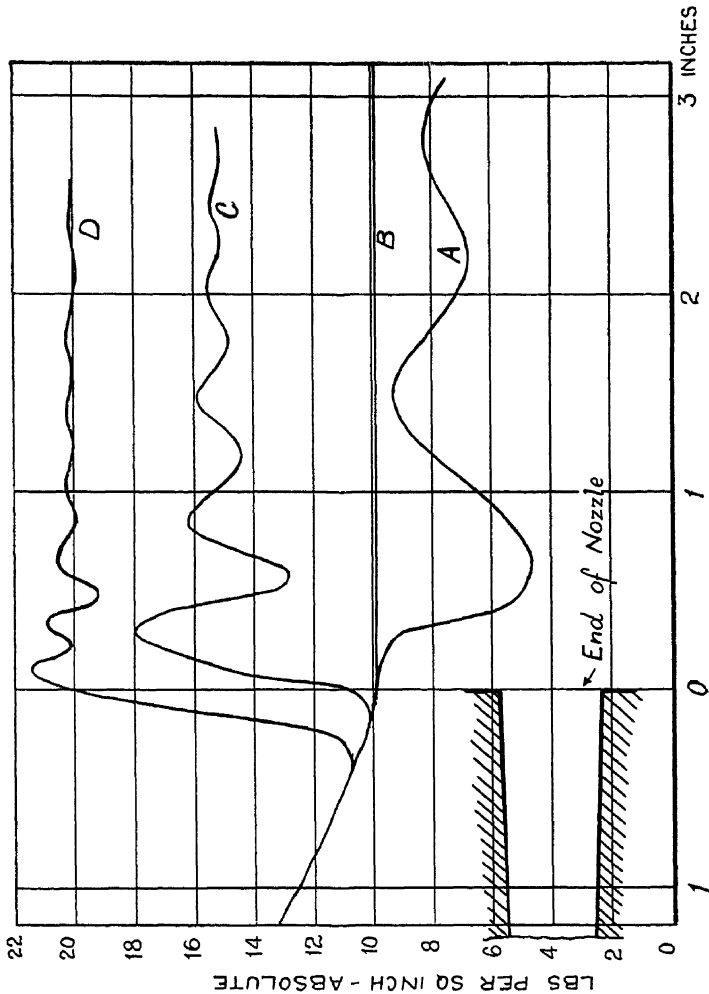


Fig 53—(Stodola)

pressure first dropped suddenly and then continued to oscillate, as shown by curve A. With a pressure higher than the correct back pressure, there was considerable impact followed by pressure oscillations.

Generally, the conclusions to be drawn from Stodola's experiments are that in a convergent-divergent nozzle in which the throat and mouth areas are correctly proportioned for the conditions

of working and which has a good form, the losses due to shock will be a minimum, but that if the outlet pressure is either higher or lower than that for which the nozzle is designed, considerable shock losses will occur.

57. Experimental Results obtained with Reaction Apparatus. In the year 1900 Mr. W. Rosenhain (2) published the results of some experiments made on simple forms of nozzles, including the convergent-divergent type. The velocity of efflux was determined by the reaction method.

Dr. P. Christlein (3) made experiments with apparatus similar to that shown in Fig. 49. One set of experiments was made on a single convergent-divergent turbine nozzle having a throat of circular cross-section changing to a rectangular outlet section oblique to the nozzle axis. The velocity coefficient reached its maximum value under the conditions of steam pressure and temperature, etc., for which the divergent part of the nozzle was designed, the value of η_v then being 0.946. When the conditions were allowed to deviate from the design conditions, the velocity coefficient and hence the nozzle efficiency fell away rapidly. Christlein also experimented with a group of three convergent turbine nozzles of the type in which the guide vanes are cast-in. He found that η_v increased with increasing velocity up to a theoretical steam velocity of about 1,800 ft. per sec., η_v then attaining a maximum value of about 0.93.

More recently, Messrs. Warren and Keenan (4) carried out experiments with an improved reaction apparatus. In tests on a simple convergent nozzle with a parallel outlet part the value of η_v was 0.99. In further tests on a group of four turbine nozzles having machined guide vanes the value of η_v was practically constant over a wide range of velocities and equal to 0.982. Great care was taken in these experiments to avoid errors due to circulation currents in the apparatus, but the values of the velocity coefficient appear to be rather high.

58. Institution of Mechanical Engineers Steam Nozzles Research. In the year 1914 a Committee of the Institution of Mechanical Engineers was formed in order to report upon a possible programme of research work relating to the flow of steam through nozzles. A large amount of preliminary work was undertaken, but was much interfered with by the war, and actual experimenting was not begun until September, 1921. From that time until 1930 a very considerable amount of work was done, the results being embodied in six reports (5), to which the reader is referred for fuller information. The aim of the Committee was to determine the velocity coefficient η_v for practical forms of impulse turbine nozzles, for reaction turbine blades, and also for so-called "elementary" nozzles, i.e. nozzles of simple form having the outlet plane perpendicular to the nozzle axis. Subsidiary experiments were also made to

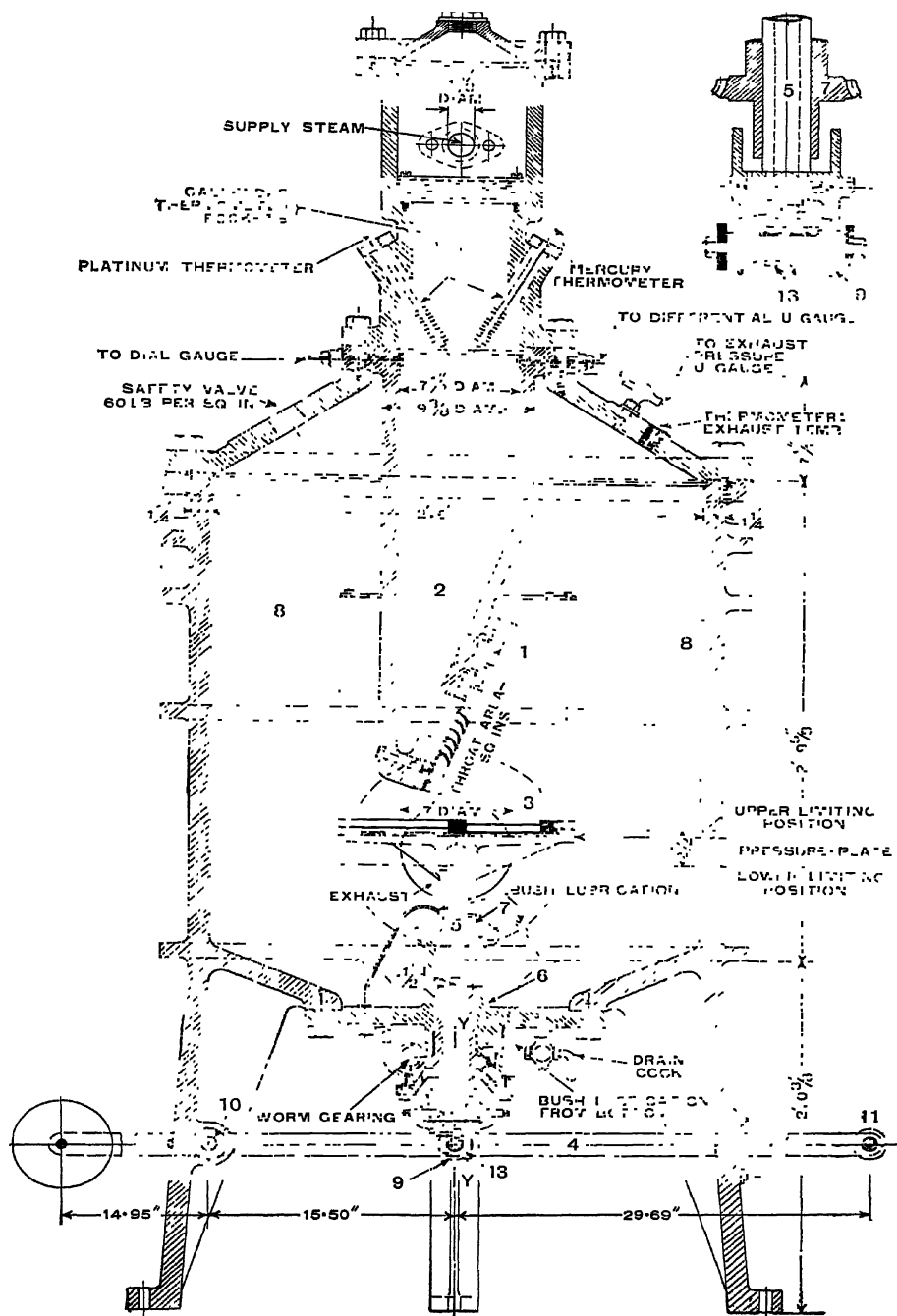


FIG 54

Cross-sectional View of Impulse-type Nozzle Tester used by the Institution of Mechanical Engineers Steam Nozzles Research Committee

determine the effects of surface roughness in straight convergent-parallel nozzles and of curvature in impulse turbine nozzles. The impulse method (jet momentum) was adopted for measuring the jet velocities and much effort was initially expended in order

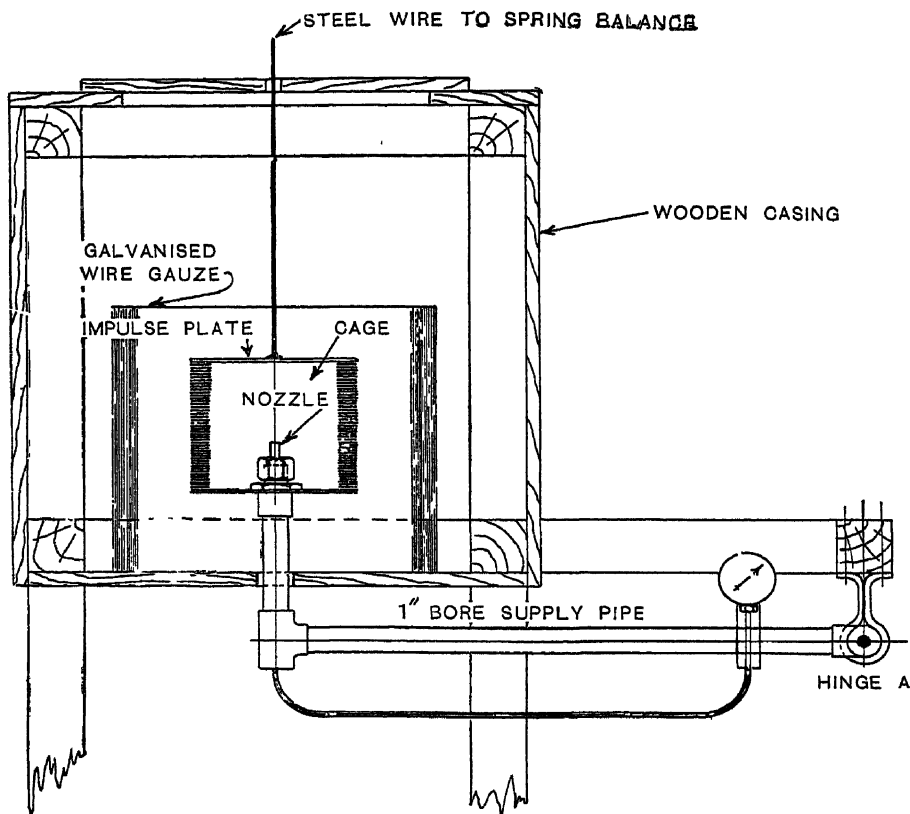


FIG. 55

Arrangement of Apparatus for Testing Impulse Cage

to eliminate the anomalies associated with the method of measurement.

59. Apparatus. The apparatus used is shown in Fig. 54. The impulse plate is replaced by a cage 3, built up of thin strips of aluminium separated by thin washers, and carried on the plate proper, which is faced by layers of wire mesh and a layer of fine gauze. The object of such a design is to cause the steam to leave the cage at a low velocity and in directions entirely at right angles to the plate. The impulse is transmitted to a weighing lever 4, by means of a spindle 5, which slides in a frictionless (rotating) guide.

The nozzles to be tested are bolted to the steam chest 2. The steam exhausted from the tester is condensed in a small surface condenser and weighed.

In the preliminary experiments, the cage was tested by the method illustrated in Fig. 55. The nozzle and cage were attached rigidly together and carried by a pipe, hinged at A, supplying air under pressure. A steel wire connected the impulse plate to a sensitive spring balance. It will readily be seen that the force due to impulse opposes that due to reaction, and any difference between the two would affect the reading of the spring balance. In the early tests there was some difference between the impulse and reaction due to eddies, but when the plate was covered with felt and wire netting, the difference disappeared. For steam experiments, the felt was replaced by wire gauze.

60. Results of Tests. *Angle of Efflux* In order that the nozzle might be set correctly in the tester, i.e. so that the jet would impinge normally on the plate, it was necessary to observe the exact angle of efflux of the jet. This was done by means of a balanced light metal vane placed in the steam jet. Some of the results are shown in Fig. 56. In all these cases and in all other nozzles tested, with the exception of the built-up impulse nozzle, the angle of efflux is less than the nominal angle of the nozzle. This appears to be due to the effect of the flat surface of the vane outlet edge in causing a suction on that part of the jet which is inclined to it at an acute angle. As the steam velocity is gradually increased it seems as though the greater momentum of the steam is more and more able to overcome this effect, and the jet efflux angle approaches nearer to the geometrical angle. In Parsons blades, where the back and front surfaces are curved and gradually approach each other at the outlet edge, the deviations of the jet from the geometrical angle are not so great.

Tests on Impulse Nozzles. Numerous tests were made on impulse turbine nozzles in which the guide vanes are cast in position. The geometrical angles of the nozzles were chosen as 12° and 20° . Some of the nozzles had thin vanes, only 0.04 in. thick, while others had vanes $\frac{3}{16}$ in. thick. A section through the latter is shown in Fig. 57. In all cases the perpendicular distance between the nozzle plates was $\frac{1}{4}$ in. The length of the parallel throat was originally $\frac{3}{4}$ in. in all cases, giving a so-called throat ratio of 3/1. In subsequent tests, the face of the whole nozzle block was machined so as to give successive throat ratios of 2/1, 1/1, and 0/1. In the last case it will be clear that the nozzle passage was entirely convergent without any parallel portion at all.

Some of the test results for the 20° nozzles are shown in Fig. 58. It will be observed that a large ratio of throat length to width is somewhat detrimental owing, probably, to the increased surface friction.

Effect of Chamfering Nozzle Plate. One important conclusion of this research work is that nozzles having thin vanes are more efficient than those having thick vanes. This is shown, for instance, by the curves in Fig. 58. The reason is probably due to the effects,

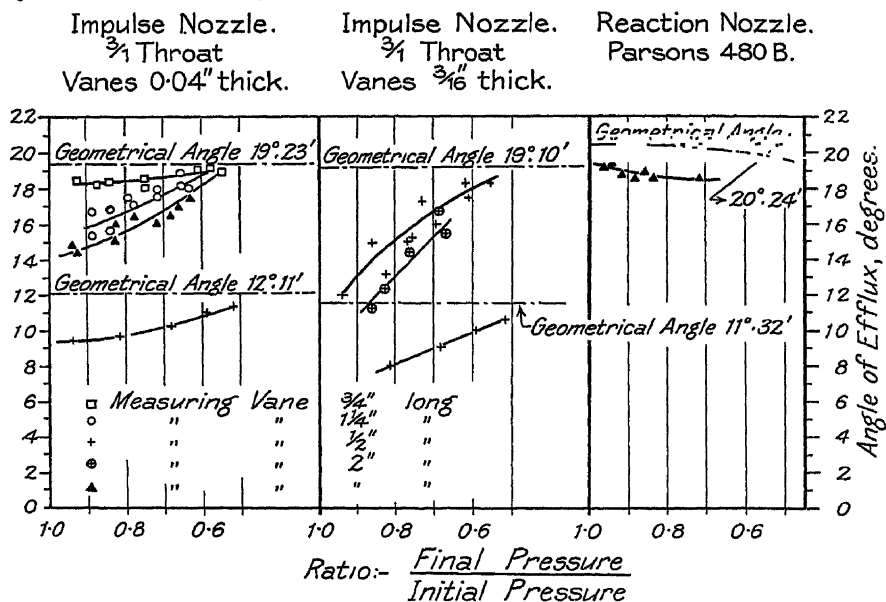


FIG. 56

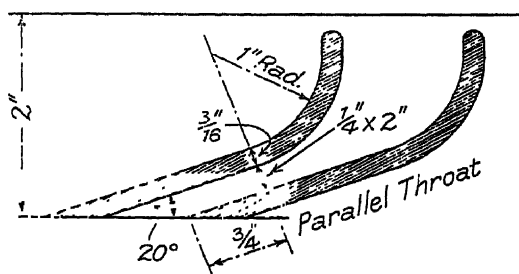


FIG. 57

already mentioned above, of the flat surface of the outlet edge of the vane, in producing deviation and instability of the jet. This flat surface may be entirely eliminated by chamfering the exit edge of the vane as shown in Fig. 59.

As thicker vanes give greater strength and rigidity to impulse turbine diaphragms, and also a greater margin of material in the event of corrosion or erosion, tests were made to determine whether the efficiency of thick-plate nozzles would be affected by chamfering

the nozzle plates in such a way as to reduce the thickness of the outlet edge to a very small dimension. Some of the results of these tests are given in the table on the next page and show that the

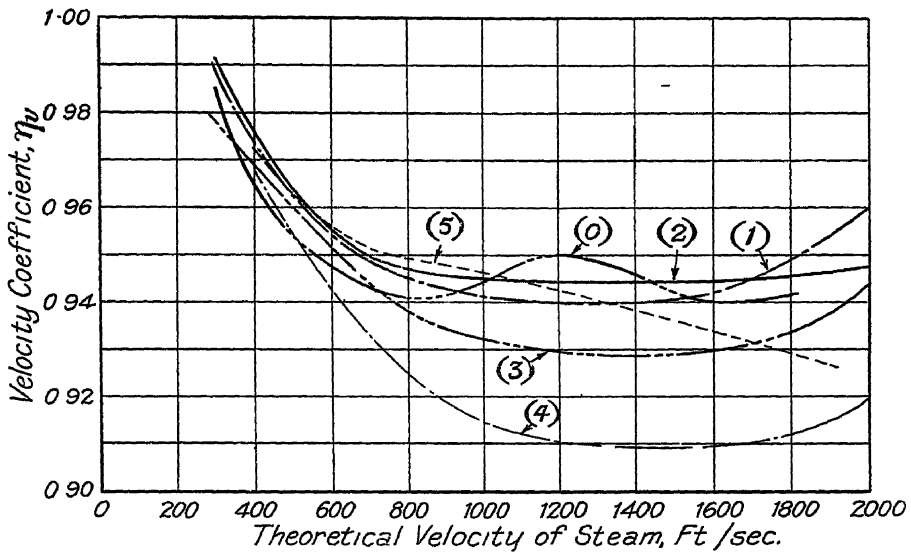


FIG 58 —Velocity Coefficients for 20° Turbine Nozzles

Curve (0)	relates to impulse nozzle with vanes	0.04 in thick,	0/1 throat	
" (1)	"	"	"	0.04 in 1/1 "
" (2)	"	"	"	0.04 in 2/1 "
" (3)	"	"	"	0.04 in 3/1 "
" (4)	"	"	"	3/16 in (Mean curve) Vane unchamfered
" (5)	relates to Parsons reaction blade	1 in wide		

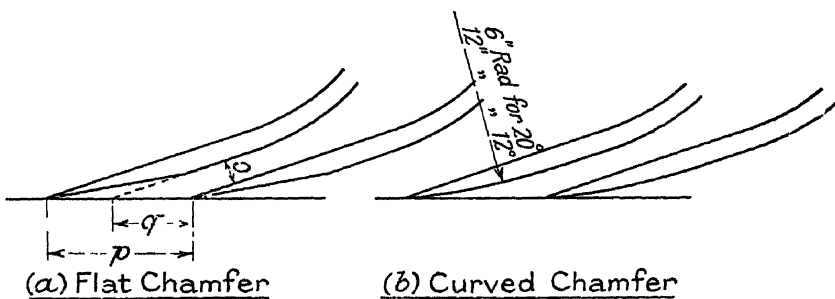


FIG 59 —Chamfering of Outlet Edges of Nozzle Plates

chamfered nozzle plate gives results practically as good as those obtained with the thin plate, irrespective of the angle of the nozzle.

As might be expected, the chamfering exerts quite an appreciable influence on the angle of efflux of the jet. The effect is to reduce the angle, and observations showed that the actual angle of efflux for pressure drops less than the critical was more nearly equal to

AVERAGE VELOCITY COEFFICIENTS FOR IMPULSE NOZZLES FOR THE
VELOCITY RANGE 1,000 TO 1,700 FT. PER SECOND

	20° Angle.		12° Angle.	
	3/1 Throat.	0/1 Throat.	3/1 Throat.	0/1 Throat.
Thin plate	0.932	0.946	0.905	0.944
Thick plate	0.913	0.915	0.894	0.933
Thick plate, radius chamfer	0.941	0.936	0.928	0.941
Thick plate, flat chamfer	—	0.945	—	—

the angle whose sine is o/p , Fig. 59, than the geometrical nozzle angle whose sine is o/q . With a geometrical nozzle angle of $19^{\circ} 10'$, the observed efflux angle of the chamfered nozzle with a throat ratio of 0/1 was about $10^{\circ} 40'$ at low velocities and about 12° at the critical velocity. Similarly, for a nozzle having a geometrical angle of $11^{\circ} 32'$, the observed angles of efflux at low velocities were about $5^{\circ} 45'$ for a 0/1 throat ratio and about 5° for a 3/1 throat ratio, whilst at about the critical velocity the observed angles were about $8^{\circ} 20'$, and $7^{\circ} 30'$ for the 0/1 and 3/1 throats respectively. These deviations from the nominal angle are of some importance in turbine theory and design because, as will be readily appreciated, the smaller angles lead to a greater theoretical or "diagram" efficiency and also affect the blade heights necessary to prevent the steam from blowing across the top of the blades.*

Tests of Built-up Impulse Nozzles. Some tests were made on a set of three built-up nozzles similar to those frequently used in the high-pressure stages of modern impulse turbines. A section through the test nozzle is shown in Fig. 60. The nozzle components are machined all over, so that it is possible to obtain a better surface finish than is generally obtainable with the usual "cast-in" construction, in which two of the four metal surfaces bounding the jet are rough castings. The length of parallel throat is extremely small, being only $\frac{1}{16}$ in. In making tests with this type of nozzle, the apparatus was modified so as to give the steam admission to the nozzle with less sudden changes in direction. The mean velocity coefficient for this nozzle was about 95 per cent.

Effects of Roughness. The roughness of the surface in the various nozzles tested by the Committee varied to some extent from that of well-finished machined steel, through that of drawn brass to that of a cast-iron surface. Tests undertaken to determine the effect of roughness on the velocity coefficient at various steam velocities were

* See Art. 139.

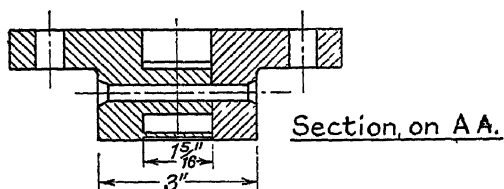
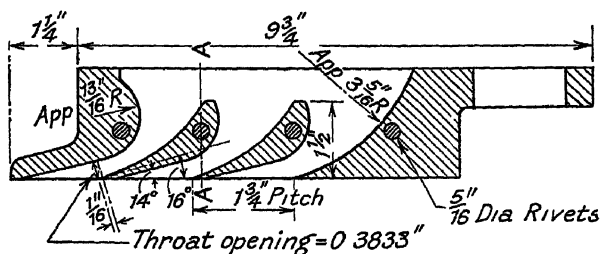


FIG 60 — Built-up Nozzle

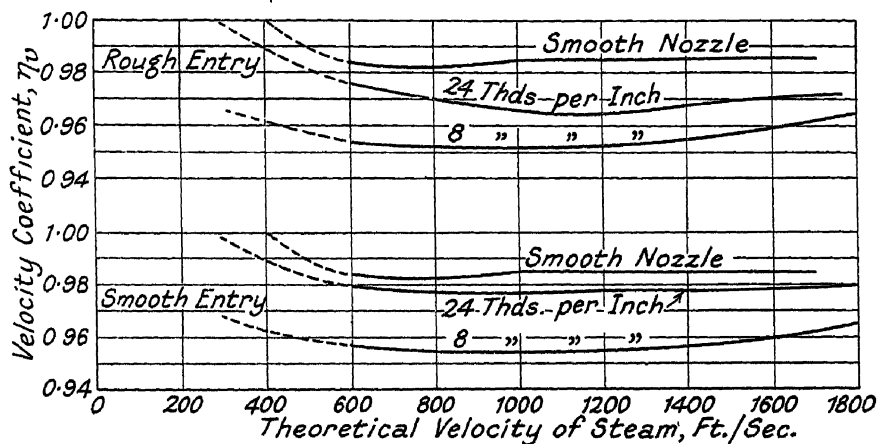
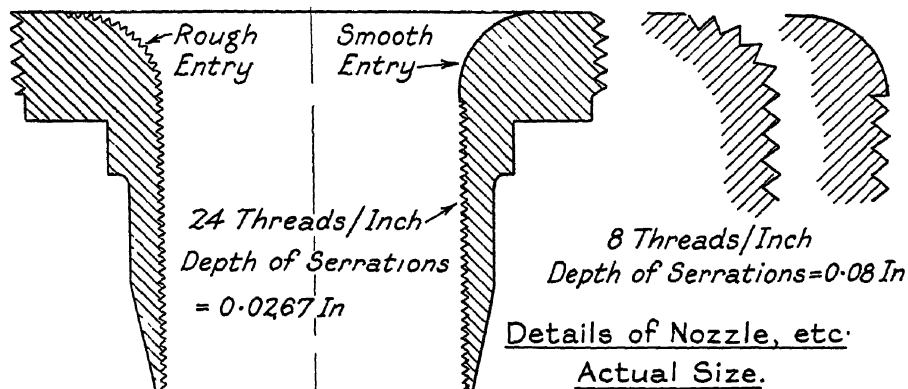


FIG 61 — Tests on Artificially Roughened Nozzles

made on straight "elementary" nozzles with a rounded inlet, the roughness being produced by means of standard Whitworth threads of 24 and 8 to the inch respectively. In one case the entry was left smooth (see Fig. 61), and in the other a number of concentric grooves corresponding to the thread in the parallel part of the nozzle were turned on the rounded inlet. The mean results are shown in Fig. 61, from which it will be seen that roughness has quite an important influence on nozzle efficiency, especially when it occurs

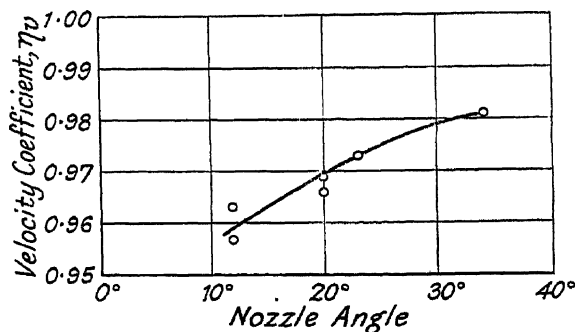


FIG. 62.—Influence of Nozzle Angle on Velocity Coefficient

at the curved inlet. It is possible that the roughness which might develop on the working surfaces of actual steam turbine nozzles, due to corrosion, etc., might be so serious as to be simulated by serrations of a Whitworth thread 24 to the inch, but it is hardly likely.

Further, the roughnesses met in practical nozzles are distributed in a much more irregular fashion than those artificially produced in the experiments. Certainly the 8 threads per inch would represent a very bad case.

Effect of Nozzle Angle. The function of a turbine nozzle is not only to convert available heat energy into energy of motion, but also to direct the jet formed in a particular direction. In the actual nozzle both these operations are carried out simultaneously. It might well be imagined that the greater the angle through which the steam turns during its passage through the nozzle, the greater the inequalities in the flow and the greater the losses. In the course of its work, the committee made a number of tests on straight circular convergent nozzles with a rounded inlet and an almost parallel throat. The mean velocity coefficient for this type was about 98 per cent. In other tests with short rectangular convergent nozzles having well-rounded inlets, the mean velocity coefficient is 99 per cent. The relationship between the geometrical or nominal angle of the nozzle and the mean velocity coefficient is shown in Fig. 62, from which it will be seen that the outlet angle exerts quite an appreciable influence on nozzle efficiency. Some additional data

obtained at the works of Messrs. Metropolitan-Vickers Electrical Co., Ltd., have been used in deriving Fig. 62.

Tests to Determine the Effect of Entrance Conditions. In the final report of the Committee, an account is given of interesting tests on nozzles having various shapes of inlet. In the first place, straight nozzles of 1 in. square section and rectangular section (2 in. $\times$ $\frac{1}{2}$ in.) were tested with sharp entrances, then with two edges rounded and lastly with four edges rounded. As might be expected, the sharp edges were detrimental to efficiency. Over the range of velocity from 400 to 1,200 ft. per sec, the velocity coefficient had a mean value of 0.86 to 0.87 with the sharp edges, whereas with two and four edges rounded the coefficient had mean values of about 0.96 and 0.97 respectively. With higher velocities the coefficient for the sharp-edged inlet increased steadily, and at 2,000 ft. per second it was almost as high as that for rounded inlet edges.

Other tests on impulse turbine nozzles, with cast-in plates having a radius of curvature of $1\frac{3}{4}$ in. as compared with $\frac{7}{8}$ in. and 1 in. in the earlier types, were also made and showed improvements varying from 0.2 to 1.4 per cent in favour of the larger radius of curvature.

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THE FLOW OF STEAM THROUGH IMPULSE
TURBINE BLADES

61. Velocity Diagrams for Impulse Turbines. It has already been pointed out in Chapter I that the essential parts of an impulse steam turbine are the nozzles and blades. In the former, the expansive property of the steam is utilized to produce a jet of steam moving with a comparatively high velocity. The function or purpose of the blades is to change the direction, and hence the momentum, of the jet or jets of steam and so produce a force which propels the blades. It is a matter of prime importance that we should be able to estimate the propelling force that would be applied to a turbine rotor under any given set of conditions, for by this means we can calculate the work done and hence the power. Since the force is due to a change of momentum caused mainly by a change in the direction of flow, it becomes essential to draw diagrams showing how the velocity of the steam varies during its passage through the blades.

It will be realized that velocity is a vector quantity; i.e. it has (1) magnitude, (2) direction, and (3) sense in that direction. Thus we can represent a velocity by a straight line, and indicate (1) its magnitude by the length of that straight line to a suitable scale; (2) its direction by the direction of the said straight line with reference to some fixed direction; and (3) its sense by an arrow placed on the straight line. Moreover, we can add and subtract velocities graphically. In explaining the velocity diagram the author will adopt the notation proposed by Professor R. H. Smith (1) for the study of mechanisms.

Fig. 63 shows the nozzles and blades either of a single-stage impulse turbine or of one stage of a multi-stage turbine. Steam enters the nozzles at pressure P_0 and issues at pressure P_1 . Due to the fall of pressure in the nozzles the steam issues with a velocity which we shall suppose is uniform and equal to V_1 . Moreover, we shall assume that the many steam filaments which flow from the nozzle issue at a uniform angle α . Then we represent this velocity by the vector V_1 , Fig. 63, and, using Smith's notation, by the letters $c-s_1$ —i.e. the velocity of the *steam* relative to the *casing* at inlet. This would be drawn to a convenient scale and the arrow shows the actual direction in which the steam is moving. By Smith's notation, however, this is sufficiently indicated by the letters $c-s_1$. Now draw $c-b$ to represent the velocity of the *blades* relative to the *casing*. Let this be u ft. per sec. Join $b-s_1$. Then, according to the notation used, $b-s_1$ is the velocity of the *steam*

relative to the *blades* at inlet. Thus, although the steam is actually moving towards the blade along the line $c-s_i$, owing to the fact that the blade itself is moving with the velocity $c-b$, the steam appears to enter the blade with the direction $b-s_i$ and with the relative velocity at inlet of U_1 ft per sec

In a pure impulse turbine the steam pressure P_2 in the space immediately adjacent to the blades on their outlet sides is equal to the pressure P_1 in the clearance space between the nozzles and blades. The steam will flow out of the blades with a certain relative velocity which we shall assume to be uniform and equal to U_2

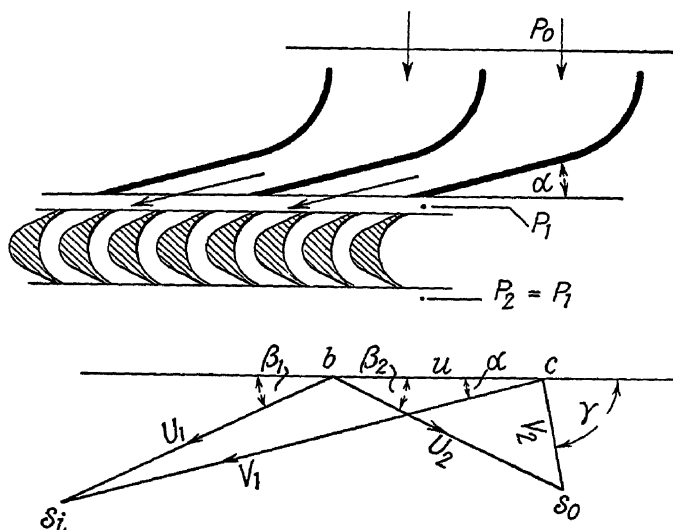


FIG 63

If there is no friction loss in the blade channels then U_2 will be equal to U_1 , if there is friction, U_2 will be less than U_1 . We shall assume that the steam filaments flowing from the blades issue at a certain angle which is uniform and equal to β_2 , which we shall term the blade outlet angle. Now draw $b-s_o$ to represent the velocity of the *steam* relative to the *blade* at outlet. By joining c and s_o we obtain the vector $c-s_o$, which represents the velocity of the *steam* relative to the *casing* at outlet. This is represented by the symbol V_2

The velocities V_1 and V_2 , being relative to the fixed casing of the turbine, will be referred to as *absolute* velocities; the velocities U_1 and U_2 , being relative to the moving blades, will be referred to as *relative* velocities.

The velocity diagram may be explained by the following alternative method. Considering Fig. 64, suppose a steam particle occupies a position A in the nozzle a very short interval of time δt before it actually enters the blade at B. The distance AB equals $V_1 \cdot \delta t$

At the same instant the blade must be at C, where the peripheral distance CB is equal to $u \cdot \delta t$. Then, to an imaginary observer on the blade the steam must appear to be approaching the blade in the direction AC at the angle β_1 to the wheel plane. Moreover, the distance AC must equal $U_1 \cdot \delta t$. Completing the parallelogram, it is obvious that $AD = BC$, $DB = AC$, and that the triangle ADB is similar to cbs_i of Fig. 63. Considering the velocities at outlet, a steam particle entering the blade at B follows an absolute path BE, and leaving the blade at E will move a distance $EF = U_2 \cdot \delta t$, due to its relative velocity U_2 , and a distance $EG = u \cdot \delta t$, due to

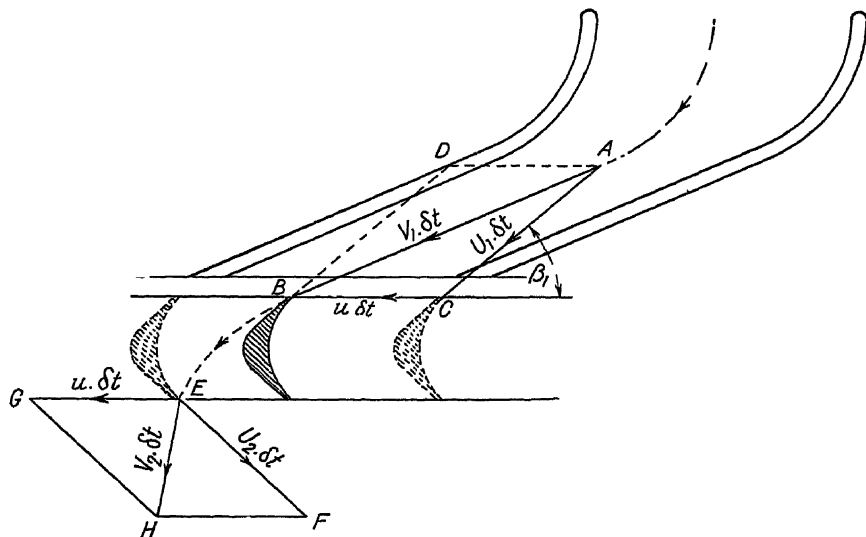


FIG. 64

the peripheral velocity u . Combining these displacements, the resulting displacement is EH and this must equal $V_2 \cdot \delta t$ where, as before, V_2 is the absolute velocity of the steam at outlet from the blade. Again it is apparent that the triangle EGH is similar to the triangle cbs_o of Fig. 63.

62. Forces on Blades. Having explained the diagram of relative velocities by means of Smith's notation, we shall have no further use for this notation. In subsequent work we shall use the velocity diagram shown in Fig. 65, together with the following standard notation—

V_1 = steam velocity at outlet from nozzles.

U_1 = steam velocity relative to blades at inlet.

U_2 = steam velocity relative to blades at outlet.

V_2 = absolute velocity of steam at outlet from blades.

u = peripheral velocity of blades (unless otherwise stated, this is the mean value).

α = jet angle.

β_1 = inlet angle of blades as given by diagram

β_2 = outlet angle of blades

γ = absolute direction of steam leaving blades.

Since V_1 is the initial absolute velocity and V_2 is the final absolute velocity of the steam, the change of velocity which the steam undergoes in passing through the blades is represented by the vector BD.

Let w = steam flow through blades, lb. per sec

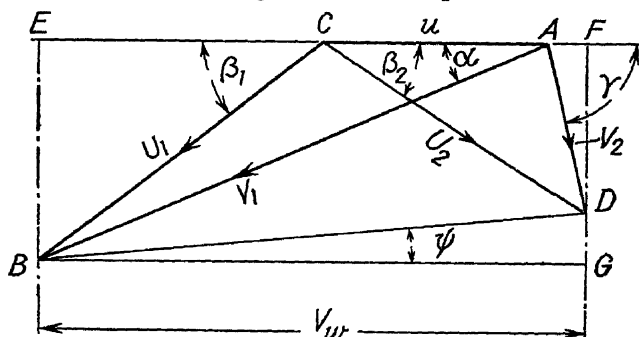


FIG 65

Then the resultant force necessary to produce the change of velocity BD is

$$\frac{w}{g} \times BD \text{ lb weight}$$

This is inclined at the angle ψ to the plane in which the wheel rotates. In accordance with Newton's Third Law, the force exerted by the steam on the blades is equal and opposite to that exerted by the

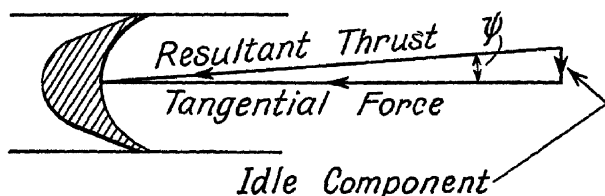


FIG 66 — Forces on Impulse Blade

blades on the steam. This is shown in Fig. 66. The resultant force may be resolved into two components, viz.—

(1) A useful thrust or tangential force parallel to the wheel plane and equal to $\frac{w}{g} \cdot BG = \frac{w}{g} V_w$, where V_w is the tangential component of the change of velocity and is usually termed the velocity of whirl.

(2) An idle thrust, perpendicular to the wheel plane, i.e. parallel to the turbine axis. This is equal to $\frac{w}{g} \cdot DG$.

We can now calculate the power developed and the blade efficiency. Since the blades are moving with a mean velocity u ft. per sec.—

Work done per sec.

$$= \frac{w \cdot u \cdot V_w}{g} \text{ ft.-lb.} \quad . \quad . \quad . \quad (66a)$$

$$= \frac{w \cdot u \cdot V_w}{gJ} \text{ B.Th.U.} \quad . \quad . \quad . \quad (66b)$$

$\therefore$ Horse-power developed

$$= \frac{w \cdot u \cdot V_w}{550g} \quad . \quad . \quad . \quad (67)$$

This will be termed the “rim horse-power” (r.h.p.) to distinguish it from the actual power transmitted to the shaft.

Since the energy of the steam entering the blades is

$$\frac{wV_1^2}{2g} \text{ ft.-lb. per sec.}$$

the efficiency of the blades alone is

$$\eta_b = \frac{w \cdot u \cdot V_w}{g} \div \frac{w \cdot V_1^2}{2g}$$

i.e.

$$\eta_b = \frac{2u \cdot V_w}{V_1^2} \quad . \quad . \quad . \quad (68)$$

EXAMPLE 8. Steam issues from the nozzles of a de Laval turbine with a velocity of 3,000 ft. per sec. The nozzle angle is 20° , the mean diameter of the blades is 10 in., and the speed of rotation 20,000 r.p.m. The mass flow through the turbine nozzles and blading is 0.4 lb. of steam per sec. If the friction loss in the blade channels is 33 per cent of the kinetic energy corresponding to the relative velocity at inlet to the blades, draw the velocity diagram and derive or calculate the following—

- (1) Velocity of whirl.
- (2) Tangential force on blades.
- (3) Axial force on blades.
- (4) Work done on blades per sec.
- (5) Horse-power of wheel.
- (6) Efficiency of blading.
- (7) Inlet angle of blades for shockless inflow of steam.

Assume that the outlet angle of the blades is equal to the inlet angle.

$$\begin{aligned}\text{Blade speed} \quad u &= \frac{\pi \times 10}{12} \times \frac{20,000}{60} \\ &= 873 \text{ ft per sec}\end{aligned}$$

The velocity diagram ABC, Fig. 67, may now be constructed and the following results obtained either graphically or by calculation $U_1 = 2,200$, axial velocity at inlet $V_{a1} = 1,026$, $CE = U_1 \cos \beta_1 = 1,946$, and $\beta_1 = 27^\circ 48'$

Since the friction loss in the blade channel is $0.33 \times \frac{U_1^2}{2g}$, then

$$\begin{aligned}U_2^2 &= 0.67 U_1^2 \\ \text{i.e.} \quad U_2 &= 0.8185 U_1 \\ &= 1,800 \text{ ft per sec.}\end{aligned}$$

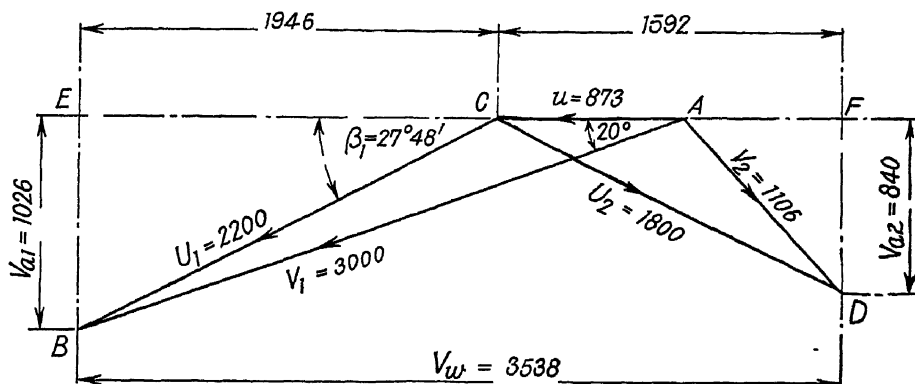


FIG 67

The velocity diagram may now be completed, bearing in mind that $\beta_2 = \beta_1$. The additional results obtained are $V_{a2} = 0.8185 \cdot V_{a1} = 840$, $CF = U_2 \cos \beta_2 = 1,592$, $V_2 = 1,106$

Then $V_w = 1,946 + 1,592 = 3,538$ ft per sec

Tangential force on blades

$$\begin{aligned}&= \frac{0.4 \times 3538}{32.2} \\ &= 43.95 \text{ lb}\end{aligned}$$

Axial force on blades

$$\begin{aligned}&= \frac{0.4}{32.2} (1026 - 840) \\ &= 2.3 \text{ lb}\end{aligned}$$

Work done on blades per sec.

$$= 43.95 \times 873 = 38,350 \text{ ft.-lb}$$

Horse-power developed

$$= \frac{38,350}{550} = 69.72$$

Kinetic energy supplied by steam jets per sec.

$$= \frac{0.4 \times 3000^2}{64.4} = 55,900 \text{ ft.-lb.}$$

$\therefore$ Efficiency of blading

$$= \frac{38,350}{55,900} \\ = 68.6 \text{ per cent}$$

Inlet angle of blades for shockless inflow of steam

$$= \beta_1 = 27^\circ 48'$$

63. Influence of Ratio of Blade Speed to Steam Speed on Blade Efficiency in Single-stage Turbine. Consider now the special case

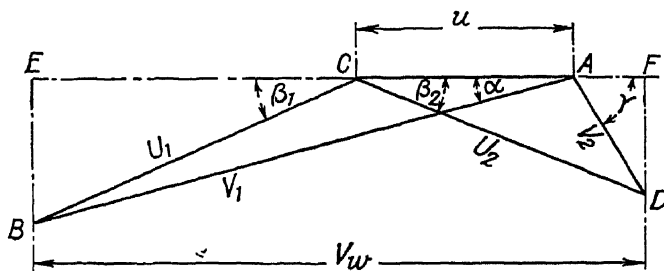


FIG. 68

of a single-stage impulse turbine consisting of a single nozzle or a set of nozzles expanding steam to the exhaust pressure and a single wheel with axial-flow impulse blades of the normal type, as illustrated in Fig. 63. The velocity diagram is shown in Fig. 68.

We shall take into account the effects of friction in the blade channels by assuming that U_2 is less than U_1 , and that $\frac{U_2}{U_1} = k$, where k is a constant less than unity. As will be shown later in this chapter, the blade outlet angle β_2 depends on the height of the blade, and vice versa. In many cases in practice the angles β_1 and β_2 are equal; in other cases β_2 is somewhat smaller than β_1 , but in pure impulse turbines the difference between the angles is always small.

It has been shown in Art. 62 that the blade efficiency

$$\eta_b = \frac{2u \cdot V_w}{V_1^2}$$

Now
$$\begin{aligned} V_w &= CE + CF \\ &= U_1 \cos \beta_1 + U_2 \cos \beta_2 \\ &= U_1 \cos \beta_1 \left(1 + \frac{U_2 \cos \beta_2}{U_1 \cos \beta_1} \right) \\ &= U_1 \cos \beta_1 (1 + kC) \end{aligned}$$

where $C = \frac{\cos \beta_2}{\cos \beta_1}$. As stated above, β_1 and β_2 are usually nearly

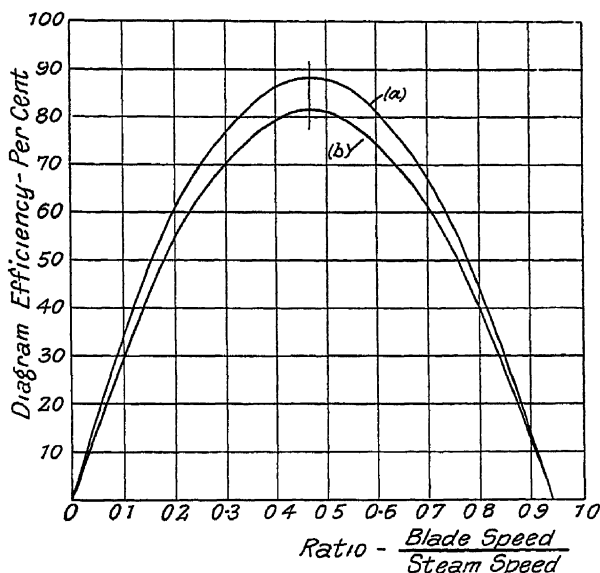


FIG. 69

(a) Diagram Efficiency without Losses
(b) " " " with " "

equal, and for the purpose of this analysis it will be sufficiently accurate to assume that C is a constant

But $U_1 \cos \beta_1 = V_1 \cos \alpha - u$
 $\therefore 2u \cdot V_w = 2u(1 + kC)(V_1 \cos \alpha - u)$

Let $\rho = \frac{u}{V_1}$ This is known as the blade speed ratio and is the most important factor in turbine theory and design, as the efficiency of a steam turbine depends almost wholly upon its value.

Then $u = \rho \cdot V_1$
 and $2uV_w = 2V_1^2(1 + kC)(\rho \cos \alpha - \rho^2)$
 or $\eta_b = 2(1 + kC)(\rho \cos \alpha - \rho^2)$ (69)

This is the equation to the parabolas (a) and (b) of Fig. 69. In

drawing these it has been assumed that $\alpha = 20^\circ$, and $\beta_2 = \beta_1$. Curve (a) is for frictionless flow while (b) is for flow with friction, the value of k being 0.84. The curves show that the blade efficiency increases as ρ is increased until ρ reaches a value approximately one-half, after which η_b diminishes.

If α , k , and C are assumed to be constant, then η_b depends on the value of ρ . Differentiating (69), we have—

$$\frac{d\eta_b}{d\rho} = 2(1 + kC)(\cos \alpha - 2\rho)$$

$$\frac{d^2\eta_b}{d\rho^2} = -4(1 + kC)$$

For a maximum or a minimum value of η_b we equate the first differential coefficient to zero, and since the second differential coefficient is negative the value obtained is the maximum.

$$\therefore \cos \alpha - 2\rho = 0$$

i.e. optimum value of ρ

$$\rho_{\text{opt.}} = \frac{\cos \alpha}{2} \quad (70)$$

Substituting this value in (69), we have

$$\eta_b(\text{max.}) = (1 + kC) \frac{\cos^2 \alpha}{2} \quad . \quad . \quad . \quad (71)$$

64. Gross Stage Efficiency. This may be defined as the ratio of the work done on the blades per lb. of steam flowing through the nozzles and blades to the adiabatic heat drop in the stage.

Let η_{gs} = gross stage efficiency.

H_a = adiabatic heat drop in stage (in this case in the nozzles).

By eq. (66*b*), Art. 62, the work done on the blades per lb. of steam is $\frac{u \cdot V_w}{gJ}$ B.Th.U.

$$\eta_{gs} = \frac{u \cdot V_w}{g \cdot J \cdot H_a}$$

$$= \left(\frac{2u \cdot V_w}{V_1^2} \right) \times \left(\frac{V_1^2}{2gJ \cdot H_a} \right)$$

or

$$\eta_{gs} = \eta_b \cdot \eta_n \quad . \quad . \quad . \quad . \quad (72)$$

In words, the gross stage efficiency is the product of the blading efficiency and the nozzle efficiency.

For any given nozzle angle, since η_n would remain constant, the gross stage efficiency will be a maximum when η_b is a maximum. Values of the optimum blade speed ratio, of the maximum blade efficiency, and of the maximum gross stage efficiency are given in

Table II for various nozzle angles. In the last section of the table an attempt has been made to allow for the influence of nozzle angle on the nozzle efficiency, data being taken from Fig. 62, Art. 60. The resulting figures show the importance of using the smallest practicable nozzle angle in single-stage turbines. However, such turbines are usually small power machines used for driving auxiliary machinery and the like and, in consequence, the attainable efficiency is not so important as in turbines of high power.

TABLE II
INFLUENCE OF NOZZLE ANGLE ON OPTIMUM BLADE SPEED RATIO,
MAXIMUM BLADE EFFICIENCY, AND MAXIMUM GROSS STAGE EFFICIENCY
IN SINGLE-STAGE TURBINES
Values based on $k = 0.86$

Nozzle Angle α	12°	16°	20°	25°	30°
ρ_{opt} η_b (max), per cent	0.489 88.98	0.481 85.92	0.470 82.13	0.453 76.40	0.433 69.74
Maximum value of η_{gs} when η_n is assumed constant and equal to 0.93, per cent	82.75	79.91	76.38	71.06	64.86
η_v for nozzle $\eta_n = \eta_v^2$	0.9590 0.919	0.9645 0.930	0.9691 0.938	0.9750 0.950	0.9790 0.9585
Maximum value of η_{gs} when account is taken of the probable variation of η_n with variation of nozzle angle, per cent	81.77	79.91	77.03	72.57	66.85

65. Efficiency of Multi-stage Impulse Turbine with Single-row Wheels. In the first stage of a multi-stage impulse turbine, the initial velocity of the steam is so low that the steam may be considered to start from rest. Of the mechanical energy of the steam issuing from the first-stage nozzles, a certain proportion is utilized in the blades, a smaller proportion is absorbed by friction, and the remainder is represented by the kinetic energy of the steam issuing from the blades with the final absolute velocity V_2 . In a single-stage turbine the last-mentioned item becomes important, and is the chief reason why the efficiency falls as the nozzle angle is increased.

In a multi-stage turbine, however, the kinetic energy of the steam emerging from one set of blades is conveyed by the steam into the succeeding set of nozzles, and, provided the inlet angle of the said nozzle guide vanes is such as to reduce impact and other losses to a minimum, then the adiabatic heat drop necessary to produce a given outlet velocity V_1 will be correspondingly smaller.

Thus the loss due to the velocity V_2 will be largely eliminated, except in the last stage of the turbine where there is little or no possibility of utilizing any of the kinetic energy corresponding to the final absolute velocity of the steam in that stage.

Consider a series of stages, all having the same nozzle outlet angle α , the same mean diameter, and the same steam velocity V_1 . Assume further that the velocity coefficient k is the same in all stages and that the blades have the same outlet angles. It follows that for any value of the blade velocity u (equal in all stages) the corresponding velocities will be identical in all stages. Fig. 70 shows the velocity diagrams for any two intermediate stages.

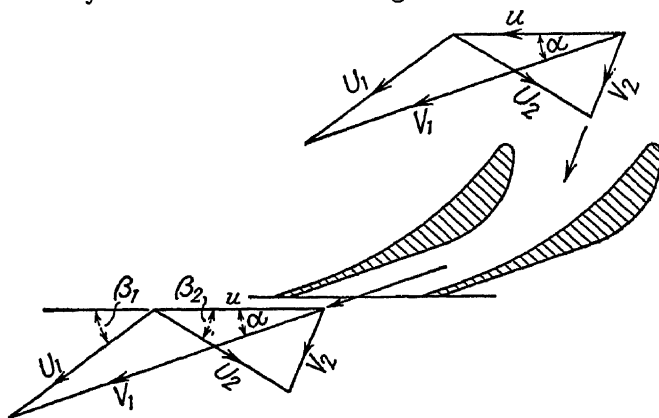


FIG. 70

Let η_n = nozzle efficiency, defined as the ratio

$$\frac{\text{Increase in Kinetic Energy in Nozzle}}{\text{Adiabatic Heat Drop}}$$

ϕ = "carry-over" coefficient, i.e. the fraction of the kinetic energy corresponding to V_2 which is available for utilization in the nozzles of the succeeding stage.

Then

$$\eta_n = \frac{V_1^2 - \phi \cdot V_2^2}{2gJ \cdot \Delta H} \quad (73)$$

where ΔH represents the adiabatic heat drop in one set of nozzles.

It will be assumed, in considering the influence of $\rho = u/V_1$ on efficiency, that the inlet angle of the nozzle guide vanes will be so adjusted (see Fig. 70) as to give ϕ its maximum value. Moreover, it will be assumed that this value is constant.

$$\begin{aligned} \text{Then } V_2^2 &= U_2^2 + u^2 - 2u \cdot U_2 \cdot \cos \beta_2 \\ &= k^2 \cdot U_1^2 + u^2 - 2k \cdot u \cdot U_1 \cos \beta_2 \\ &= k^2(V_1^2 - u^2 - 2u \cdot U_1 \cos \beta_1) + u^2 - 2k \cdot u \cdot U_1 \cos \beta_2 \\ &= k^2 V_1^2 + u^2(1 - k^2) - 2uU_1 \cos \beta_1(k^2 + kC) \end{aligned}$$

where $C = \frac{\cos \beta_2}{\cos \beta_1}$ As already mentioned, this factor is always nearly equal to unity and will be assumed constant for all values of ρ

$$\begin{aligned} \therefore V_2^2 &= k^2 V_1^2 + u^2(1 - k^2) - 2u(V_1 \cos \alpha - u)(k^2 + kC) \\ &= V_1^2\{k^2 + \rho^2(1 - k^2) - 2(\rho \cos \alpha - \rho^2)(k^2 + kC)\} \\ \therefore \eta_n &= \frac{V_1^2\{1 - \phi[k^2 + \rho^2(1 - k^2) - 2(\rho \cos \alpha - \rho^2)(k^2 + kC)]\}}{2gJ \cdot \Delta H} \end{aligned}$$

Therefore, the required adiabatic heat drop per stage

$$\Delta H = \frac{V_1^2\{1 - \phi[k^2 + \rho^2(1 - k^2) - 2(\rho \cos \alpha - \rho^2)(k^2 + kC)]\}}{2gJ \cdot \eta_n} \quad (74)$$

Since the work done on the blades is equal to

$$\frac{V_1^2}{g}(1 + kC)(\rho \cos \alpha - \rho^2)$$

the gross stage efficiency is

$$\begin{aligned} \eta_{gs} &= \{2\eta_n(1 + kC)(\rho \cos \alpha - \rho^2)\} \\ &\quad - \{1 - \phi[k^2 + \rho^2(1 - k^2) - 2(\rho \cos \alpha - \rho^2)(k^2 + kC)]\} \quad (75) \end{aligned}$$

The effect of taking into account the "carry-over" from the previous stage is best shown by working out examples Table III gives the efficiencies calculated for the following assumed data $k = 0.86$, $C = 1$ (equiangular blades), $\phi = 0.90$. The work of

TABLE III
CALCULATED GROSS STAGE EFFICIENCIES OF MULTI-STAGE IMPULSE
TURBINES

Assumed data $k = 0.86$, Equiangular blades, $\phi = 0.90$

Case	1	2	3	4	5
Nozzle angle α	12°	16°	20°	25°	30°
η_v	0.959	0.9645	0.9691	0.975	0.979
$\eta_n = \eta_v^2$	0.919	0.930	0.938	0.950	0.958
$\rho = 0.1$	0.511	0.512	0.510	0.501	0.493
0.2	0.688	0.691	0.687	0.683	0.670
0.3	0.770	0.776	0.773	0.768	0.755
0.4	0.819	0.825	0.820	0.813	0.797
0.5	0.845	0.851	0.845	0.833	0.814
0.6	0.857	0.860	0.853	0.834	0.804
0.7	0.853	0.850	0.836	0.802	0.747
0.8	0.810	0.802	0.769	0.699	0.562
0.9	0.676	0.645	0.517	0.130	—

Ackeret, Keller, and Salzmann (2) has shown that when the conditions of entry into the nozzle channel are favourable, an even higher value of ϕ may be realized. The nozzle efficiency is taken as a function of the angle as given in Fig. 62. This is not quite correct for all values of ρ because the angle through which the jet is deflected

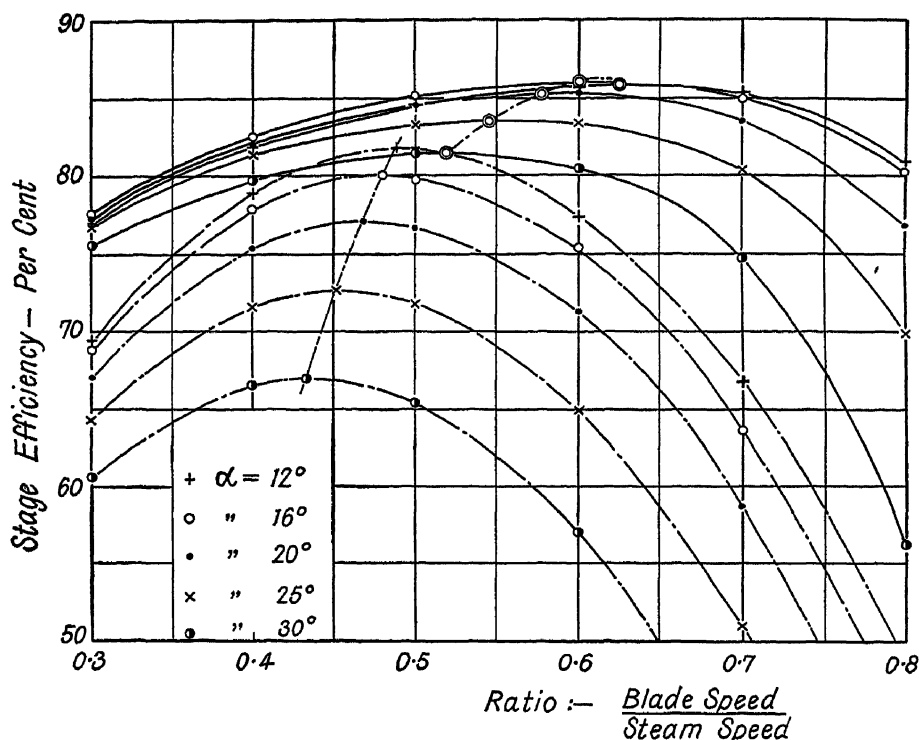


FIG. 71.—Curves of Stage Efficiency for Single stage and Multi-stage Impulse Turbines with Single-row Wheels

Full lines refer to multi-stage turbines, broken lines to single-stage turbines. Cross curves are loci of points of maximum efficiency. See Table III for relevant data.

by the nozzle will vary according to the value of ρ , but it serves to give relative values of the efficiency.

The results are plotted in Fig. 71 for the more useful range of values of ρ . Values of η_{ss} for a single-stage turbine are shown in the same figure. It will be seen that when the carry-over is taken into account—

(1) The efficiency curves are appreciably flattened, and variation of ρ on either side of the optimum value does not affect the efficiency to the same extent as when carry-over is neglected.

(2) The maximum efficiency of the multi-stage turbine is definitely greater than that of the single-stage turbine for the same nozzle

angle For a normal angle of 16° , the maximum value of η_{gs} is raised from 0.799 to 0.860. For the relatively large angle of 30° , which would be used in the latter stages of a large output turbine, the value is raised from 0.668 to 0.815.

(3) The value of ρ corresponding to the maximum efficiency is

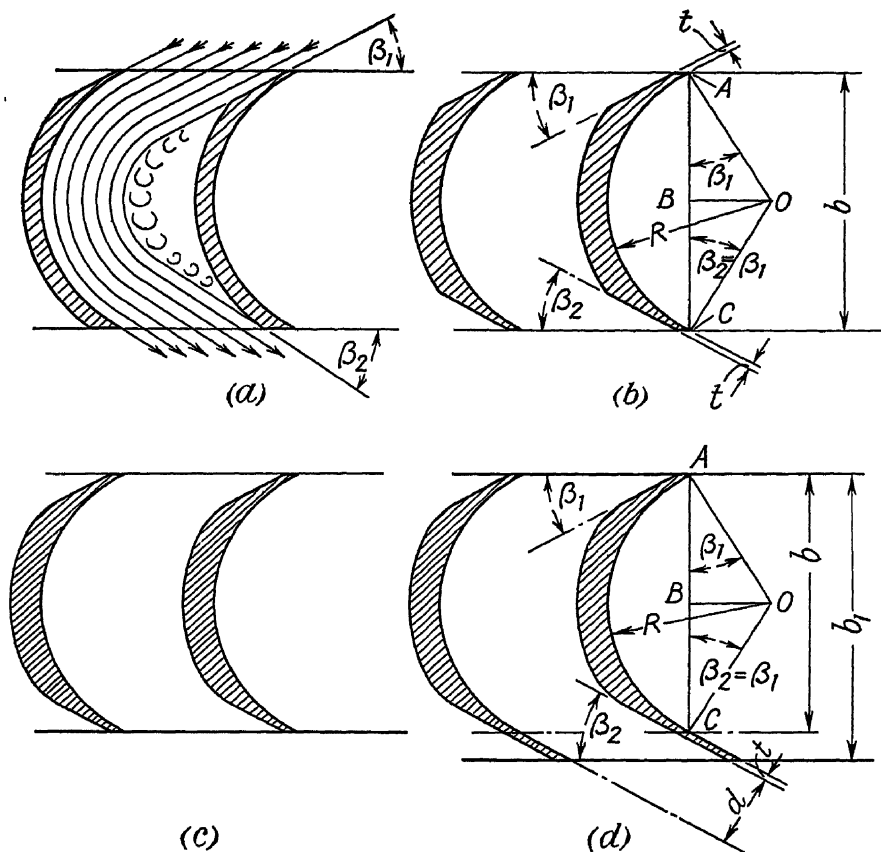


FIG 72 —Plate Blades

higher. When α is 16° , the optimum values are respectively 0.48 and 0.6 for the single-stage and multi-stage turbines.

66. Impulse Blade Sections. Impulse blades may be classified broadly into two types: (1) the so-called "plate blade," and (2) the "profile blade."

Various forms of plate blade are shown in Fig. 72. Fig. 72 (a) shows an early form of Rateau turbine blade constructed from sheet metal. In order to reduce impact losses at inlet, the inlet edge is chamfered, leaving an edge thickness of 0.01 in. Fig. 72 (b)

shows a symmetrical plate blade. The inlet and outlet angles are equal. The concave side of the blade is usually an arc of a circle, the blade is of constant thickness, and both inlet and outlet edges are chamfered. Fig. 72 (c) shows a further modification. The blade section shown in Fig. 72 (d) is provided with a straight outlet edge of

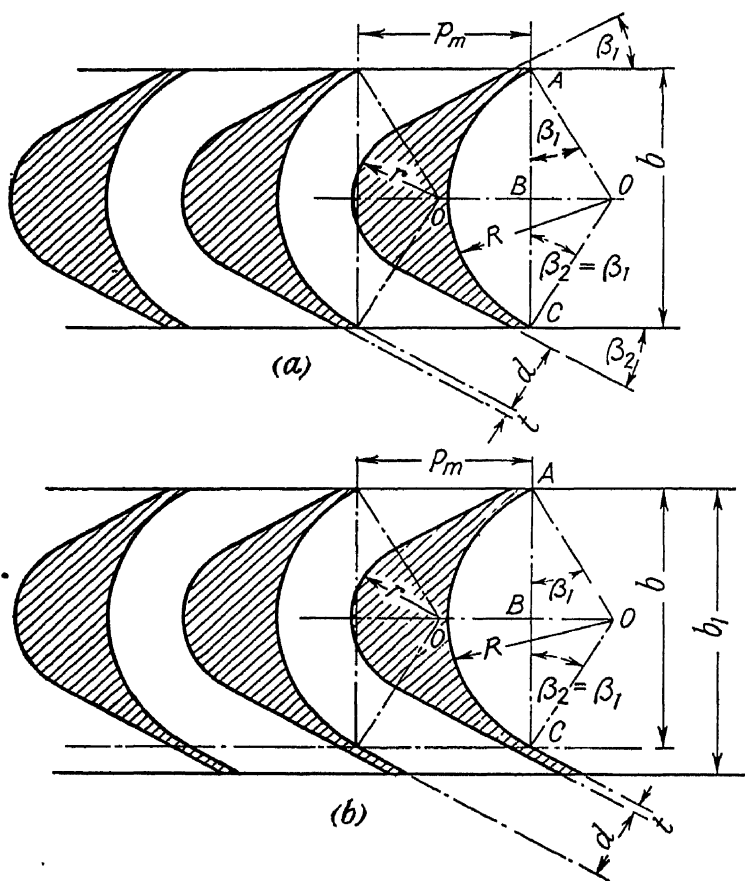


FIG. 73.—Profile Blades

uniform thickness t . The object of this is to give better control of the jet of outflowing steam. The plate type of blade is simple to manufacture, and because of the comparatively light section the centrifugal force on the blade and hence the stresses imposed on the rotor due to edge loading are relatively light. A further result of this is that the attachment of the blade to the rotor is considerably simplified. On the other hand, the channel area provided in the blade for the flow of steam rapidly increases from inlet to the centre, and then rapidly diminishes from the centre to the outlet

(see Fig. 72 (a)), with the result that the steam breaks away from the blade surface and an eddy space is formed on the convex side of the blade.

The eddy space could be more or less entirely avoided if the section of the blade were extended so as to occupy the eddy space. This is done in the profile blade, as shown in Fig. 73. A symmetrical equiangular blade is shown in Fig 73 (a). By making the centres of curvature of the convex and concave sides of adjacent blades coincide, the channel width is kept constant. Fig 73 (b) shows an equiangular blade with a straight outlet edge to give a "controlled" jet. It is necessary to give the blade a certain thickness t at the inlet and outlet edges so that it will have the necessary strength and rigidity to withstand the strains incidental to machining. The impact of the high-velocity

jet of steam on the inlet edge, however, causes a disturbance in the flow. This may be almost entirely eliminated by using the blade section shown in Fig. 74. This, again, is a blade having equal inlet and outlet angles. The angle made by the flat leading edge on the inlet side is taken to be the inlet angle β_1 , but the tangent to the concave arc makes an angle $\beta_1 + \psi$ with the wheel plane, ψ being a small angle of the order 2 to 4 degrees. As the leading and trailing surfaces thus

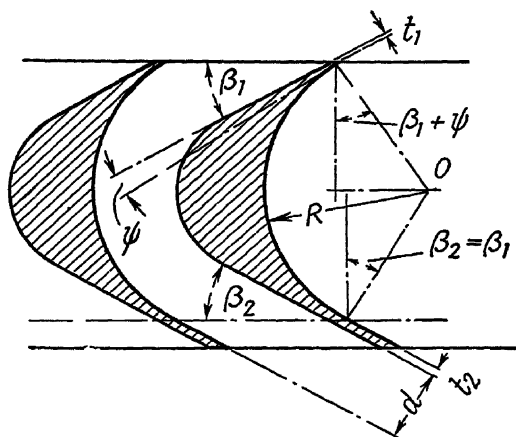


FIG 74

diverge rather rapidly, a smaller thickness t_1 may be used at the inlet edge. In fact, the inlet edge may be made almost knife-edged. The thickness t_2 of the outlet edge would be about 0.02 to 0.03 in., according to the size of the blade and the blade angle. The resulting blade, though nominally equiangular, is unsymmetrical.

In the equiangular blades shown, excepting Fig 74, it will be clear that

$$b = AB + BC = 2R \cos \beta_1$$

from which

$$R = \frac{b}{2 \cos \beta_1}$$

In the majority of impulse blades the outlet angle is smaller than the inlet angle. Such a blade is shown in Fig. 75. Assuming that

the concave side of the blade is a circular arc extending from A to D, then the width

$$b = R(\cos \beta_1 + \cos \beta_2)$$

$$R = \frac{b}{\cos \beta_1 + \cos \beta_2}$$

In setting out the blade profile it is preferable to calculate the exact

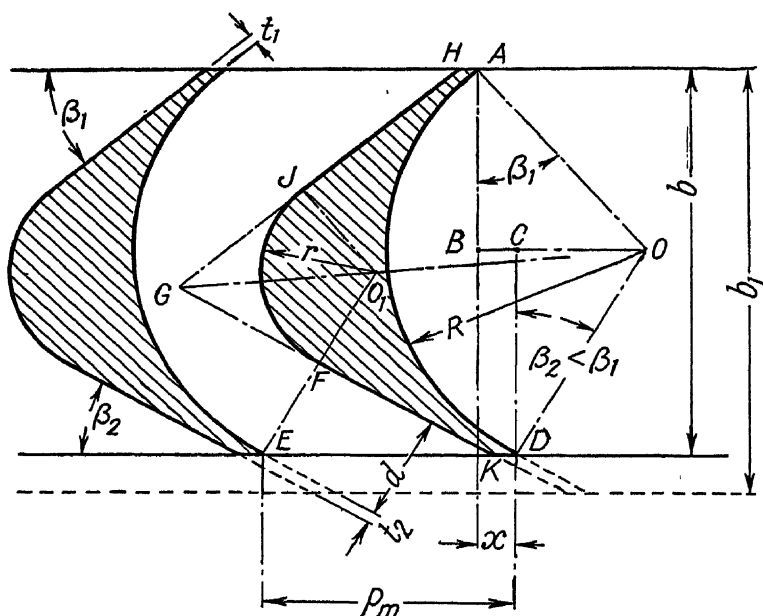


FIG. 75.—Profile Blade with Unequal Angles

position of the point D and, having calculated the radius R, to locate the centre O—

$$x = BC = R(\sin \beta_1 - \sin \beta_2)$$

The centre of curvature O_1 of the convex part of the profile may be found by drawing EF perpendicular to KF and producing it to intersect the bisector of the angle HGK in O_1 . The profile may be modified, as shown by the dotted lines, by extending the outlet edge.

67. Choice of Blade Angles. It is essential that the steam should flow into the blade with the minimum of loss. Consequently, the flat edge HJ, Fig. 75, should make an angle β_1 with the plane of the wheel, where β_1 is the *relative* direction of steam flow into the blade as given by the velocity diagram. The diagram is usually based on the steam conditions either at full load or at the economical load in the case of large central station turbines. Under fractional load

conditions the heat drop per stage will be smaller than under full load conditions, the greatest reduction in stage adiabatic heat drop occurring in the last one or two stages. Consequently, the jet velocity AB' at partial load, Fig 76, will be smaller than the value AB at full load, and the inlet angle β_1' will be larger than β_1 . It is obviously impossible to legislate for all conditions of load, but when a turbine is likely to operate for considerable periods at partial loads the blade inlet angle may be made greater than the diagram angle by the following amounts—

In the last stage	5–10 degrees
In the penultimate stage	4– 5 „
In the other stages	2– 3 „

The outlet angle must be found from a consideration of the area required at outlet for the flow of steam.

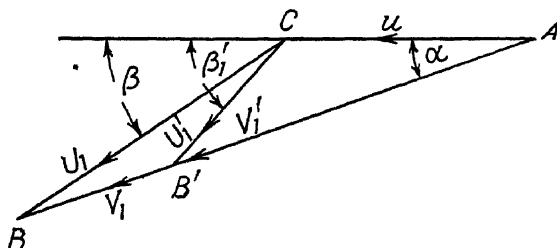


FIG 76

Let v_2 = specific volume of steam at outlet from blade.

l_2 = radial length of blade on the outlet side, inches

p_m = mean circumferential pitch of blades, inches.

n = number of blades through which steam is flowing.

With full admission this is obviously equal to

$$\frac{\pi \cdot D_m}{p_m}. \text{ With partial admission and a nozzle arc } \lambda,$$

$$n = \frac{\lambda}{p_m}.$$

t = thickness of blade outlet edge, inches

Then the mean effective width of the blade channel at outlet (Figs 72, 73, 74, and 75) is

$$d = p_m \sin \beta_2 - t$$

Total area through active blades

$$= n \cdot l_2 (p_m \sin \beta_2 - t) \text{ sq. in.}$$

$$\text{Area required} = \frac{144w \cdot v_2}{U_2} \text{ sq. in.}$$

Equating these, we have

$$\sin \beta_2 = \frac{1}{p_m} \left\{ \frac{144 \cdot w \cdot v_2}{n \cdot l_2 \cdot U_2} + t \right\} \quad (76)$$

If the calculated angle should not prove suitable, e.g. if it cannot be accommodated by available or standard blade sections, then it is preferable to use a blade section with a somewhat smaller angle than one with a slightly larger angle, because the slight restriction in area will bring about a small amount of reaction in the blade

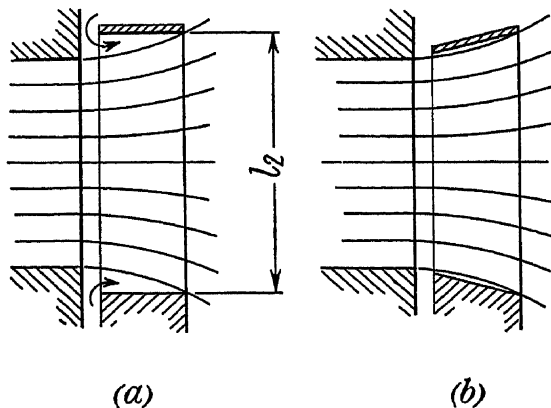


FIG. 77

which is normally beneficial from the point of view of blade performance.

The blade height l_2 is, to some extent, an arbitrary dimension. It will be clear from eq. (76) above that if l_2 be increased the angle β_2 may be reduced, thus giving an increased velocity of whirl and a slight apparent gain in efficiency. In a blade of constant height, such as that shown in Fig. 77 (a), the height l_2 must be selected with due regard to the conditions at inlet. It is necessary to make the radial height of the blade at inlet slightly greater than that of the nozzles in order to prevent the steam jets striking the shrouding strip or the blade root. Quite a small allowance should be sufficient for this. If a large allowance is made then a certain amount of steam is drawn into the blade channels by frictional entrainment from the space between the blade and the diaphragm, and this involves a certain amount of loss which may be avoided by the more expensive construction shown in Fig. 77 (b).

68. Experiments on Impulse Turbine Blades. Most of the experiments on impulse turbine blades have been made with the object of finding the ratio $k = U_2/U_1$, i.e. the ratio of the relative velocity at

outlet to that at inlet to the blade. Furthermore, most of the experiments have been made on a small group of stationary blades, termed a "packet," a nozzle or nozzles being used having an outlet angle equal to the blade inlet angle. The force on the blade packet is measured, and from the known inlet velocity, steam flow, and blade angles the value of k can be calculated. Such experiments are open to certain criticisms. The number of blades is small and some of the blade channels may be only partially filled with steam. Moreover, it is unlikely that steam will behave in the same manner in rapidly moving blades as it does in stationary blades.

Rateau (3) carried out a number of experiments on stationary blades of the type shown in Fig. 72 (a). The inlet angle was 31° and the outlet angle 25° , and the pitch was one-half the width of the blades in all cases. The general results of Rateau's experiments indicate that k decreases with wider blades, the value varying from 0.65 with blades 26 mm. (1 in.) wide to 0.83 with blades 12 mm. ($\frac{1}{2}$ in.) wide. The effect of varying the relative velocity was small, but the experiments showed that k increased as the steam velocity was increased. N. Briling (4) carried out extensive experiments on blades of the type shown in Fig. 72 (b). Contrary to the results of Rateau's experiments, Briling found that the blade loss is greater with narrow blades than with wide blades. This is, perhaps, to be expected since, although the steam path is longer with a wider blade and the pure friction losses therefore greater, the shock losses arising from the sudden deflection of the high-velocity steam jet are likely to be less. Briling made experiments on the effect of varying the blade pitch and found that the best pitch was given by the expression

$$p = \frac{R}{2 \sin \beta_1} \quad (77)$$

With equiangular blades, it follows that since

$$R = \frac{b}{2 \cos \beta_1}, \text{ then } p = \frac{b}{2 \sin 2\beta_1} \quad (78)$$

No such results are available for profile blades, but the draughting of such blades shows that a suitable pitch would be from 1.0 to 1.2 times the pitch given by Briling's formula.

Dr. P. Christlein (5) adapted the pendulum type nozzle tester shown diagrammatically in Fig. 49. The blades under test were attached to the pendulum, steam flowing into the blades from a fixed nozzle. For steam velocities ranging from 600 to 1,400 ft. per sec.

$$k = 0.676 + \frac{U_1}{10,500}$$

Dr. Ing. H. Faltin (6) carried out experiments on a single-stage experimental turbine having a mean blade ring diameter of 50 in.,

and blades 2.756 in. in height. Faltin found that the value of k depends on the amount of reaction in the wheel. For pure impulse working, the value of k first diminishes with increasing velocity until the velocity reaches 1,400 ft. per sec., after which k increases again. These results suggest that k would reach high values at low velocities. The following values of k have been interpolated from Faltin's published curves for pure impulse conditions.

Steam Velocity (U_1)	k	Steam Velocity (U_1)	k
650	0.913	1,300	0.747
700	0.894	1,400	0.742
800	0.858	1,500	0.743
900	0.825	1,600	0.752
1,000	0.798	1,700	0.765
1,100	0.777	1,800	0.783
1,200	0.760	1,900	0.806

Published information on the subject of impulse blade performance is very meagre and incomplete. Practical testing consists in finding the blade profile which, when tested in a multi-stage experimental turbine in conjunction with efficient nozzles covering, preferably, the full circumference, will yield the highest value of the stage efficiency. The corresponding value of k could then be obtained approximately by working backwards and assuming reasonable values of the nozzle efficiency and carry-over factor.

69. Velocity-compounded Impulse Turbine. This has already been briefly described in Chapter I. Fig. 78 shows the details of a wheel carrying three rows of moving blades and commonly termed a "three-row wheel." (a) is a longitudinal section, and (b) is a part development through the line cc . A is the nozzle, B representing one of the vanes. C is the rim of the wheel, which is grooved to carry the three rows of moving blades M_1 , M_2 , and M_3 . There are two rows of fixed blades— F_1 , F_2 —which may be attached to the inside of the cylinder or to a segmental casting or forging D. The passage of the steam through the fixed and moving blades takes place at constant pressure, but, as the velocity diagram will show, the velocity of the steam is steadily reduced as the steam passes forward through the blades. Moreover, the losses which take place in the blade channels cause the steam to be reheated, and hence its specific volume increases. The increased volume and reduced velocity (the latter influence predominating) call for a considerable increase in the area required for flow. This is partly provided by the successive increase in blade length, but the increase in area due to this expedient is quite insufficient, and it becomes necessary to use blades having greater outlet angles. The inlet angles are, of course, automatically increased by the nature of the velocity diagram.

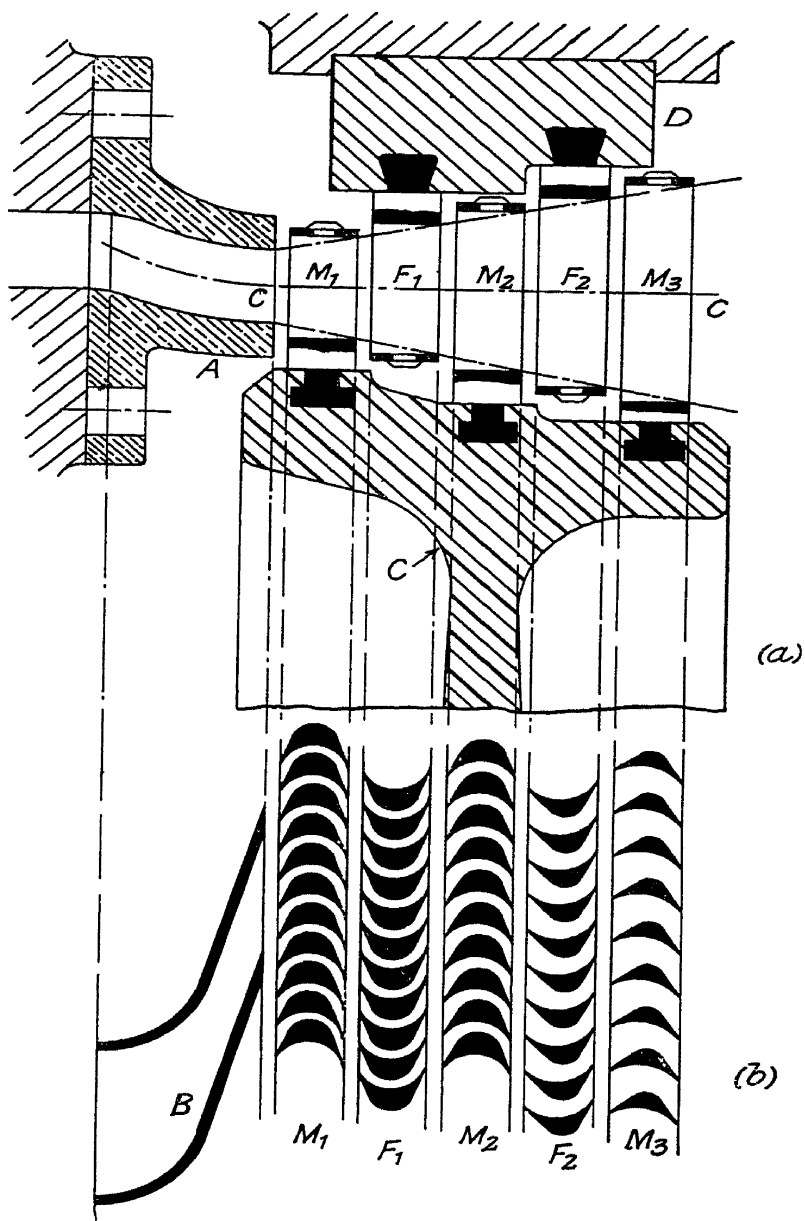


FIG. 78

70. **Velocity Diagram for Typical Velocity-compounded Wheel.** Fig. 79 shows the velocity diagram for a three-row wheel in what, in the author's opinion, is the most convenient, most easily understood and most useful form. The method employed in drawing the diagram is merely an extension of the method used for a single-row wheel. In the first row, for instance—

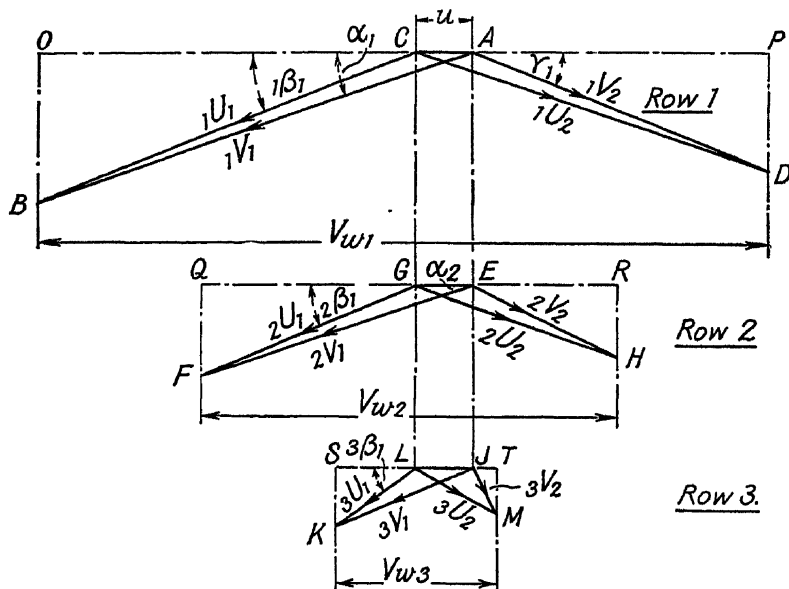


FIG. 79.—Velocity Diagrams for Three-row Wheel

${}_1V_1$ = absolute velocity of steam entering blades.

${}_1U_1$ = relative velocity of steam entering blades.

${}_1U_2$ = relative velocity of steam leaving blades.

${}_1V_2$ = absolute velocity of steam leaving blades.

OCB = blade inlet angle.

PCD = blade outlet angle.

Thus ${}_1V_2$ is the velocity with which the steam enters the first row of fixed blades, and the angle PAD is the inlet angle of the fixed blades if the steam is to enter them with the minimum shock loss.

In the velocity diagram for the second row of blades, ${}_2V_1$ is the absolute velocity of the steam issuing from the first row of fixed guide blades. Owing to friction in those blades, ${}_2V_1$ is less than ${}_1V_2$ and we may write

$${}_2V_1 = k \cdot {}_1V_2$$

where k represents the blade velocity coefficient. The remainder of the diagrams will be sufficiently clear without further explanation.

The velocities of whirl in the three rows are, respectively, V_{w1} , V_{w2} , and V_{w3} . If w is the mass of steam passing through the blades per second, then

$$\text{Work done per second in first row} = \frac{w}{g} \cdot u V_{w1}$$

$$\text{Work done per second in second row} = \frac{w}{g} \cdot u V_{w2}$$

$$\text{Work done per second in third row} = \frac{w}{g} \cdot u V_{w3}$$

$$\begin{aligned} \text{Total work done per second} &= \frac{wu}{g} (V_{w1} + V_{w2} + V_{w3}) \\ &= \frac{wu}{g} \cdot \Sigma V_w \end{aligned} \quad (79)$$

$$\text{Total power developed} = \frac{wu}{550g} \Sigma V_w \text{ r h p.} \quad (80)$$

$$\text{Initial kinetic energy per second} = \frac{w \cdot {}_1V_1^2}{2g}$$

$\therefore$ Blade efficiency

$$\eta_b = \frac{2u}{{}_1V_1^2} \Sigma V_w \quad (81)$$

If η_n is the nozzle efficiency, then the gross stage efficiency

$$\eta_{gs} = \frac{2u}{{}_1V_1^2} \cdot \eta_n \cdot \Sigma V_w \quad (82)$$

71. Most Economical Ratio of Blade Speed to Steam Speed for a Two-row Wheel. As in the case of the single-row impulse wheel, there is a certain ratio of blade speed to steam speed which results in the greatest blade efficiency. This ratio will now be determined analytically, the following assumptions being made—

(1) That the blades in each row are equiangular

(2) That the velocity coefficient k is the same for each row of blades, i.e. the friction loss in the blades, fixed as well as moving, is directly proportional to the square of the velocity relative to the blade.

The velocity diagrams are shown in Fig 80. Let ${}_1V_1$ be the absolute velocity of the steam entering the first row of blades, and let it be assumed that for each value of u the blade inlet angles are adjusted to secure shockless inflow of the steam. Then the following relations may be written down—

$$AL = {}_1V_1 \cos \alpha \qquad CL = {}_1V_1 \cos \alpha - u$$

$$CM = k ({}_1V_1 \cos \alpha - u)$$

$$AM = k ({}_1V_1 \cos \alpha - u) - u.$$

$$\begin{aligned} \therefore V_{w1} &= LA + AM \\ &= {}_1V_1 \cos \alpha + k {}_1V_1 \cos \alpha - ku - u \end{aligned}$$

Since $EF = k \cdot AD$, and angle $NEF =$ angle MAD

$$EN = k^2({}_1V_1 \cos \alpha - u) - ku$$

$$GN = k^2({}_1V_1 \cos \alpha - u) - ku - u$$

$$GO = k^3({}_1V_1 \cos \alpha - u) - k^2u - ku$$

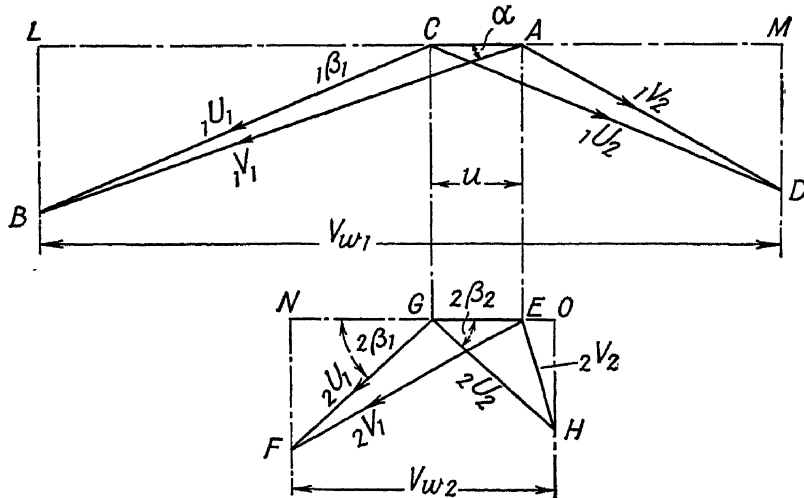


FIG. 80

$$\begin{aligned} \therefore V_{w2} &= NG + GO \\ &= k^3 \cdot {}_1V_1 \cos \alpha + k^2 \cdot {}_1V_1 \cos \alpha - k^3u - 2k^2u - 2ku - u \end{aligned}$$

$$\begin{aligned} \therefore V_{w1} + V_{w2} &= K_1 \cdot {}_1V_1 \cos \alpha - K_2u \\ \text{where} \quad K_1 &= 1 + k + k^2 + k^3 \\ \text{and} \quad K_2 &= 2 + 3k + 2k^2 + k^3 \end{aligned} \quad (83)$$

$$\therefore \text{Blade efficiency } \eta_b = \frac{2u}{{}_1V_1^2} (K_1 \cdot {}_1V_1 \cos \alpha - K_2u)$$

$$\text{Writing } \rho = \frac{u}{{}_1V_1}, \text{ i.e. } u = \rho \cdot {}_1V_1$$

$$\text{then } \eta_b = 2(K_1 \cdot \rho \cos \alpha - K_2 \cdot \rho^2) \quad (84)$$

$$\text{For maximum efficiency, } \frac{d\eta_b}{d\rho} = 0$$

$$\text{i.e. } 2K_1 \cdot \cos \alpha - 4K_2\rho_{\text{opt.}} = 0$$

$$\text{or } \rho_{\text{opt.}} = \frac{K_1}{K_2} \cdot \frac{\cos \alpha}{2} \quad (85)$$

Note that when the flow is frictionless, $k = 1$, $K_1 = 4$, and $K_2 = 8$. Hence the optimum value of ρ is $\frac{\cos \alpha}{4}$, i.e. just one-half the value for a single-row wheel.

Returning to the general case and substituting the optimum value of ρ in eq. (84), we have—

$$\eta_b (\text{max.}) = \frac{K_1^2}{K_2} \frac{\cos^2 \alpha}{2} \quad (86)$$

and
$$\eta_{gs} (\text{max.}) = \frac{K_1^2}{K_2} \eta_n \frac{\cos^2 \alpha}{2} \quad (87)$$

Data relative to the values of ρ , $\eta_b (\text{max.})$, etc., are compiled in Table IV for impulse wheels with one and two rows of moving blades.

TABLE IV
COMPARATIVE EFFICIENCIES OF SINGLE-ROW AND TWO-ROW IMPULSE WHEELS

Assumptions (1) $\alpha = 16^\circ$, (2) Blades in each row equiangular, (3) $\eta_n = 0.9$, (4) $k = 0.86$

Hence $K_1 = 3.236$, $K_2 = 6.695$

Rows of Moving Blades	1	2
Optimum value of ρ	$\frac{\cos \alpha}{2}$	$\frac{\cos \alpha}{4.14}$
Optimum value of ρ	0.480	0.232
$\eta_b (\text{max.})$	$0.930 \cos^2 \alpha$	$0.782 \cos^2 \alpha$
$\eta_b (\text{max.})$	0.859	0.723
Gross stage efficiency	0.773	0.651
Steam velocity V_1 when $u = 600$ ft per sec	1,250	2,585
$\frac{1}{2g} \frac{V_1^2}{J}$ B Th U per lb	31.2	133.3
Stage adiabatic heat drop (carry-over neglected)	34.7	148.1
Relative number of stages	4.27	1

The optimum value of ρ for a three-row velocity-compounded wheel may be shown to be approximately equal to $\frac{\cos \alpha}{6}$ and the maximum gross stage efficiency to be about 55 per cent. With four rows of moving blades the optimum blade speed ratio is about 0.12 and the maximum stage efficiency about 49 per cent.

72. Blade Heights in Velocity-compounded Wheels. Consider the flow of w lb. of steam per second through the nozzles and blading of a velocity-compounded turbine or stage, and assume, for example, that there are two rows of moving blades. Let l_n , l_1 , l_2 , and l_3 represent the radial heights at the outlet side of the nozzle, first row, fixed row, and second row respectively (Fig. 81). It may be

assumed, with little error, that the specific volume v is constant throughout the blade system. Actually, owing to friction losses in the blade channels, the steam will be reheated and v will be slightly increased; but the change in volume is not large.

Suppose that λ is the effective length of arc over which steam is flowing. The nozzle angle is α . Then by the equation of continuity

$$wv = {}_1V_1 \cdot \lambda \cdot l_n \cdot \sin \alpha$$

Let p_1 = mean circumferential pitch of blades in first row.

p_f = mean circumferential pitch of blades in fixed row.

p_2 = mean circumferential pitch of blades in second row.

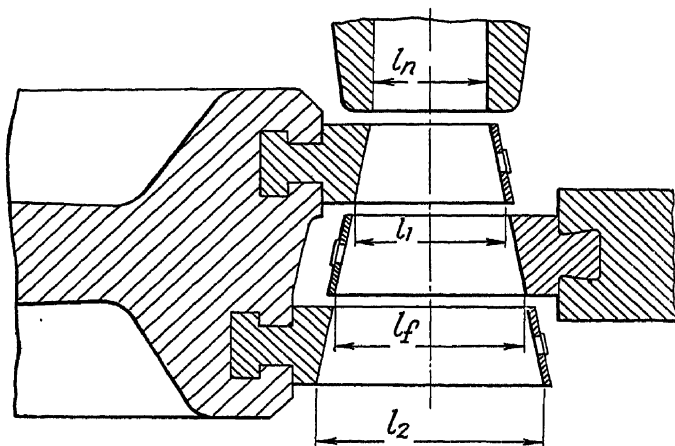


FIG. 81

It is reasonable to assume that the arc covered by the steam as it passes through the blade system remains sensibly constant. Then the number of blade channels through which steam is flowing in the first moving row is $\frac{\lambda}{p_1}$, and the total area through the blade channels at outlet is

$$\frac{\lambda}{p_1} \cdot l_1 (p_1 \sin {}_1\beta_2 - t_1)$$

where t_1 is the thickness of the outlet edge of the blades in the first row.

$$\text{Then} \quad wv = {}_1U_2 \cdot \frac{\lambda}{p_1} \cdot l_1 (p_1 \sin {}_1\beta_2 - t_1) \quad . \quad . \quad (88)$$

Thus the angle ${}_1\beta_2$ may be calculated for any given value of l_1 or, alternatively, l_1 may be calculated for any assumed value of ${}_1\beta_2$. The latter is, perhaps, the more fundamental process. For instance,

${}_1\beta_2$ may be made equal to the inlet angle ${}_1\beta_1$, Fig. 82, or ${}_1\beta_2$ may be made a few degrees smaller than ${}_1\beta_1$, as is often done in practice.

In a similar manner, it may be shown that

$$wv = {}_2V_1 \cdot \frac{\lambda}{p_f} \cdot l_f(p_f \cdot \sin \alpha_2 - t_f) \quad (88a)$$

where α_2 is the outlet angle of the fixed guide blades and t_f is the thickness of the outlet edge of the same

$$\text{Also} \quad wv = {}_2U_2 \cdot \frac{\lambda}{p_2} \cdot l_2(p_2 \cdot \sin {}_2\beta_2 - t_2) \quad (88b)$$

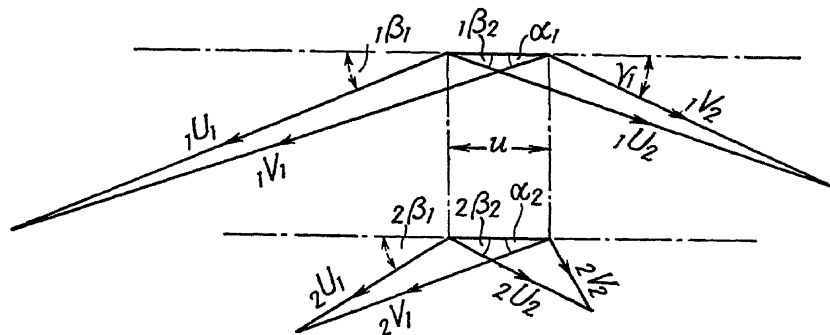


FIG 82

In practice the height ratio, i.e. the ratio of the blade height in the last moving row to the nozzle height, has the following values—

Two-row wheels	1.8 to 2.5
Three-row wheels	2.5 to 3.5

EXAMPLE 9. The nozzles in a two-row velocity-compounded impulse stage are $\frac{3}{4}$ in in height and extend over an arc of 30° . The effective angle at outlet is 16° and the specific volume of the steam is 10.0 cub. ft. per lb. The mean diameter of the blades is 28 in. and the rotational speed 3,000 r.p.m. The steam velocity at outlet from the nozzles is 1,830 ft. per sec.

The height of the second row of moving blades is 2.5 times the radial height of the nozzle, i.e. $1\frac{3}{4}$ in., and the blade heights increase in arithmetical progression.

Calculate the blade inlet and outlet angles, assuming that the blades have an axial width of 1 in., that the thickness of the outlet edge is 0.02 in., and that the blade velocity coefficient $k = 0.86$.

The mean circumferential pitch of the blades may be calculated by means of Brilong's formula

$$p_m = \frac{b}{2 \sin 2\beta_1}$$

First Moving Row.

$$u = \frac{\pi D}{12} \times \frac{N}{60} = \pi \times \frac{28}{12} \times \frac{3000}{60} = 367 \text{ ft. per sec.}$$

$$wv = {}_1V_1 \cdot \lambda \cdot l_n \cdot \sin \alpha$$

$$\therefore \frac{wv}{\lambda} = {}_1V_1 \cdot l_n \sin \alpha$$

$$= 1,830 \times \frac{3}{4 \times 12} \times 0.2756 = 31.5$$

The velocity diagram is shown in Fig. 83.

$${}_1U_1 = 1,481; {}_1U_2 = 1,273; {}_1\beta_1 = 20^\circ$$

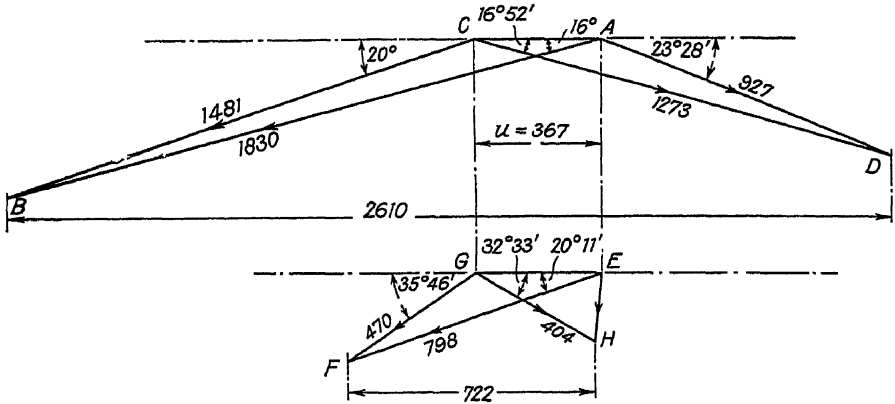


FIG. 83

Circumferential mean pitch of blades

$$p_1 = \frac{b}{2 \sin {}_1\beta_1} = \frac{1.0}{2 \sin 40^\circ} = 0.778 \text{ in.}$$

From eq. (88)

$$p_1 \cdot \sin {}_1\beta_2 = \frac{wv}{\lambda} \cdot \frac{p_1}{{}_1l_1 \cdot {}_1U_2} + t_1$$

$$\text{or} \quad \sin {}_1\beta_2 = \frac{wv}{\lambda} \cdot \frac{1}{{}_1l_1 \cdot {}_1U_2} + \frac{t_1}{p_1}$$

But l_1 (Fig. 84) is $1\frac{1}{8}$ in.

$$\begin{aligned} \therefore \sin {}_1\beta_2 &= \frac{31.5 \times 12}{1.125 \times 1273} + \frac{0.02}{0.778} \\ &= 0.290 \end{aligned}$$

from which

$${}_1\beta_2 = 16^\circ 52'$$

CD (Fig 83) is drawn at an angle of $16^{\circ} 52'$ to represent 1,273 ft. per sec, giving ${}_1V_2$ as 927 ft per sec, and the theoretical inlet angle of the guide blades as $23^{\circ} 28'$

Fixed Row

$$p_f = \frac{1.0}{2 \sin 46^{\circ} 56'} = 0.684$$

From eq (88a)

$$\begin{aligned} \sin \alpha_2 &= \frac{31.5 \times 12}{1.5 \times 798} + \frac{0.02}{0.684} \\ &= 0.345 \end{aligned}$$

from which $\alpha_2 = 20^{\circ} 11'$ This is the outlet angle of the fixed blades. EF is drawn at $20^{\circ} 11'$ to the straight line EG and to represent the velocity 798 ft per sec

Second Moving Row FG is joined, giving ${}_2U_1 = 470$

$$\therefore {}_2U_2 = 404, {}_2\beta_1 = 35^{\circ} 46'$$

$$\therefore p_2 = \frac{1.0}{2 \sin 71^{\circ} 32'} = 0.528$$

From eq (88b)

$$\begin{aligned} \sin {}_2\beta_2 &= \frac{31.5 \times 12}{1.875 \times 404} + \frac{0.02}{0.528} \\ &= 0.538 \end{aligned}$$

from which ${}_2\beta_2 = 32^{\circ} 33'$

The blade sections are shown in Fig 84 The actual blade inlet angles have been made somewhat larger than the inlet angles given by the velocity diagrams The circumferential pitch is, of course, an arbitrary dimension and could have been based on the actual blade inlet angle The effect of using asymmetrical blades may be gauged by calculating the stage efficiency, assuming a nozzle efficiency of 90 per cent

$$\begin{aligned} \eta_s &= \frac{2 \cdot \eta_n \cdot u}{{}_1V_1^2} \cdot \Sigma V_w \\ &= (2 \times 0.9 \times 367 \times 3332) \div 1830^2 \\ &= 65.75 \text{ per cent.} \end{aligned}$$

With equiangular blades and other things equal the stage efficiency is 64 per cent.

73. Advantages and Disadvantages of Velocity-compounding. The velocity-compounding principle leads to relatively large heat drops per stage and, therefore, to a comparatively small number of

stages. This is clearly shown by the data set out in Table IV, page 145. Thus the first cost of a velocity-compounded turbine is likely to be low. Against this advantage must be set the serious disadvantage of a low efficiency ratio and therefore a high steam consumption. For many years the vertical-spindle Curtis turbine having four two-row wheels was a favourite type of steam turbine

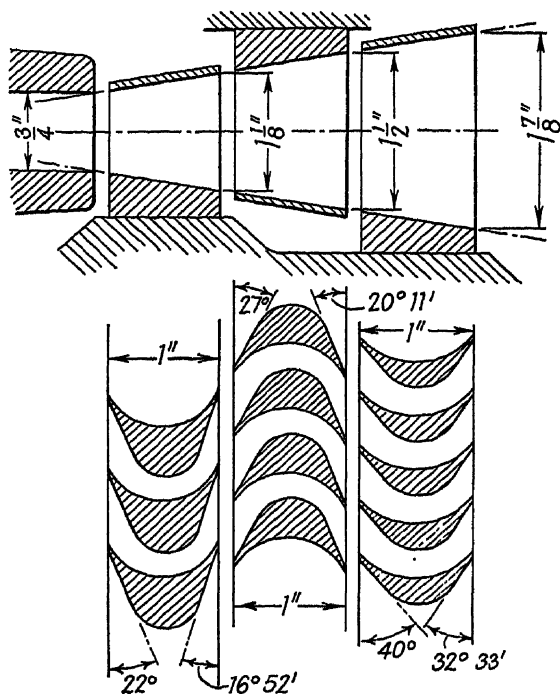


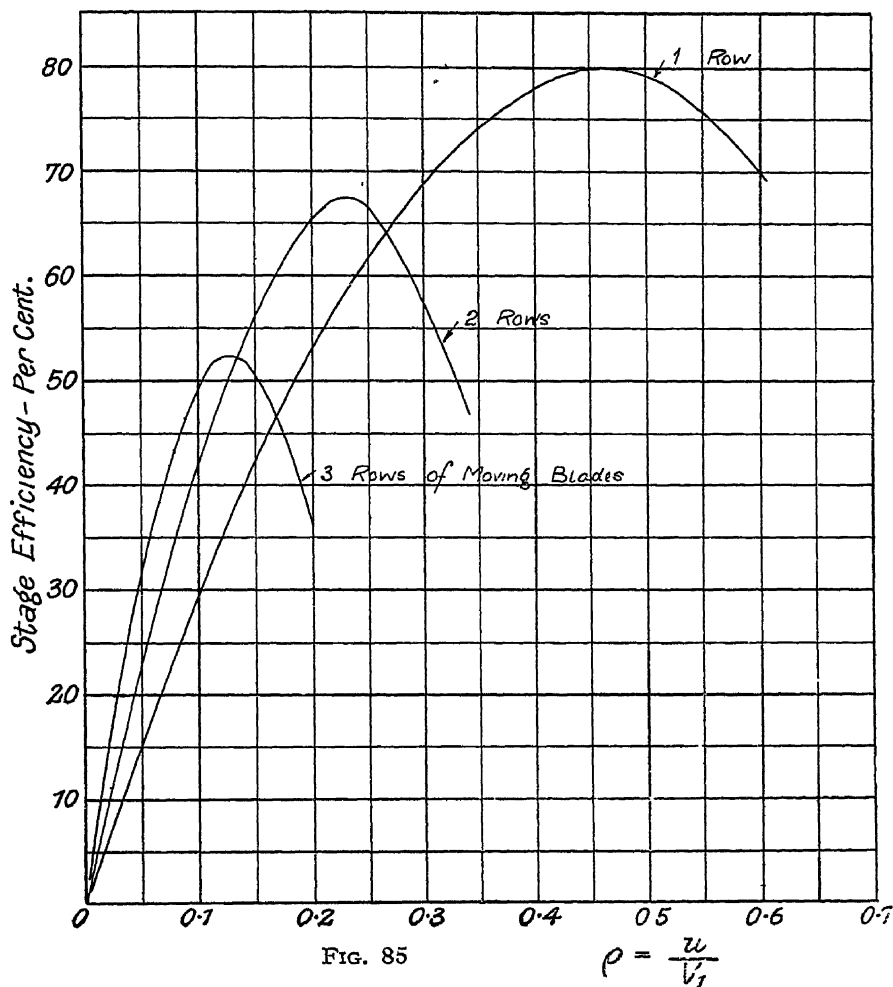
FIG. 84

in electricity generating stations, but it was gradually superseded by the more efficient turbine with single-row wheels. Approximate values of the stage efficiency of turbines having one, two, and three rows of moving blades are shown in Fig. 85.

There is, however, still one important application of velocity-compounding, namely, in the first stage of impulse or reaction turbines. The large heat drop which can be adopted with the two-row wheel enables a large drop in pressure to take place in the nozzles of the first stage. Comparative data are set out in Table V.

It will be seen that when a single-row wheel is used the gauge pressure in the turbine cylinder is 188.3 lb. per sq. in., whereas with a two-row wheel this is only 58.8 lb. per sq. in. An equally important point is the steam temperature. With a single-row wheel, the steam temperature is 548°F. , and this would be approximately

the temperature reached by the metal of the cylinder in that stage Ordinary cast iron may not be employed for temperatures in excess of 450° F owing to the phenomenon of "growth," and here a cast-steel cylinder would have to be used By employing a two-row



wheel, however, the steam temperature is sufficiently low to allow of cast iron being used, provided the steam chest is constructed of steel to withstand the high temperature

The efficiency ratio of the whole turbine would be reduced from about 77 per cent to about 73 per cent by the use of the two-row wheel, and this must be set off against the advantages mentioned above.

TABLE V

COMPARISON OF CONDITIONS IN THE FIRST-STAGE WHEEL CHAMBER OF AN IMPULSE TURBINE WITH ONE-ROW AND TWO-ROW WHEELS

Basic Data—

Initial Steam Pressure	250 lb. per sq. in. abs.
„ „ Temperature	600° F.
Blade Velocity	550 ft. per sec.
Nozzle Angle	16 deg.
Nozzle Efficiency	90 per cent
Stage Efficiencies: Single-row wheels	77 per cent
Two-row „	65 per cent

Rows of Blades	1	2
$p_{opt.}$	0.480	0.232
V_1	1146	2370
Corresponding heat drop, B.Th.U./lb.	26.2	111.9
Mean heat drop, B.Th.U./lb.	29.16	124.4
Pressure at nozzle outlet	203.0	73.5
Gauge pressure in cylinder	188.3	58.8
Heat equivalent of work done in stage	22.4	80.9
Value of I of steam in wheel chamber	1297.6	1239.1
Corresponding steam temperature, ° F.	548	410
Drop of pressure in stage	47	176.5
Drop of temperature in stage, ° F.	52	190

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EXAMPLES VII

1. Steam issues from the nozzles of a de Laval turbine with a velocity of 3,500 ft. per sec. The nozzle angle is 20° , the mean blade velocity is 1,200 ft. per sec., and the inlet and outlet angles of the blades are equal. The mass of steam flowing through the turbine per hour is 1,800 lb. Calculate—

- (a) The blade angles.
- (b) The relative velocity of the steam entering the blades.
- (c) The tangential force on the blades.
- (d) The horse-power developed.
- (e) The blade efficiency.

Assume that $k = 0.8$.

2 The mean diameter of the blades of an impulse turbine with a single-row wheel is 42 in. and the speed 3,000 r p m. The nozzle angle is 18° , the ratio of blade speed to steam speed is 0.42, and the ratio of the relative velocity at outlet from the blades to that at inlet is 0.84. The outlet angle of the blade is to be made 3° less than the inlet angle. The steam flow is 16 lb per sec.

Draw the velocity diagram for the blades and derive the following—

- (a) Resultant thrust on blades
- (b) Tangential thrust on blades
- (c) Axial thrust on blades
- (d) Horse power developed in blades
- (e) Blading efficiency

3. In a stage of an impulse turbine provided with a single-row wheel, the mean diameter of the blade ring is 32 in. and the speed of rotation is 3,000 r p m. The steam issues from the nozzles with a velocity of 900 ft per sec, and the nozzle angle is 20° . The rotor blades are equiangular and, due to friction in the blade channels, the relative velocity of the steam at outlet from the blades is 0.86 times the relative velocity of the steam entering the blades.

What is the power developed in the blading when the axial thrust on the blades is 26.9 lb?

4 Steam flows from the nozzles of a single-row impulse turbine with a velocity of 1,300 ft per sec, the nozzle angle being 16° . The final absolute velocity at outlet from the blades is 308 ft per sec in a direction $98^\circ 12'$ with the direction of motion of the blades. If the blades are equiangular, calculate the blade horse-power when the mass flow of steam through the blading is 16.1 lb per sec, and also calculate the friction loss per lb of steam in the blade channels.

5. Show that in a single-stage impulse turbine with equiangular blades, provided k , the ratio of the relative velocity at outlet from the moving blades to the relative velocity at inlet, is constant, the most efficient ratio of blade velocity to steam jet velocity is independent of the value of k .

Hence derive the optimum ratio of blade speed to steam speed and the corresponding blade inlet angle when the nozzle angle is 14° . What is the maximum blade efficiency in this case when $k = 0.9$? Assume that the inlet and outlet angles of the blade are equal.

6 The following particulars relate to a single stage of an impulse turbine—

Steam velocity at nozzle mouth	.	1,000 ft per sec.
Nozzle angle	.	18°
Mean blade velocity	.	480 ft per sec

Draw to a suitable scale the diagram of relative velocities for the steam, assuming that the outlet angle of the blades is 3° less than the inlet angle, and that the relative velocity of the steam at outlet from the blades is 0.84 times the relative velocity of the steam entering the blades.

If the power to be developed in that stage only is to be 1,000 kW, calculate the weight of steam which must pass through the turbine per second. Neglect disc friction and leakage losses.

7. Deduce a general expression for the blade efficiency of a stage of an impulse turbine with single-row wheels, assuming equiangular blades, a nozzle angle α , and that k is the ratio of the relative velocity at outlet from the moving blades to the relative velocity at inlet. What is the maximum efficiency if $\alpha = 20^\circ$, and $k = 0.83$?

If the blade efficiency is to be 90 per cent of the maximum value, what are the possible ratios of blade speed to steam speed? Draw the velocity diagrams for each case and state the blade angles.

8. The nozzles in a stage of an impulse turbine are of the convergent type and are formed by "casting-in" vanes of uniform thickness. The radial height of the nozzles is 6 in., the mean diameter 46 in., the nozzle angle 18° , the vane thickness 0.116 in., and the number of nozzles in the complete ring is 72. Given that the specific volume of the steam at the nozzle outlet is 246 cub. ft. per lb., and that the velocity of the steam at the nozzle outlet is 1,200 ft. per sec., calculate the weight of steam flowing through the nozzles per second.

Calculate the power developed in the blades of the given stage, assuming that there is a single row of moving blades having equal inlet and outlet angles, that the ratio of the relative outlet velocity to the relative inlet velocity is 0.86, and that the rotor turns at 3,000 r.p.m.

9. The adiabatic heat drop in a given stage of a multi-stage impulse turbine is 9.5 B.Th.U. per lb. of steam. The nozzle outlet angle is 16° , and the efficiency of the nozzle, defined as the ratio of the actual gain of kinetic energy in the nozzle to the adiabatic heat drop, is 92 per cent. The mean diameter of the blades is 58 in. and the revolutions per minute 1,500. Given that the "carry-over" factor ϕ is 0.88, that the blades are equiangular, and that the blade velocity coefficient is 0.87, calculate the steam velocity at outlet from the nozzles, the blade angles, and the gross stage efficiency.

10. The following particulars relate to a single-stage impulse turbine with a single row of moving blades—

Nozzle angle	18°
Steam velocity at nozzle outlet	800 ft. per sec.
ϕ	0.85
Steam flow	50 lb. per sec.
Horse-power developed in blades	754

What are the possible values of the mean blade velocity, assuming that the blade outlet angle is equal to the angle made by the steam at inlet to the blades?

If the nozzle efficiency is 92 per cent and the "carry-over" coefficient ϕ is 0.9, calculate the gross stage efficiency of a multi-stage turbine operating under the same conditions with the higher blade velocity.

11. The following particulars relate to a two-row velocity-compounded impulse wheel which forms the first stage of a combination turbine—

Steam velocity at nozzle outlet	1,900 ft. per sec.
Mean blade velocity	380 ft. per sec.
Nozzle outlet angle	16°
Outlet angle, first row of moving blades	18°
Outlet angle, fixed guide blades	22°
Outlet angle, second row of moving blades	36°
Steam flow, per second	5.3 lb.

The ratio of the relative velocity at outlet to that at inlet is 0.84 for all blades.

Determine for each row of moving blades the following—

- The velocity of whirl.
- The tangential thrust on the blades.
- The axial thrust on the blades.
- The power developed.

What is the efficiency of the wheel as a whole?

12. A velocity-compounded impulse wheel has two rows of moving blades with a mean diameter of 28 in. The speed of rotation is 3,000 r.p.m., the nozzle angle is 16° , and the estimated steam velocity at the nozzle outlet is

FLOW OF STEAM THROUGH TURBINE BLADES 155

1,820 ft per sec. The mass of steam passing through the blades per second is 11.2 lb

Assuming that the energy loss in each row of blades (moving and fixed) is 24 per cent of the kinetic energy of the steam entering the blades and referred to as the relative velocity, and that the outlet angles of the blades are (1) first row of moving blades 18° , (2) intermediate guide blades 22° , (3) second row of moving blades 38° , draw the diagram of relative velocities and derive the following—

- (a) Blade inlet angles
- (b) Power developed in each row of blades
- (c) Efficiency of the wheel as a whole

THE FLOW OF STEAM THROUGH IMPULSE-REACTION
TURBINE BLADES

74. **The Meaning of Impulse-reaction.** In Art. 67 of the preceding chapter it was shown that for a pure impulse turbine the outlet angle of the moving blades in any given stage must be such that

$$\sin \beta_2 = \frac{1}{\phi_m} \left(\frac{144w \cdot v_2}{n \cdot l_2 \cdot U_2} + t \right)$$

Although this expression was applied to the pure impulse turbine there is nothing in its derivation which restricts its application to that type. It is, in fact, the equation of continuity in another form and must be satisfied under all conditions. In the pure impulse turbine, U_2 has a somewhat special value; it is the relative outlet velocity when the steam pressures at inlet to, and outlet from, the blade are identical.

Suppose that the blade outlet angle is actually made smaller than the angle given by the above relation, the blade height remaining unaltered at l_2 . Assuming that ϕ_m , w , n , and t remain the same, then if β_2 be reduced the values of v_2 and U_2 must be varied in such a way that the equation is satisfied. Neglecting, for the moment, any change in steam pressure and, therefore, in v_2 , a reduction of β_2 is only possible by increasing U_2 , and the latter can only be brought about by a drop in steam pressure taking place in the blade channel between inlet and outlet, because with pure impulse the value of U_2 is limited to $k \cdot U_1$. Such a fall in steam pressure must be accompanied by an increase in the specific volume v_2 , this demanding a further increase in U_2 . The latter, in turn, brings about a still further increase in specific volume, and so on. Actually, when the steam pressure is reduced, U_2 increases at a much greater rate than v_2 and an equilibrium point is reached at which the equation of continuity is satisfied.

Consider the velocity diagram shown in Fig. 86. Let $AB = V_1$ be the jet velocity at outlet from the fixed blades or nozzles, and $AC = u$ the blade velocity. Then $CB = U_1$ is the relative velocity at inlet to the blades. The angle β_2 is made appreciably less than β_1 . With pure impulse working, the relative velocity of the steam at outlet from the blades would be $CD = k \cdot U_1$. In this case, the final absolute velocity would be AD and the change of velocity suffered by the steam would be $\overline{BD}$. This would give rise to the force db , which equals $\frac{w}{g} \cdot DB$ and is parallel to DB . This force is the "impulse" on the blades.

The steam velocity is increased from CD to CE by expansion. This gives rise to a reaction ed , which equals $\frac{w}{g} \cdot ED$. Combining the impulse db and the reaction ed , we obtain the resultant force eb , which equals $\frac{w}{g} \cdot EB$. This agrees with the velocity diagram, since in the actual blade the final absolute velocity is AE and the change of velocity is $\overline{BE}$, giving rise to a resultant thrust $\frac{w}{g} \cdot EB$. This is

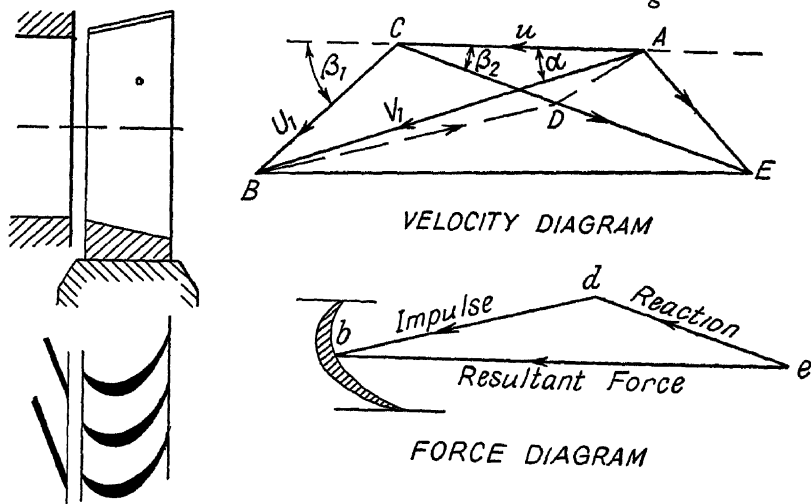


FIG 86

the principle of the impulse-reaction turbine, commonly termed a reaction turbine.

75. Degree of Reaction. Consider a number of stages of an impulse-reaction turbine, and let the mean diameter of all the stages be the same, as shown in Fig 87. Assume, moreover, that the sections of the blades in each row of stationary blades are identical, and that those in each row of blades are also identical, but not necessarily the same as those of the fixed blades. Furthermore, assume that the velocity ratios in each row are the same. Then, since the blade velocity is uniform, the absolute velocities V_1 and V_2 and the relative velocities U_1 and U_2 will be the same for each "pair" of blades comprising one fixed row and one moving row of blades.

Since the steam is undergoing expansion in each row of blades its specific volume is gradually increasing, and since there is a constant rate of steam flow through the blading, the blade heights must be increased progressively, as shown in Fig. 87, in order to satisfy the above conditions.

In each row of fixed blades except the first the steam enters with a kinetic energy equal to

$$\frac{V_2^2}{2gJ} \text{ B.Th.U. per lb.}$$

Owing to losses at inlet, the available energy will be reduced to

$$\phi \cdot \frac{V_2^2}{2gJ}$$

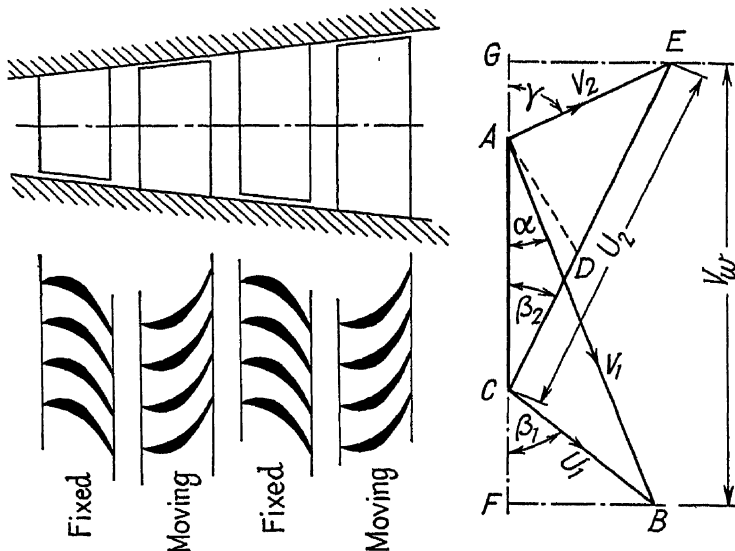


FIG. 87

where ϕ is a "carry-over" coefficient. The gain of kinetic energy in the fixed blades is then

$$\frac{V_1^2 - \phi \cdot V_2^2}{2gJ}$$

and if η_n is the efficiency of the blades when considered as nozzles, i.e. the expansion efficiency, then the adiabatic heat drop in each row of fixed blades (or nozzles) will be—

$$h_n = \frac{V_1^2 - \phi \cdot V_2^2}{2g \cdot J \cdot \eta_n} \text{ B.Th.U. per lb.} \quad (89)$$

But

$$\begin{aligned} V_2^2 &= U_2^2 + u^2 - 2u \cdot U_2 \cdot \cos \beta_2 \\ &= U_2^2(1 + \rho_2^2 - 2\rho_2 \cdot \cos \beta_2) \end{aligned}$$

where

$$\rho_2 = \frac{u}{U_0}$$

$$\therefore h_n = \{V_1^2 - \phi U_2^2(1 + \rho_2^2 - 2\rho_2 \cdot \cos \beta_2)\} \div 2g \cdot J \cdot \eta_n. \quad (89a)$$

Without any reaction the velocity CD would be kU_1 , where k represents the velocity factor, or we may write for the available energy

$$\frac{\phi \cdot U_1^2}{2gJ}$$

where ϕ has the same meaning as above. If η_n is again the efficiency of the blades as nozzles (assumed for the sake of simplicity to be the same for fixed and moving blades) the required adiabatic heat drop in the moving blades is

$$h_b = \frac{U_2^2 - \phi \frac{U_1^2}{\eta_n}}{2g} \text{ B Th U per lb} \quad (90)$$

$$\begin{aligned} \text{But } U_1^2 &= V_1^2 + u^2 - 2uV_1 \cos \alpha \\ &= V_1^2(1 + \rho_1^2 - 2\rho_1 \cos \alpha) \end{aligned}$$

$$\text{where } \rho_1 = \frac{u}{V_1}$$

$$\therefore h_b = \{U_2^2 - \phi V_1^2(1 + \rho_1^2 - 2\rho_1 \cos \alpha)\} \div 2gJ \eta_n \quad (90a)$$

The degree of reaction R is defined as the ratio of the heat drop in the moving blades to the sum of the heat drops in the fixed and moving blades

$$\text{i.e. } R = \frac{h_b}{h_n + h_b} \quad (91)$$

The velocity of whirl $V_w = CF + GC$

$$= V_1 \cos \alpha - u + U_2 \cos \beta_2$$

Therefore the work done per lb. of steam

$$\begin{aligned} &= \frac{u}{gJ} (V_1 \cos \alpha - u + U_2 \cos \beta_2) \text{ B Th U} \\ &= \frac{1}{gJ} \{V_1^2(\rho_1 \cos \alpha - \rho_1^2) + \rho_2 \cdot U_2^2 \cos \beta_2\} \quad (92) \end{aligned}$$

The gross stage efficiency is then

$$\eta_{gs} = \{V_1^2(\rho_1 \cos \alpha - \rho_1^2) + \rho_2 U_2^2 \cos \beta_2\} \div gJ(h_n + h_b) \quad (93)$$

The actual stage efficiency would be lower than the gross value because of leakage of steam through the clearances between the fixed blades and the rotor and between the moving blades and the cylinder

76. Impulse-reaction Turbine with Similar Blade Sections and Half-degree Reaction (Parsons Turbine). In the well-known Parsons turbine the section of the blades is the same in both fixed and moving rows of blades. The reason for this is that the blades are often made from long-sectioned bars which are extruded through

standard dies. The longer blades are sometimes machined from forgings and demand special consideration.

If, as is frequently the case, the blades of a turbine are divided into groups or "expansions" containing several rows of blades having the same mean diameter and the same radial height, then since the specific volume of the steam is increasing as the pressure falls, the velocity of the steam also increases, and in addition to the variation in the blade speed ratio from row to row, the heat drop

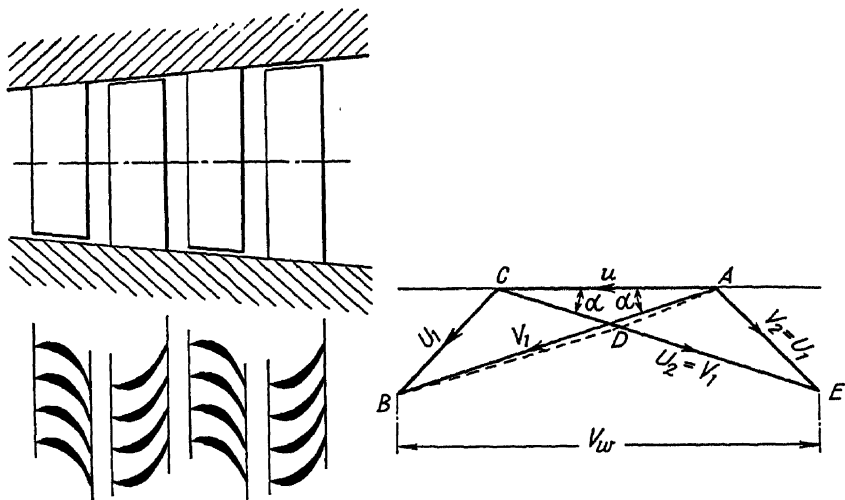


FIG. 88

in each row of blades is somewhat greater than that in the row preceding it, and thus the degree of reaction is rather more than one-half.

If, however, we assume that the mean diameter of the blades is kept constant, and that the height of the blades is so increased that the steam velocity at exit from each row of blades is uniform throughout the group, then the velocity triangles ABC and ACE (Fig. 88) will be congruent.

This case is so important that we shall consider it apart from the relations already obtained for the general case.

With pure impulse, the steam would flow from the moving blades with a velocity CD equal to $k \cdot U_1$. As the area at outlet from the blade channel is restricted by the section of the running blade used, the velocity is increased, by expansion, to the value $U_2 = V_1$. Then the kinetic energy supplied in the moving blades is equal to

$$\frac{V_1^2 - k^2 \cdot U_1^2}{2gJ} = \frac{V_1^2 - \phi \cdot U_1^2}{2gJ}$$

where $\phi = k^2$, and if we let η_n represent the efficiency of the

blades when considered as nozzles, the required adiabatic heat drop must be

$$h_b = \frac{V_1^2 - \phi U_1^2}{2gJ \cdot \eta_n} \text{ B Th U. per lb.}$$

But

$$U_1^2 = V_1^2 + u^2 - 2u \cdot V_1 \cos \alpha$$

and writing

$$\rho = \frac{u}{V_1}$$

we have

$$U_1^2 = V_1^2(1 + \rho^2 - 2\rho \cos \alpha)$$

$\therefore$ Heat drop in each row of moving blades

$$h_b = \frac{V_1^2}{2gJ \cdot \eta_n} \{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)\} \quad (94)$$

In each row of fixed blades except the first, the steam enters the blades with a velocity $V_2 (= U_1)$ and leaves with a velocity V_1 . If ϕ represents the fraction of the incoming energy which is rendered available, then the kinetic energy to be supplied by expansion is

$$\frac{V_1^2 - \phi U_1^2}{2gJ}$$

and the adiabatic heat drop in the nozzles would be

$$h_n = \frac{V_1^2 - \phi \cdot U_1^2}{2g \cdot J \cdot \eta_n}$$

$$\therefore h_b = h_n, \text{ and } R = 0.5$$

The total adiabatic heat drop in a "pair" of blades is clearly

$$h_n + h_b = \frac{V_1^2}{gJ \cdot \eta_n} \{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)\} \quad (95)$$

The work done per lb of steam is

$$\begin{aligned} \frac{u}{gJ} \frac{V_w}{J} &= \frac{u}{gJ} (V_1 \cos \alpha + V_1 \cos \alpha - u) \\ &= \frac{V_1^2}{gJ} (2\rho \cos \alpha - \rho^2) \text{ B Th U} \end{aligned} \quad (96)$$

$\therefore$ Gross stage efficiency

$$\eta_{gs} = \frac{\eta_n(2\rho \cos \alpha - \rho^2)}{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)} \quad (97)$$

The gross stage efficiency is shown in Fig 89 for the following data—

$$\alpha = 20^\circ; \phi = 0.86^2 = 0.74, \text{ and } \eta_n = 0.90$$

The curve for the single-row impulse turbine and for the same basic data is also shown in the same figure for comparison.

The gross stage efficiency is a maximum when

$$\rho = \cos \alpha = 0.94^*$$

but the efficiency curve is very flat and the value of ρ may be

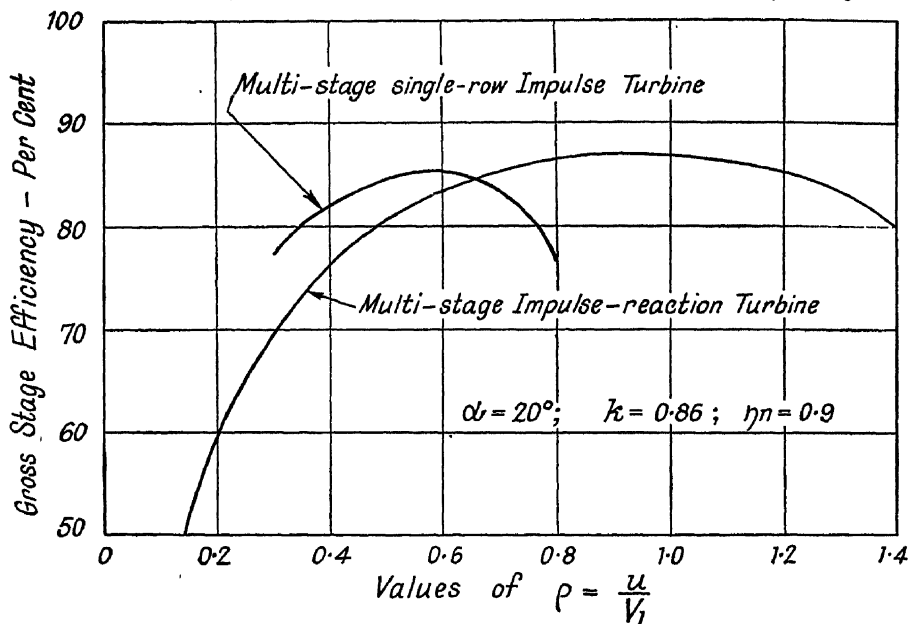


FIG. 89

reduced to about 0.75 for a reduction of only 1 per cent in the efficiency. Substituting $\rho = \cos \alpha$ in eq. (97), we have

$$\eta_{gs} (\text{max.}) = \frac{\eta_n \cdot \cos^2 \alpha}{1 - \phi(1 - \cos^2 \alpha)} \quad (98)$$

EXAMPLE 10. The following particulars refer to a group of blading in a Parsons turbine—

Mean diameter of blades	27 in.
R.p.m.	3,000
Number of rows of fixed blades	4

* The author is indebted to Mr. G. G. Elliott, lecturer in Mechanical Engineering in the University of Cape Town, who has pointed out to the author that eq. (97) may be rewritten

$$\eta_{gs} = \frac{\eta_n}{\frac{1 - \phi}{2\rho \cos \alpha - \rho^2} + \phi}$$

This is clearly a maximum when $2\rho \cos \alpha - \rho^2$ is a maximum, whatever the values of η_n and ϕ , i.e. when $\rho = \cos \alpha$.

Number of rows of moving blades	.	.	.	4
Weight of steam passing through blading per second	.	.	.	30 lb.
Mean ratio: $\frac{\text{Blade Speed}}{\text{Steam Speed}}$	.	.	.	0.75
Blade outlet angle	.	.	.	20°

If the value of ϕ is 0.74 and the efficiency of conversion of heat energy into kinetic energy in the blade channels is 0.92, calculate—

- Total heat drop in the group, B.Th U per lb. of steam.
- Horse-power developed in the group
- Gross stage efficiency

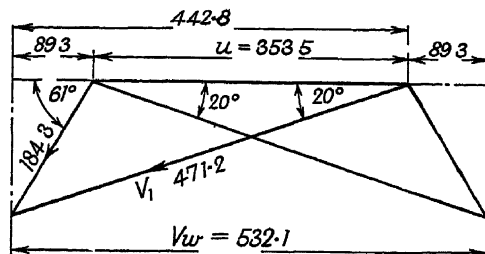


FIG 90

$$\begin{aligned} \text{Mean blade speed } u &= \frac{\pi \times 27}{12} \times \frac{3000}{60} \\ &= 353.5 \text{ ft per sec} \end{aligned}$$

$$\therefore \text{Steam velocity } V_1 = \frac{353.5}{0.75} = 471.2$$

The velocity diagram is shown in Fig. 90, the inlet angle of the blades being 61°.

Kinetic energy supplied in the blade channels of each row

$$\begin{aligned} &= \left(\frac{471.2}{223.8} \right)^2 - 0.74 \left(\frac{184.3}{223.8} \right)^2 \\ &= 4.434 - 0.502 \\ &= 3.932 \text{ B Th U per lb} \end{aligned}$$

$\therefore$ Adiabatic heat drop in the group

$$= 8 \times \frac{3.932}{0.92} = 34.19 \text{ B.Th.U. per lb. of steam}$$

Tangential thrust on moving blades

$$\begin{aligned} &= \frac{w}{g} \cdot V_w \\ &= \frac{30 \times 532.1}{32.2} = 495.7 \text{ lb.} \end{aligned}$$

Work done in group (four moving rows) per second

$$= 4 \times 495.7 \times 353.5 \\ = 700,900 \text{ ft.-lb.}$$

$$\therefore \text{H.p. developed} = \frac{700,900}{550} = 1275$$

Heat equivalent of work done per second

$$= \frac{700,900}{777.8} = 901.4 \text{ B.Th.U.}$$

$$\therefore \text{Gross stage efficiency} = \frac{901.4 \times 100}{30 \times 34.19} \\ = 87.9 \text{ per cent}$$

77. Height of Reaction Blading. The height of the blades in a reaction turbine must be calculated in somewhat the same manner as the height of a nozzle with complete or all-round admission.

Some allowance must be made for the leakage of steam over the blade tips in reaction turbines where ordinary unshrouded blading is used. The quantity of steam passing over the blade tips may be taken as

$$3 \left(\frac{c}{l} \right) \times \text{full steam flow}$$

where l is the radial height of the blade,
and c is the radial tip clearance.

If w = total steam flow per second

w_1 = "working" steam flow per second, i.e. steam flow through blades only

then
$$w_1 = w \left(1 - \frac{3c}{l} \right), \text{ approximately}$$

Fig. 91 shows the cross-section of a typical reaction turbine blade. As may be seen, the profile is made up entirely of curves. The tangents to the leading and trailing surfaces at outlet make angles α' and α'' , respectively, with the wheel plane. Provided the difference between these angles is not large the arithmetic mean of the two angles may be taken as the true outlet angle α of the blade. Owing to the fact that the streams on each side of the outlet edge of the blade converge on each other, no allowance need be made for the thickness of the blade. Hence if p_m be the mean circumferential pitch of the blades and l the radial height, then the area of the outlet for each blade will be

$$p_m \cdot l \cdot \sin \alpha \text{ sq. in.}$$

As already pointed out in the previous article, if a reaction turbine were designed for a constant ratio of blade speed to steam

speed and a constant outlet angle α , the blade lengths would have to be increased to accommodate the increasing volume of steam. A gradual increase of blade length is not always provided in practice. The total expansion from the initial pressure to the condenser pressure is divided into a certain number of smaller "expansions". The total number of these varies from twelve to about thirty. In the high-pressure part of the turbine, where the mean diameter is comparatively small, such an expansion might consist of five to seven pairs of blades, whereas in the low-pressure part of the turbine an expansion from the initial pressure comprises only one ring of fixed blades and one of moving blades. In each of the expansions containing more than one pair of blades the radial height of the blade

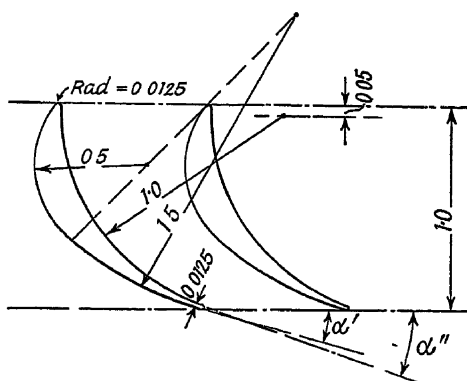


FIG 91 — Approximate Section of Reaction Turbine Blade
($\alpha' = 16^\circ$, $\alpha'' = 18.4^\circ$, Mean Outlet Angle = 17.2°)

is made constant. In these cases, one of two possible conditions will obtain—

(1) If the outlet angle be constant (i.e. if the area for flow be constant), the steam speed must increase as expansion proceeds. Hence the value of ρ diminishes from inlet to outlet.

(2) If a constant value of ρ is desired, then a progressive increase in area may only be obtained by increasing α . In such a case, the normal angle of 20° would be reduced in the first half of the expansion and increased in the second half. This is rarely done in practice.

Assuming condition (1), i.e. a uniform blade angle, the necessary blade height may be based on the steam conditions at the middle of the expansion.

Let v_s = actual specific volume at the middle of the expansion,
cub. ft. per lb.

D = mean diameter of blade ring, in.

l = blade height, in.

N = r.p.m.

$$\text{Number of blades per ring} = \frac{\pi D}{p_m}$$

$$\begin{aligned}\therefore \text{Total area for flow} &= \frac{\pi D}{p_m} \times p_m \cdot l \cdot \sin \alpha \\ &= \pi D \cdot l \cdot \sin \alpha \text{ sq. in.}\end{aligned}$$

$$\text{Total area required} = \frac{144 \cdot w_1 v_s}{V_1}$$

$$\therefore \pi D \cdot l \cdot \sin \alpha = \frac{144 \cdot w_1 v_s}{V_1}$$

$$\text{But } V_1 = \frac{u}{\rho} = \frac{\pi D N}{720 \rho}$$

$$\therefore D^2 l = \frac{144 \times 720}{\pi^2} \times \frac{w_1 \cdot v_s \cdot \rho}{N \cdot \sin \alpha}$$

$$\text{or } D^2 l = \frac{10,510 w_1 \cdot v_s \cdot \rho}{N \cdot \sin \alpha} \quad . \quad . \quad . \quad (99)$$

Generally in any "expansion" the quantities w_1 , v_s , ρ , N , and $\sin \alpha$ are known. Hence $D^2 l$ may be found and, when D is known, l may be found.

If the diameter of the rotor be D_r in., then $D = D_r + l$, and

$$D^2 l = (D_r + l)^2 l$$

In such a case, when $D^2 l$ and D_r are known, the value of l is best found by trial and error.

EXAMPLE 11. In a reaction steam turbine of the Parsons type, the pressure at the middle of the expansion is 20 lb. per sq. in. abs., and the dryness fraction is 91 per cent. $w_1 = 13$ lb. per sec., $\alpha = 20^\circ$, $N = 3,000$ r.p.m., and $\rho = 0.65$. If the diameter of the rotor drum is 26 in., calculate the required blade height.

$$\begin{aligned}\text{From the steam tables, } v_s &= 0.91 \times 20.08 \\ &= 18.273 \text{ cub. ft. per lb.}\end{aligned}$$

$$\begin{aligned}\text{Hence } (26 + l)^2 \cdot l &= \frac{10,510 \times 13 \times 18.273 \times 0.65}{3000 \times 0.342} \\ &= 1581\end{aligned}$$

By trial and error it is found that $l = 2.01$ in.

78. Stage Efficiency of Impulse-reaction Turbine with Half-degree Reaction. Assuming, as before, that w represents the total steam flow through the turbine and w_1 the "working steam," i.e. the steam actually flowing through the blades, each in lb. per sec., then by eq. (96), p. 161—

Work done per second

$$= \frac{w_1 \cdot V_1^2}{gJ} (2\rho \cos \alpha - \rho^2)$$

Total adiabatic heat drop per second

$$= w(h_n + h_b) \\ = \frac{w \cdot V_1^2}{g \cdot J \eta_n} \{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)\}$$

Since the leakage loss is practically the only loss affecting the efficiency of a reaction turbine stage—apart from blade friction and shock losses which have already been considered and whose effects are included in the above expressions—we may write—

Stage efficiency

$$\eta_s = \frac{w_1}{w} \frac{\eta_n(2\rho \cos \alpha - \rho^2)}{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)} \quad (100)$$

If ordinary unshrouded blades are used then, since

$$\frac{w_1}{w} = 1 - \frac{3c}{l} \\ \eta_s = \left(1 - \frac{3c}{l}\right) \frac{\eta_n(2\rho \cos \alpha - \rho^2)}{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)} \quad (101)$$

EXAMPLES VIII

1 The following particulars refer to a stage of a Parsons steam turbine comprising one ring of fixed blades and one ring of moving blades—

Mean diameter of blade ring	26 in
R p m	3,000
Steam velocity at exit from blades	525 ft per sec
Blade outlet angle	20°
Steam flow through blades	15.5 lb per sec

Draw the velocity diagram and derive the following—

- Blade inlet angle
- Tangential force on ring of moving blades
- Horse-power developed in stage.

2 In a stage of an impulse-reaction turbine operating with half-degree reaction, the fixed and moving blades are of identical section and the outlet angle is 18°. The absolute velocity of discharge from the moving blades is 130 ft per sec. in a direction 110° to the direction of motion of the blades, and the change of velocity produced by the moving blades is parallel to that direction.

Draw the velocity diagram and derive—

- Mean velocity of the blades.
- Ratio of blade velocity to the velocity of efflux from the fixed blades.
- Work done per lb. of steam

3. If the total tangential thrust on the blades of one ring of a Parsons turbine is 293 lb. when the blade speed is 350 ft. per sec., the weight of steam flowing through the blading per sec. is 16 lb., and the blade outlet angle is 20° , calculate the steam velocity at outlet from the blades.

If the friction losses which would occur with pure impulse are 33 per cent of the kinetic energy corresponding to the relative velocity at entry to each ring of blades, and if the expansion losses are 14 per cent of the heat drop in the blades, what would be the heat drop per ring, and the gross stage efficiency?

4. The outlet angle of the blade of a Parsons turbine is 20° and the axial velocity of flow of the steam is 0.5 times the mean blade velocity. Draw the velocity diagram for a stage consisting of one fixed and one moving row of blades, given that the mean diameter is 28 in. and that the speed of rotation is 3,000 r.p.m. State the inlet angle of the blades if the steam is to enter the blade channels without shock.

If the blade height be 2.5 in., the mean steam velocity 80 ft. per sec., the steam dry and saturated ($v = 5.466$ cub. ft. per lb.), find the horsepower developed per pair of blades.

5. A blade group in a reaction turbine is of constant height. The power of the turbine is 2,000 kW, the speed of rotation 3,000 r.p.m., initial steam pressure 160 lb. per sq. in. gauge, steam initially dry and saturated, vacuum $27\frac{1}{2}$ in. (30 in. barometer), and the efficiency ratio 67 per cent. The mean blade velocity in the group is to be 300 ft. per sec. and the average specific volume of the steam in the group may be taken as 10.25 cub. ft. per lb. If the ratio of the blade velocity to the steam velocity be 0.55, find the height of the blades in the group if the outlet angle is 18.5° and the effect of tip leakage be neglected.

Draw the velocity diagram for these blades and calculate the heat drop required for the group if the coefficients η_n and ϕ are 0.92 and 0.8 respectively, and there are five pairs of blades in the group.

6. The following particulars refer to a stage of an impulse-reaction turbine—

Outlet angle of fixed blades	.	.	.	20°
Outlet angle of moving blades	.	.	.	30°
Radial height of fixed blades	.	.	.	4 in.
Radial height of moving blades	.	.	.	4 in.
Mean blade velocity	.	.	.	450 ft. per sec
Ratio of blade speed to steam speed	.	.	.	0.625
Specific volume of steam at fixed blade outlet	.	.	.	20.0
Specific volume of steam at moving blade outlet	.	.	.	21.2

Calculate the degree of reaction, the adiabatic heat drop in a pair of blade rings, and the gross stage efficiency, given the following coefficients which may be assumed to be the same in both fixed and moving blades: $\eta_n = 0.90$; $\phi = 0.86$.

INTERNAL LOSSES IN STEAM TURBINES

79. Introductory. The internal losses in steam turbines may be enumerated as follows—

- | | |
|----------------------------|-------------------------------|
| (1) Nozzle losses | (4) Partial admission losses. |
| (2) Blade friction losses. | (5) Gland leakage losses |
| (3) Disc friction losses. | (6) Residual velocity loss. |

Items (1) and (2) have already been dealt with in preceding chapters, and we now pass on to a brief consideration of the other items in turn

80. Disc Friction. When a disc such as those used in impulse turbines is rotating in a slightly viscous fluid like steam there is a true surface friction loss due to relative motion between the disc and the steam particles. As a result, the moving disc surface exerts a drag on the steam which sets it in motion and produces a definite circulation, the character of which depends on the shape of the space enclosing the disc. Fig 92 shows the circulation currents set up when a simple disc is rotating in an enclosure of simple shape

It is beyond the scope of this book to consider the subject in greater detail, but brief references will be made to the more important work on the subject

Professor Stodola (1), as the result of some experiments made on discs rotating in free air, proposed the following formula for the disc friction loss—

$$H p_d = \frac{C}{10^6} \cdot D^2 \cdot u^3 \cdot \rho$$

$H p_d$ = horse-power loss due to disc friction

D = diameter of disc, ft.

u = peripheral velocity of disc, ft. per sec

ρ = density of fluid in which disc is rotating, lb. per cub. ft

C = 0.071 for *air* only

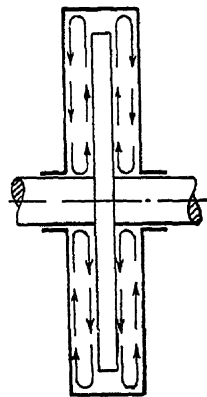


FIG 92

Stodola states that the loss for an encased wheel would be less than that for a wheel rotating in free air, varying from one-quarter to one-half of that amount, according to conditions. Odell (2) also carried out experiments on discs rotating in free air. An important mathematical paper on the subject was published in 1921 by Professor von Kármán (3). In 1934, the well-known Swiss firm of

Brown Boveri & Co. published the results (4) of some experiments made on discs in a closed chamber filled with various gases.

From an analysis of these and other experimental investigations, the author has deduced the following simple relation for the power absorbed by discs and wheels that are well encased.

$$\text{H.p.}_a = C \cdot D^2 \left(\frac{u}{100} \right)^3 \rho \quad . \quad . \quad . \quad (102)$$

the constant C being given by the expression

$$\log_{10} C = 1.90 - 0.2 \log_{10} \left(\frac{D \cdot u \cdot \rho}{\mu} \right) \quad . \quad . \quad (103)$$

D , u , and ρ have the same meanings as before and μ is the coefficient of viscosity of the steam, the unit being lb. per ft. sec.

Between 15 and 500 lb. per sq. in., the coefficient of viscosity of dry and saturated steam, according to the recent work of K. Sigwart (5), is given by the following simple relation—

$$\mu = (0.143t + 56.0) \times 10^{-7} \frac{\text{lb.}}{\text{ft.-sec.}} \quad . \quad (104)$$

where t is the saturation temperature in degrees Fahr. Provided the steam temperature is below 700° F. the above expression may be used for superheated steam at temperature t , since the viscosity of steam is hardly affected by pressure.

EXAMPLE 12. Calculate the power absorbed in overcoming disc friction in a stage of an impulse turbine in which the disc diameter is 46 in.; r.p.m. 3,000; steam pressure 120; superheat 200° F.

From steam tables, saturation temperature = 341.3, $\therefore$ steam temperature = 541.3° F. Substituting in eq. (104)

$$\mu = 133.5 \times 10^{-7}$$

By Callendar's steam tables, $v = 4.846$

$$D = \frac{46}{12} = 3.833 \text{ ft.}; u = 602.0$$

$$\therefore \log_{10} C = 1.90 - 0.2 \log_{10} \left(\frac{3.833 \times 602.0 \times 10^7}{133.5 \times 4.846} \right)$$

$$= 1.90 - 1.5104 = 2.3896$$

$$\therefore C = 0.0245$$

$$\text{Then} \quad \text{H.p.}_a = \frac{0.0245}{4.846} (3.833)^2 \cdot (6.02)^3$$

$$= 16.2$$

It is usual to drill from four to six pressure-balancing holes in turbine discs; the disturbance due to these will probably increase the disc friction loss.

81. Partial Admission Losses. Fig 93 (a) shows a transverse section through a high-pressure stage of a steam turbine, in which stage, owing to the comparatively small area required for the nozzles, the latter extend over only a fraction of the circumference. For example, in Fig. 93 (a) the nozzles are divided into three groups, each group containing different numbers of nozzles. Allowing for the peripheral displacement due to the axial clearance between the nozzle guide blades and the blade inlet edges, the blade positions at steam inlet will be A, C, and E, and at steam outlet B, D, and F. The shaded areas in Fig. 93 (a) represent the active arcs, the unshaded

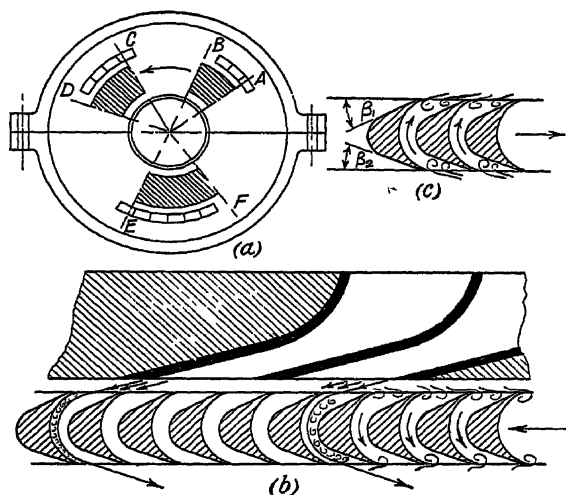


FIG 93 —Diagrams showing Causes of Partial Admission Loss

areas represent arcs over which there is no steam flow through the blades

Considering Fig 93 (b), it will be evident that there will always be six blades which will be partially filled with steam and in which the flow will be considerably disturbed.

The movement of the blades through steam in the inactive arcs BC, DE, and FA leads to further loss. There will be certain eddies produced in the channels of the idle blades, as shown in Fig. 93 (b). If the blades are driven in the reverse direction, as in a marine astern turbine when the turbines are running in the ahead direction, then, due to the scooping action of the blades, the eddies produced are much more violent. Professor Stodola found that the power absorbed in driving a bladed wheel in free air in the reverse direction was five to six times that required for the normal direction. If the wheel was encased, the ratio of the loss was reduced to 1.2.

If the blade outlet angle β_2 is less than the inlet angle β_1 and if the wheel is running in the reverse direction (Fig. 93 (c)), then due

to the scoop action the blades act as an axial-flow fan and a flow is produced in the direction shown by the arrows. When blades having unequal angles are moving in the normal direction (Fig. 93 (b)), a flow is set up in the direction shown by the arrows.

The sum of the losses due to partial admission is variously described as "blade ventilation" loss, or "vane action" loss. We prefer to use the term "blade windage," which is not quite so specific.

For an unencased wheel with a single row of blades rotating in *air* in the normal (ahead) direction, Stodola gives the following expression for the blade windage loss—

$$\text{H.p.}_{bw} = 0.458D \cdot l^{1.5} \left(\frac{u}{100} \right)^3 \cdot \rho$$

If the nozzle arc extend over a fraction λ of the whole circumference and if this arc is continuous, then we may introduce a correction factor $(1 - \lambda)$ and so obtain the expression—

$$\text{H.p.}_{bw} = 0.458(1 - \lambda)D \cdot l^{1.5} \left(\frac{u}{100} \right)^3 \cdot \rho \quad . \quad (105)$$

D = mean diameter of blade ring, ft.

l = effective blade height, in.

u = mean blade velocity, ft. per sec.

ρ = steam density, lb. per cub. ft.

Professor Wm. Kerr published (6) in 1913 the results of some very careful experiments to determine the total resistance of turbine wheels. A turbine wheel was driven round in steam at different densities, the resistance being measured by noting the rate of retardation of the rotor with the power cut off. The following expression was deduced for the *total* power absorbed in wheel friction, i.e. disc friction plus blade windage—

$$\text{H.p.}_{wf} = \{ \beta_1 \cdot D + n\beta_2(1 - \lambda)l^{1.5} \} \left(\frac{u}{100} \right)^3 \cdot D \cdot \rho \quad . \quad (106)$$

in which D , u , ρ , and λ have the same meanings as before, but l is the mean blade height in inches.

$\beta_1 = 0.0607$; $\beta_2 = 0.458$.

$n = 1$ for wheels with 1 row of blades.

$= 1.23$ for wheels with 2 rows of blades.

$= 1.8$ for wheels with 3 rows of blades.

$= 2.9$ for wheels with 4 rows of blades.

The first term in the bracket evidently relates to the disc friction, the second to the blade windage. This formula gives the same result as that of Stodola for single-row wheels.

More recently, Professor Kerr has carried out further researches (7) on the subject and, by studying the action of the disc and blades separately, he has been able to evolve formulae for each loss. The formula given for the blade windage loss is

$$H P_{bw} = \left(\frac{u}{100}\right)^3 \left(\frac{d}{10}\right)^2 \rho \left[\beta(1 - \lambda) \left\{ \left(\frac{l_1}{d}\right)^{1.5} + \left(\frac{l_2}{d}\right)^{1.5} \right\} \right] \quad (107)$$

in which

d = mean diameter of blade ring in inches

l_1 = blade height at inlet edge of first row, in.

l_2 = blade height at outlet edge of last row, in

β = 9.3 for wheels with 1 row of blades.

= 12.5 for wheels with 2 rows of blades

= 17.9 for wheels with 3 rows of blades.

The loss of power due to blade windage may be considerably reduced by enclosing the blades on both sides. Two constructions

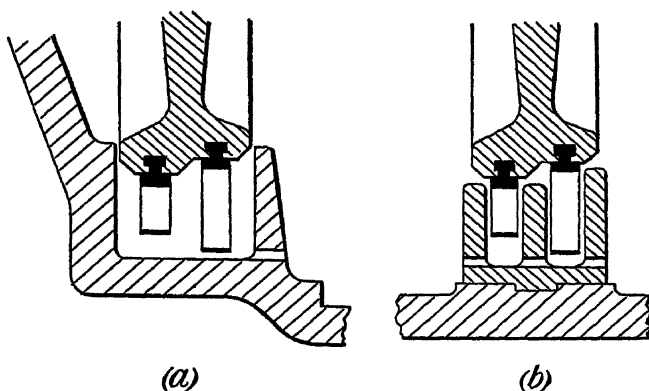


FIG 94 — Blade Shields for Reducing Windage

for effecting this are shown in Fig. 94. In the one case, the shields are formed in the cylinder casting, in the other, separate shield rings are fitted in the cylinder.

82. Gland Leakage Losses. There is a small loss of energy in each stage of an impulse turbine owing to the leakage of steam from one wheel chamber to the next through the space between the diaphragm and the shaft, or the wheel hub, as the case may be.

There is also a certain wastage of steam at the external glands where the turbine spindle passes through the ends of the turbine cylinder, or cylinders. Broadly speaking, the glands at the high-pressure end of the turbine have to minimize the leakage of steam from the turbine cylinder to the atmosphere, whereas the function

of the low-pressure gland is to prevent the inleakage of air into the exhaust branch and condenser. For this purpose steam is supplied to the centre of the gland; some of the steam leaks into the exhaust space, but there is a slight outward flow to the atmosphere which effectively prevents the inleakage of air. The steam supplied

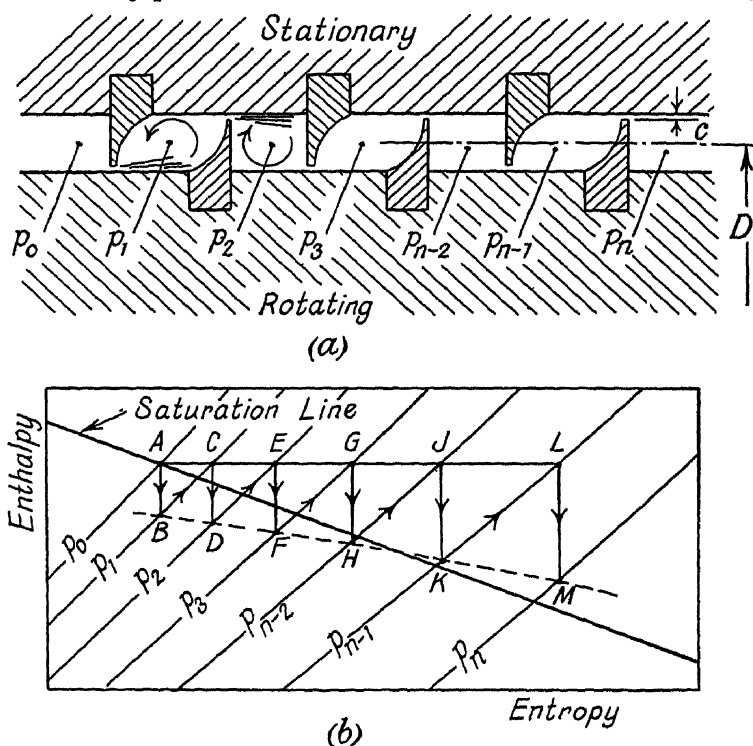


FIG. 95

to the low-pressure gland for sealing purposes may be reduced-pressure live steam or steam which has leaked through the high-pressure gland.

Under certain light-load conditions, the pressure in the first-stage wheel chamber may fall to atmospheric pressure, or below. Under these conditions it becomes necessary to supply sealing steam to the high-pressure gland to prevent an inleakage of air. It should be stated, however, that this is unnecessary when a hydraulic seal is also fitted.

Two types of gland are in general use, the carbon ring gland and the labyrinth gland. Labyrinth packing is used exclusively in diaphragm glands and in most external glands.

83. Flow of Steam through Labyrinth Packing. Fig. 95 (a) shows a section through a typical labyrinth packing. This comprises a

number of packing strips in the fixed housing and an equal number in the shaft or rotor. The inner ends of the fixed strips and the outer ends of the moving strips are reduced in thickness so that if there should be accidental contact, the thin edge of the packing strip will wear away without the generation of a large amount of heat which might cause distortion of the shaft, gland sleeve, or other part of the gland. There is a small radial clearance c between the strips and the housing or shaft, as the case may be. This forms a constriction through which the steam leaks. The small space between one constriction and the next is termed a "pocket" or "dwell space."

Suppose that the steam pressure on the up-stream side of the gland is p_0 and that on the down-stream side is p_n , there being n constrictions. Then, assuming that the areas through all the constrictions are the same, as soon as steady flow has been established, the pressures p_1, p_2 , etc., in the dwell spaces will take up definite values. In many cases, the ratio of the pressure drop at any constriction, excepting the last, to the absolute pressure at that constriction is comparatively small. In such cases, the following approximate theory is applicable.

Consider first the action in labyrinth packing. Steam expands more or less adiabatically in the first constriction from pressure p_0 to p_1 . This is represented in Fig. 95 (b) by the isentropic line AB. On entering the dwell space, the steam sets up an eddy, but the velocity of the steam is reduced to a comparatively low value. In consequence, the kinetic energy acquired by expansion is reconverted into heat at constant pressure and the enthalpy is restored, practically, to its initial value. During this process, the state point of the steam moves along the constant-pressure line BC. The expansion in the second constriction is represented by CD, the reheating in the second dwell space by DE, and so on.

A comparison of p - v curves is made in Fig. 96. In each case the relationship between p and v is fairly well represented by an equation of the type $p v^m = \text{constant}$, where m has the values 1.135 and 1.0646 for the adiabatic and saturation curves respectively. For the throttling curve ($I = \text{constant}$) m is very nearly equal to unity. Actually, the relation between p and v for the locus BDFHKM, Fig. 95 (b), is a more important relation because each of these points represents the condition of the steam at outlet from

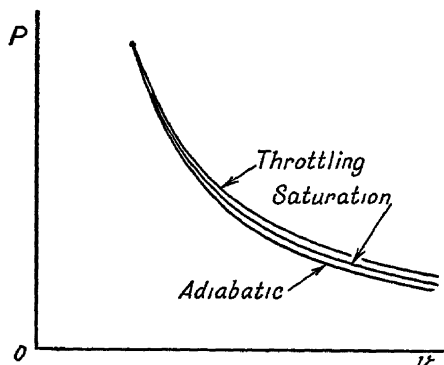


FIG. 96—Comparison of p - v Curves from Common Initial Condition

the constriction. Here, the value of m depends upon the rate of leakage, which in turn depends upon the pressure range and the number of constrictions provided. Thus m may vary between 1.02 and 1.1. Considerable simplification is achieved by assuming that m is equal to unity, and since it is rather difficult to measure the working clearance accurately it is perhaps unnecessary to attempt to calculate the leakage rate to a high degree of accuracy. The following theory must then be regarded as approximate.

Case I. Pressure Ratio for each Constriction Greater than the Critical.

Let w = mass leakage of steam, lb. per sec.

A = effective area through each constriction, sq. in.

D = mean diameter of packing strips, in.

C_c = coefficient of contraction. Owing to the one-sided contraction, this is very probably equal to about 0.8.

Then

$$A = C_c \cdot \pi D \cdot c \text{ sq. in.}$$

Consider the flow at any constriction. Let p be the pressure on the up-stream side of the constriction and $p - \delta p$ the pressure on the down-stream side. Then the velocity V reached in the constriction at pressure $p - \delta p$ will be such that

$$\frac{V^2}{2g} = 144 \cdot v_m \cdot \delta p$$

where v_m is the mean specific volume of the steam. By the equation of continuity—

$$w(v + \delta v) = \frac{AV}{144}$$

or
$$\frac{w}{A} = \frac{V}{144(v + \delta v)} = \frac{\sqrt{288g \cdot v_m \cdot \delta p}}{144(v + \delta v)}$$

If δp is small compared with p , then we may write

$$v \div v_m \div v + \delta v$$

$$\therefore \frac{w}{A} = \frac{1}{144} \sqrt{\frac{288g \cdot \delta p}{v}}$$

Let $pv = C_1$, then $v = \frac{C_1}{p}$

and
$$\frac{w}{A} = \frac{1}{144} \sqrt{\frac{288g \cdot p \cdot \delta p}{C_1}}$$

It will be seen that in a gland where A is constant, the product

$p \cdot \delta p$ is a constant, and therefore as p falls the pressure drop per constriction increases

Squaring both sides of the expression, we have

$$\left(\frac{w}{A}\right)^2 = \frac{2g}{144C_1} p \cdot \delta p$$

$$\text{or} \quad 2 \cdot 24 C_1 \left(\frac{w}{A}\right)^2 = p \cdot \delta p$$

For n rings—

$$\sum_0^n 2 \cdot 24 C_1 \left(\frac{w}{A}\right)^2 = \int_{p_n}^{p_0} p \cdot dp$$

$$\text{or} \quad 2 \cdot 24 n C_1 \left(\frac{w}{A}\right)^2 = \frac{1}{2} (p_0^2 - p_n^2)$$

$$\text{i e.} \quad n \cdot C \left(\frac{w}{A}\right)^2 = \frac{1}{2} (p_0^2 - p_n^2) \quad . \quad (108)$$

where

$$\begin{aligned} C &= 2 \cdot 24 C_1 \\ &= 2 \cdot 24 p_0 v_0 \end{aligned}$$

The rate of leakage per sq in of effective area, expressed in lb. per sec, is

$$\frac{w}{A} = \sqrt{\frac{p_0^2 - p_n^2}{2nC}} \quad . \quad (109)$$

C may be found for any given initial steam conditions. For dry and saturated steam, the constants have the following values—

p_0	v_0	$C_1 = p_0 v_0$	C
20	20.08	401.6	900
50	8.50	425.0	952
100	4.429	442.9	992
150	3.016	452.4	1012
200	2.293	458.6	1027

EXAMPLE 13. Calculate the mass rate of leakage through a high-pressure turbine gland under the following conditions—

Steam pressure in first stage	160
Steam superheat in first stage	200° F.
Steam pressure in leak-off pocket	40
Mean diameter of packing	8 in.
Radial clearance	0.02 in.
Number of constrictions	40

From the steam tables, when $p = 160$ and the superheat is 200°F. , $I_0 = 1310.0$. By Callendar's equation—

$$v_0 = \frac{1.2464}{160} (1310.0 - 835.2) - 0.002$$

$$= 3.70$$

$$C_1 = p_0 v_0 = 160 \times 3.70 = 592.0$$

$$C = 2.24 C_1 = 1327$$

$$\therefore \frac{w}{A} = \sqrt{\frac{160^2 - 40^2}{2 \times 40 \times 1327}} = 0.475 \frac{\text{lb.}}{\text{in.}^2 \text{ sec.}}$$

$$A = C_c \cdot \pi DC$$

Assuming $C_c = 0.80$, $A = 0.402 \text{ sq. in.}$

$$\therefore \text{Rate of leakage} = 0.475 \times 0.402 \\ = 0.191 \text{ lb. per sec.}$$

The pressure in any dwell space may be found from eq. (108). Thus, for the last dwell space (pressure = p_{n-1}) we have

$$(n-1)C \left(\frac{w}{A} \right)^2 = \frac{1}{2} (p_0^2 - p_{n-1}^2)$$

Substituting values, $p_{n-1} = 46.9 \text{ lb. per sq. in.}$

$$\text{Similarly } (n-2)C \left(\frac{w}{A} \right)^2 = \frac{1}{2} (p_0^2 - p_{n-2}^2)$$

from which $p_{n-2} = 53.1 \text{ lb. per sq. in.}$

These calculations confirm what has already been stated above, namely, that the pressure drop per constriction increases as the absolute pressure falls. In the foregoing example, the pressure drops at the low-pressure end of the gland are comparatively large and the calculation would not yield a very accurate result.

Under certain conditions, e.g. either a low value of the pressure p_n , or a comparatively small number of constrictions, the pressure ratio $\frac{p_n}{p_{n-1}}$ may be less than the critical value, i.e. 0.577 for saturated steam, or 0.5457 for superheated steam. In that case the foregoing theory must be modified as follows.

Case II. Pressure Ratio for the Last Constriction less than the Critical Value. In this case, the rate of flow is controlled by the pressure p_{n-1} in the last pocket.

For the last constriction, we may write by eq. (48)—

$$\frac{w}{A} = C_2 \sqrt{\frac{p_{n-1}}{v_{n-1}}}$$

If the steam is dry and saturated in the last pocket, i.e. at the pressure p_{n-1} , then according to eq. (49), C_2 would have the value 0.30. Reference to Fig. 95 (b) shows that at the beginning of the expansion in the last constriction, i.e. at point L, the steam is more or less highly superheated, and might expand down to the throat in the superheated state. Moreover, in many cases, the steam enters the gland in the superheated condition and would therefore enter the last constriction with a considerable degree of superheat. In such cases C_2 would have the value given by eq. (54), namely, 0.3155

Assuming that $p_{n-1} v_{n-1} = C_1$

$$v_{n-1} = \frac{C_1}{p_{n-1}}$$

and
$$\frac{w}{A} = \frac{0.3155 p_{n-1}}{\sqrt{C_1}} = \frac{p_{n-1}}{C_3}$$

where
$$C_3 = \frac{\sqrt{C_1}}{0.3155}$$

But for $(n-1)$ constrictions and the rate of leakage w/A , we have, by eq. (108)—

$$(n-1)C \cdot \left(\frac{w}{A}\right)^2 = \frac{1}{2} (p_0^2 - p_{n-1}^2)$$

Inserting the value $\frac{w}{A} = \frac{p_{n-1}}{C_3}$

$$(n-1)C \left(\frac{p_{n-1}}{C_3}\right)^2 = \frac{1}{2} (p_0^2 - p_{n-1}^2)$$

or
$$\frac{p_0}{p_{n-1}} = \sqrt{1 + 2(n-1) \frac{C}{C_3^2}} \quad (110)$$

Since $C = 2.24C_1$ and $C_3 = \frac{\sqrt{C_1}}{0.3155}$

$$\frac{p_0}{p_{n-1}} = \sqrt{1 + 0.446(n-1)} \quad (111)$$

For any number n , the ratio $\frac{p_0}{p_{n-1}}$ may be calculated and thus the

pressure p_{n-1} obtained. The rate of leakage may be determined from the expression

$$\frac{w}{A} = \frac{0.3155 \cdot p_{n-1}}{\sqrt{C_1}} = \frac{0.3155 \cdot p_{n-1}}{\sqrt{p_0 v_0}}$$

Values of the ratio $\frac{p_0}{p_{n-1}}$ are given in Table VI.

TABLE VI

RATIO $\frac{p_0}{p_{n-1}}$ FOR LABYRINTH GLANDS IN WHICH THE PRESSURE RATIO $\frac{p_n}{p_{n-1}}$ FOR THE LAST CONSTRICTION IS LESS THAN THE CRITICAL

Number of Constrictions	$\frac{p_0}{p_{n-1}}$	Number of Constrictions	$\frac{p_0}{p_{n-1}}$
2	1.202	17	2.852
3	1.376	18	2.929
4	1.529	19	3.004
5	1.668	20	3.077
6	1.797	21	3.149
7	1.917	22	3.220
8	2.030	23	3.288
9	2.137	24	3.355
10	2.239	25	3.421
11	2.336	26	3.485
12	2.430	27	3.549
13	2.520	28	3.611
14	2.607	29	3.673
15	2.691	30	3.733
16	2.773	31	3.791

EXAMPLE 14. Steam is supplied to the low-pressure gland of a steam turbine for sealing purposes. Calculate the rate of leakage into the exhaust branch, given the following data—

Shaft diameter	8 in.
Radial clearance	0.02 in.
Number of constrictions	20
Pressure of sealing steam	16
Pressure in exhaust branch	0.5

Assume that the steam is supplied dry and saturated.

Since there are 20 constrictions, the ratio $\frac{p_0}{p_{n-1}}$ is 3.077.

$$\therefore p_{n-1} = \frac{16}{3.077} = 5.20$$

The pressure-ratio for the last constriction is obviously much less

than the critical. Hence the flow will be controlled by the pressure in the last dwell space

$$\text{Now} \quad p_0 = 16; v_0 = 24.73$$

$$\therefore \quad \frac{w}{A} = \frac{0.3155 \times 5.20}{\sqrt{16 \times 24.73}}$$

$$= 0.0825 \frac{\text{lb}}{\text{in.}^2 \text{ sec.}}$$

and since the effective leakage area is 0.402 sq in., the approximate leakage into the exhaust branch is

$$0.0825 \times 0.402 = 0.0332 \text{ lb per sec}$$

84. Hydraulic Glands. While we are discussing methods of preventing or minimizing leakage some attention may very properly be given to the subject of hydraulic glands, even though it has little or nothing to do with internal losses

The hydraulic gland or water seal has been used by certain manufacturers for many years for sealing the low-pressure end of the turbine, and, in conjunction with a labyrinth gland, for sealing the high-pressure end. It consists of a "paddle wheel" (Fig. 97) keyed to the turbine shaft and rotating within a circular chamber which is in halves, the upper and lower halves being bolted to the upper and lower halves of the turbine cylinder respectively. The gland chamber or box is connected to an overhead tank in which a constant level of water is maintained by a float-controlled cock. The height H is normally some 15 or 20 ft, so that in this respect the diagram is not to scale. The impeller or paddle-wheel is normally provided with radial vanes, as shown in Fig. 97, or with a number of holes bored in the sides of the disc, but not, of course, passing right through, so that the rotation of the impeller causes a forced vortex to be set up in the chamber.

On the atmospheric side of the gland the pressure on the free surface of the water will be the atmospheric pressure p_a , whereas on the "vacuum" side that pressure will be the low steam pressure p_s . At a sufficient speed of rotation, the radii of the free surfaces on the atmospheric and vacuum sides will automatically adjust themselves to the values r_1 and r_3 , the pressure at the extreme radius r_2 of the vortex balancing the pressure $p_a + \rho H$ due to the head of water in the tank

Let ρ = density of water in lb. per cub in. (0.0362).

g = acceleration of gravity, 32.2×12 in. per sec.²

ω = angular velocity, radians per sec.

Considering a transverse slice of water of unit depth—

$$\text{Volume of element} = r \cdot \theta \cdot \delta r$$

$$\text{Centrifugal force on element} = \frac{\rho \omega^2 r^2}{g} \cdot \theta \cdot \delta r$$

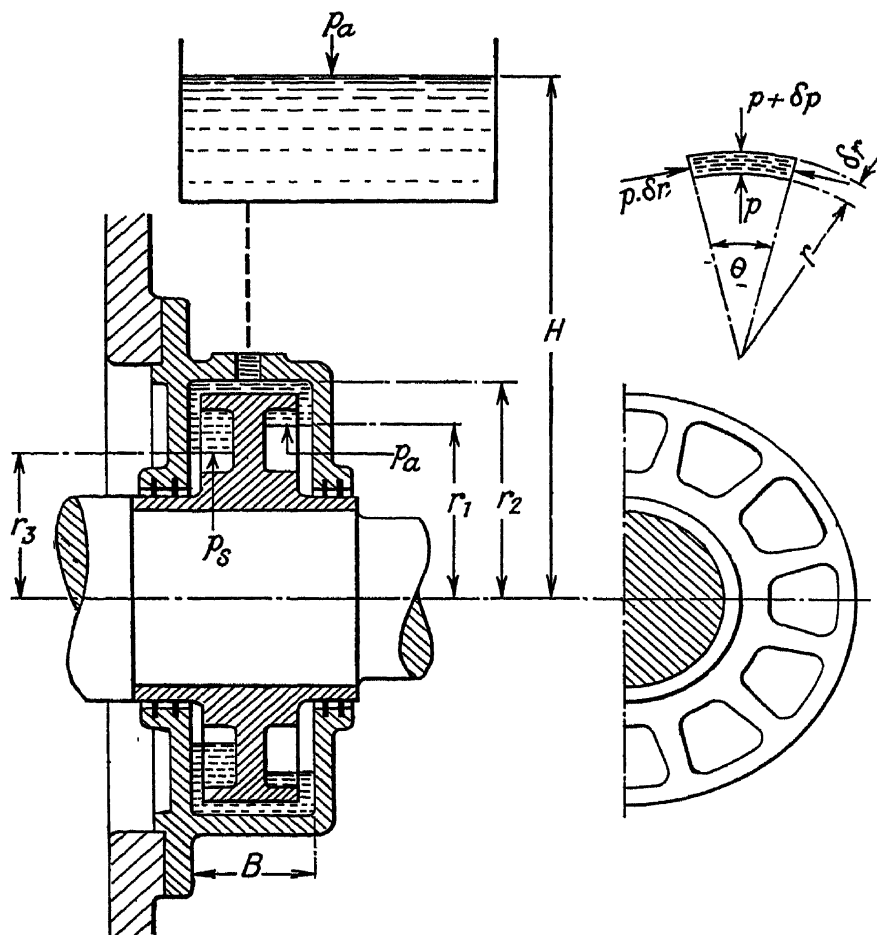


FIG. 97.—Diagrammatic Arrangement of Hydraulic Gland

Equating radial forces on element—

$$(p + \delta p)(r + \delta r)\theta = p \cdot \theta \cdot \delta r + p \cdot r \cdot \theta + \frac{\rho \omega^2 r^2}{g} \cdot \theta \cdot \delta r$$

Ignoring quantities of the second order of smallness—

$$\delta p = \frac{\rho \omega^2}{g} \cdot r \cdot \delta r$$

By integrating—

$$p_2 - p_1 = \frac{\rho \cdot \omega^2}{2g} (r_2^2 - r_1^2)$$

But

$$p_2 = p_a + \rho H, \text{ and } p_1 = p_a$$

∴

$$\rho H = \frac{\rho \omega^2}{2g} (r_2^2 - r_1^2)$$

$$H = \frac{\omega^2}{2g} (r_2^2 - r_1^2)$$

$$\text{i.e.} \quad H \text{ in } = 14.2 \left(\frac{N}{1000} \right)^2 (r_2^2 - r_1^2) \quad (112)$$

$$\begin{aligned} p_a - p_s &= \frac{\rho \cdot \omega^2}{2g} (r_1^2 - r_3^2) \\ &= 0.513 \left(\frac{N}{1000} \right)^2 (r_1^2 - r_3^2) \text{ lb per sq. in.} \quad (113) \end{aligned}$$

If a plain disc be used instead of a paddle wheel, then the mean speed of rotation of the forced vortex will probably be about one-half of the rotational speed of the shaft. An appropriate value of N will then have to be used in applying the foregoing formulae.

85. Advantages and Disadvantages of Hydraulic Glands. With ordinary labyrinth glands there is necessarily an escape of vapour into the turbine room. Under certain conditions, such vapour condenses on walls, windows and on other parts of the turbine room, and is undoubtedly a nuisance. This may be prevented absolutely by means of the hydraulic gland.

The hydraulic gland will operate automatically under all conditions of load and there is, consequently, no need to make adjustments to the gland steam supply as with certain arrangements of labyrinth glands. In turbines operating with high-temperature steam the hydraulic gland removes a considerable amount of heat from the shaft and so ameliorates conditions in the adjacent bearing.

Owing to friction losses in the gland and possibly to conduction of heat, the water in the gland is constantly being heated. Consequently, its temperature rises until it reaches the saturation temperature on the vacuum side, when a slow evaporation takes place, the vapour formed passing into the condenser. A small make-up must be constantly supplied from the overhead tank and unless a chemically pure water, such as condensate, be supplied there is a risk of scale formation in the gland.

86. Residual Velocity Loss. In a multi-stage turbine, the energy corresponding to the final absolute velocity in any stage is largely utilized in the nozzles of the succeeding stage. For a discussion of this, the reader is referred to Art 65.

In the final stage of a turbine, however, the kinetic energy corresponding to the final absolute velocity of the steam as it leaves the wheel is either wholly or partially lost. If no attempt is made to reduce the steam velocity between the last wheel and the exhaust branch, then the kinetic energy is entirely lost. In some cases, however, guide vanes are arranged in the exhaust hood, thus providing steam passages of gradually increasing area and a certain diffuser action. When such means are effective, the final velocity of the steam may be somewhat reduced and the steam pressure at outlet from the last wheel may be slightly lower than the pressure at the exhaust branch. It is doubtful, however, whether such guide vanes do more than permit the steam to turn through a right angle with less loss than if the guide vanes were not fitted.

Let V_2 = absolute velocity of steam at outlet from last-stage wheel.

H_a = adiabatic heat drop in turbine.

Then, energy loss = $\frac{V_2^2}{2gJ}$ B.Th.U. per lb. and the percentage loss

due to the terminal velocity = $\frac{V_2^2}{2gJ \cdot H_a} \times 100$ per cent.

The equivalent power loss

$$\begin{aligned} &= \frac{w \cdot V_2^2}{2g \times 550} \\ &= \frac{w \cdot V_2^2}{35,400} \text{ horse-power} \end{aligned}$$

where w = steam flow through turbine in lb. per sec.

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EXAMPLES IX

1 Calculate the power absorbed in disc friction in a de Laval turbine having a rotor 6.5 in diameter running at 25,000 r p m —

(a) When the pressure in the turbine cylinder is 2 lb per sq in abs, and the steam is dry and saturated ($v_g = 173.5$)

(b) When the pressure in the turbine cylinder is 15 lb per sq in. abs, and the steam is superheated 40° F ($v_g = 27.9$)

2 The first-stage wheel of a 20,000 kW turbine is a single-row wheel having a mean diameter 84 in. The speed of rotation is 1,500 r p m. The condition of the steam in the first stage is as follows—

Pressure	250
Superheat	200° F
Specific volume	2.43 cub. ft per lb

The blades are 1.25 in long and the active nozzles cover 40 per cent of the total circumference at full load. Calculate—

(a) Horse-power absorbed by disc friction.

(b) Horse-power absorbed by blade windage, using Kerr's 1924 formula for item (b)

3 The discs in the high-pressure cylinder of a two-cylinder steam turbine of 50,000 kW output are 36 in mean diameter, the speed being 1,500 r p m. Calculate the disc friction horse-power in the first stage, given that the steam conditions are as follow—

Steam pressure	400
Steam superheat	320° F
Steam specific volume	1.76

4 Calculate the leakage loss through the diaphragm packing in a turbine stage under the following conditions—

Steam pressure before nozzles	200
Steam superheat before nozzles	100° F
Steam pressure after nozzles	180
Shaft diameter	10 in
Radial clearance	0.015 in
Number of constrictions	4
Coefficient of contraction	0.75

Calculate also the pressures in the dwell spaces

5 In the intermediate-pressure turbine of a three-cylinder marine turbine installation, there is a diaphragm separating the intermediate-pressure ahead turbine and the high-pressure astern turbine. When the turbines are running ahead the steam pressure in the intermediate-pressure exhaust is 25 lb per sq in abs, the steam being dry and saturated, and the high-pressure astern turbine is running idly in steam at the exhaust pressure, which is 0.6 lb per sq in abs.

Calculate the approximate hourly leakage of steam through the partition gland (labyrinth type), given that the shaft diameter is 9.5 in, the radial clearance of the labyrinth packing is 0.02 in, the number of constrictions 20, and the coefficient of contraction 0.8. Neglect any heat losses.

w_i = total internal steam flow, lb. per sec.
 w_n = steam flow through nozzles, lb. per sec.
 w_g = steam flow through diaphragm gland.

$$w_i = w_n + w_a$$

a.h.p. = adiabatic horse-power of the stage, i.e. the horse-power based on the total internal steam flow and the adiabatic heat drop.

Then the stage efficiency is defined as the ratio of the shaft horse-power to the adiabatic horse-power.

Now the adiabatic work done per second by w_i lb. of steam

$$\begin{aligned} &= w_i \cdot h_n \text{ B.Th.U.} \\ &= 777.8 w_i \cdot h_n \text{ ft.-lb.} \end{aligned}$$

$$\therefore \text{a.h.p.} = \frac{777.8}{550} \cdot w_i \cdot h_n$$

or $\text{a.h.p.} = 1.414w_i \cdot h_n$ (115)

In an impulse-reaction turbine where the total adiabatic heat drop in the stage is $h_n + h_p$ —

$$\text{a.h.p.} = 1.414 \cdot w_i(h_n + h_p) \quad . \quad . \quad (116)$$

The shaft horse-power is less than the blade or rim horse-power by the amount of power absorbed in overcoming wheel friction, i.e. the sum of the disc friction and blade windage losses. Using the symbol r.h.p. to represent the "rim" or blade horse-power—

$$\text{r.h.p.} = \text{s.h.p.} + \text{h.p.}_d + \text{h.p.}_{bw}$$

88. State Point Locus on ϕ -I Diagram for Single Stage of an Impulse Turbine. Consider any stage of an impulse turbine and let

A, Fig. 98, represent the initial condition of the steam on the ϕ - I diagram. Suppose that steam expands in the nozzles from p_1 to p_2 . Then AB represents the adiabatic and isentropic expansion. Due to losses in the nozzles, the actual path of the state point during expansion is AC, as already explained in Art. 46.

The steam now enters the blades where the rim horse-power (r.h.p.) is developed. Basing the work done per lb of steam on the total internal flow w_i , the work done per lb of steam is

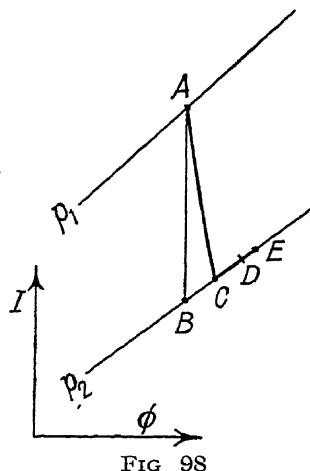
$$I_A - I_D = \frac{550 \times \text{r h p.}}{777.8 \times w_i} \text{ B Th U per lb}$$

i.e.
$$I_A - I_D = 0.707 \frac{\text{r h p.}}{w_i} \quad . \quad . \quad . \quad (117)$$

This may also be proved by the steady flow equation provided it be assumed that the absolute steam velocity at exit from the blades is identical with that of the steam entering the nozzles. The part CD of the p_2 line represents the reheating of the steam due to "blade friction" taking place, supposedly, at constant pressure. This method of showing reheating is not as accurate as it is convenient, because although the steam pressure at outlet from the blade equals that at inlet to the blade, the pressure in the blade channels is not constant.

Due to friction and blade windage (when it occurs) the steam is further reheated, the frictional reheat being

$$I_k - I_D = \frac{0.707 \text{ h p } w_f}{w_i} \quad (118)$$



The point E may be obtained directly from the shaft horse-power of the stage, for, granted the conditions stated above regarding the initial and final steam velocities, then by the steady flow equation

$$I_A - I_E = 0.707 \frac{\text{s h p}}{w_i} \quad . \quad . \quad . \quad (119)$$

i.e.
$$\text{s.h.p.} = 1.414 w_i (I_A - I_E)$$

Also, by eq (115), a.h.p. = $1.414 w_i (I_A - I_B)$

Hence
$$\eta_s = \frac{\text{s h p}}{\text{a h p}} = \frac{I_A - I_E}{I_A - I_B}$$

Thus, if η_s be known or assumed, the position of the end state point E for a stage may be readily obtained. The point E now becomes the initial state point for the succeeding stage of the turbine.

89. **State Point Locus for Multi-stage Turbine.** Suppose, for the sake of illustration, that a turbine has ten stages operating between pressure limits P_0 and P_{10} , and suppose that the stage efficiency of each stage is known and equal to η_s . Then the locus of the state point may be drawn on the ϕ -I diagram in the following manner.

Referring to Fig. 99, the initial state point A is set down according to the known initial pressure and superheat. Suppose the stage

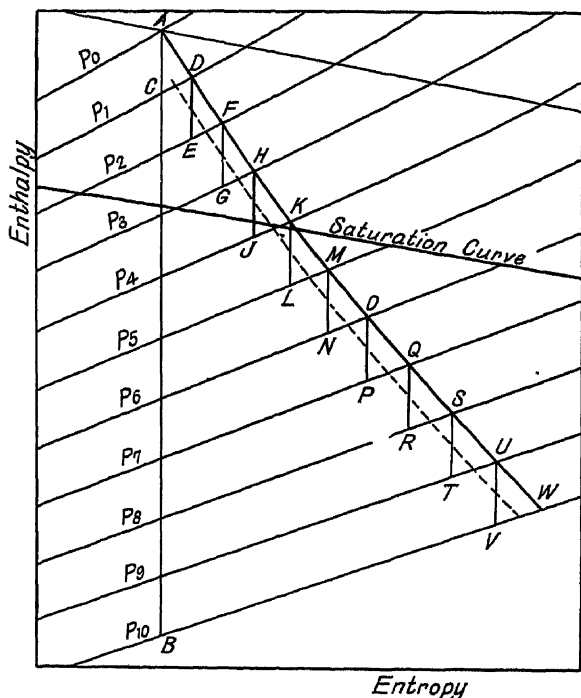


FIG. 99.—Locus of State Point on ϕ -I Diagram

pressures are $P_1, P_2, \dots, P_{10}$. AC is drawn to represent the adiabatic expansion in stage 1, and I_D is found from the relation

$$I_A - I_D = \eta_s(I_A - I_0)$$

Point D is then set down with the I_D value on the P_1 line. DE is drawn, giving $I_D - I_E$ and I_F calculated from the relation

$$I_D - I_F = \eta_s(I_D - I_E)$$

In this way the zig-zag line is drawn for the ten stages. The points D, F, H, . . . S, U, W represent the condition of the steam in the wheel chambers of the ten stages, i.e. at the blade outlets. If the nozzle efficiency is known, the condition of the steam at outlet

from the nozzles may also be obtained, giving the lower locus parallel to the curve DFH . . . UW

Such curves are essential in the design of a steam turbine because if the efficiency of the various stages is known fairly accurately, it is possible to derive from the Mollier diagram the superheat, or dryness fraction, as the case may be, and the specific volume of the steam at the assumed pressures

The sum of the adiabatic heat drops ($AC + DE + FG + \text{etc}$) from the initial steam pressure down to any given pressure is usually termed the "cumulative" heat drop and will be represented by the symbol H_c

90. Condition Curves. For the purpose of design the various quantities obtained from the Mollier diagram may be set out in the manner originally suggested by Professor Goudie (1) The method adopted by the author, however, is somewhat different from that given by Goudie and is shown in Fig 100 The base of the diagram shows values of p plotted to a log scale, the higher values being at the left-hand side since expansion proceeds from the higher pressure Plotted values of H_c then lie on a flat curve which is easily drawn with a batten. The curves showing the degree of superheat and the wetness fraction may also be drawn in the same way Instead of plotting values of v , the author plots $\log_{10} v$, thus obtaining (with a uniform stage efficiency) two straight lines, one for the superheat region and the other for the wet region, the two lines intersecting exactly at the pressure where the superheat vanishes

Fig 100 has been worked out for the following conditions—

Initial steam pressure	200 lb per sq in abs
Initial steam superheat	150° F
Final steam pressure	1 lb per sq in. abs.
Stage efficiency	0.75
Number of stages	10

In the initial stages of the calculation, when the exact heat drop per stage is not known, the condition curve may be drawn on the Mollier diagram by a step-by-step process, the adiabatic heat drop per step being taken approximately equal to the expected stage heat drop

Assuming that the curve H_c (Fig. 100) has been so obtained, the pressure distribution in the turbine may be worked out in the following manner If the mean diameter of all stages is the same then, if the ratio of blade speed to steam speed is constant, it follows that the adiabatic heat drop h_n will be the same for all stages.

$$\therefore h_n = \frac{H_c}{10} = 38.8 \text{ B Th U. per lb}$$

Thus by subdividing H_c into ten equal parts, drawing horizontals to intersect the H_c curve and projecting vertically on to the base

line, the various stage pressures are obtained. The values of v_n , the specific volume at outlet from the nozzles, and v_b , the specific

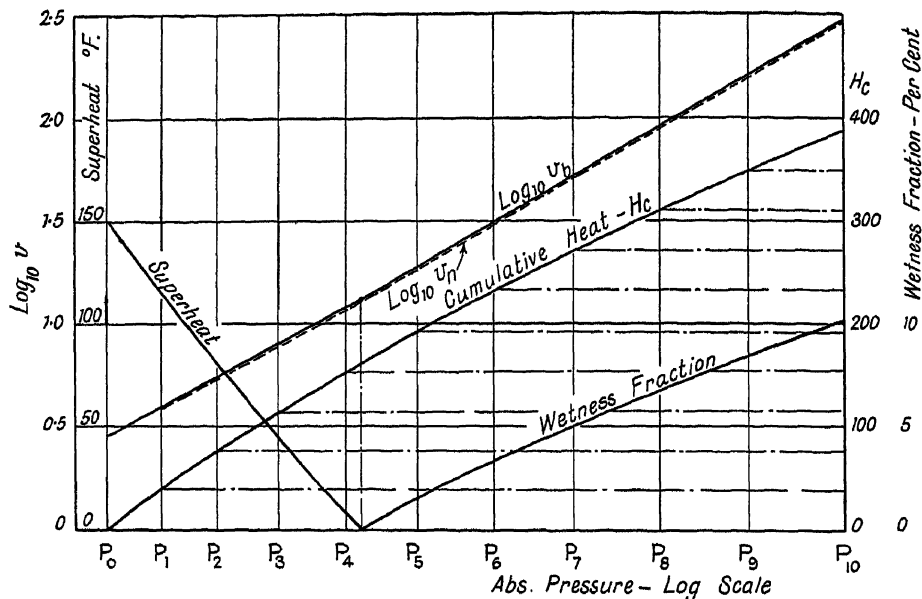


FIG. 100.—Condition Curves for Ten-stage Turbine

volume at outlet from the blades are also obtained. Such data may then be tabulated for design purposes, as shown in Table VII.

TABLE VII
VALUES OF STAGE PRESSURES, STEAM QUALITY, AND SPECIFIC VOLUMES
FOR TEN-STAGE STEAM TURBINE

Stage No.	p	v_n	v_b	Condition of Steam at Blade Outlet
1	134.9	3.855	3.908	114.2° F. Superheat
2	89.7	5.383	5.470	78.8° F. „
3	57.4	7.834	7.943	42.7° F. „
4	35.3	11.67	11.86	7.1° F. „
5	21.0	18.58	18.79	1.55% Wetness
6	12.2	30.48	30.90	3.40% „
7	6.87	51.05	51.76	5.11% „
8	3.72	89.33	90.36	6.82% „
9	1.98	158.9	160.7	8.47% „
10	1.00	297.2	299.4	10.11% „

91. Correction for Terminal Velocity. In certain cases, e.g. the L.P. stages of turbines of large output, the final absolute velocity

at outlet from the blades in a stage is greater than the inlet velocity to the nozzles of that stage. Applying the steady flow equation, and referring to Fig 98—

$$\begin{aligned} I_A + \frac{V_A^2}{2gJ} &= I_B + \frac{V_B^2}{2gJ} + \text{Work done} \\ &= I_B + \frac{V_B^2}{2gJ} + 0.707 \frac{\text{s h p.}}{w_i} \end{aligned}$$

$$\text{from which} \quad I_A - I_B = 0.707 \frac{\text{s h p.}}{w_i} + \frac{V_B^2 - V_A^2}{2gJ} \quad (120)$$

Considering the turbine as a whole, we may ignore the kinetic energy of the steam entering the first-stage nozzles. Then if V_2 represents the final absolute velocity of the steam leaving the last-stage wheel, and p_0 and p_n are the initial and final pressures, respectively, Fig 101, then considering first the frictionless adiabatic expansion we have

$$I_A = I_B + \frac{V_2^2}{2gJ} + \text{Work done per lb}$$

$$\text{i.e. Work done} = (I_A - I_B) - \frac{V_2^2}{2gJ}$$

This is another way of considering the residual velocity loss dealt with in Art 86.

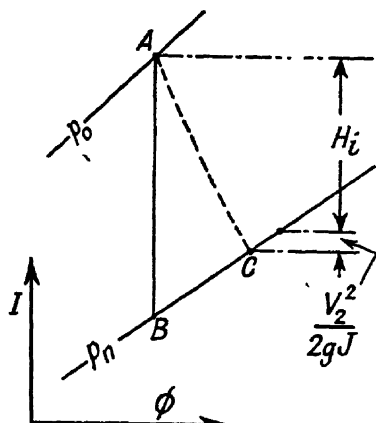


FIG 101

Let H_i = total internal work done in turbine, B.Th U. per lb of steam

$$\text{i.e.} \quad H_i = \sum_0^n 0.707 \frac{\text{s h p.}}{w_i}$$

$$\text{Then} \quad I_A = I_C + \frac{V_2^2}{2gJ} + H_i$$

$$\text{or} \quad I_A - I_C = H_i + \frac{V_2^2}{2gJ} \quad (121)$$

92. Reheat Factor. If a state point locus or condition curve ACDE . . UVW be drawn as shown in Fig. 99, and if the cumulative heat $H_c = \sum h_n$ be worked out, it will be found that H_c is somewhat greater than the adiabatic heat drop $H_a = AB$.

$$\text{The ratio} \quad R = \frac{H_c}{H_a} \quad (122)$$

is known as the reheat factor. The value of R depends upon the turbine stage efficiency, the initial pressure and superheat, and on the final pressure. Since the number of stages in the turbine will depend on H_e and the Mollier diagram only gives directly the value of H_a , the reheat factor becomes an important quantity in the initial stages of design. Before giving expressions for the value of the reheat factor, we shall endeavour to explain it.

Consider first of all the expansion of initially dry and saturated steam in a turbine, and assume that expansion takes place in thermal equilibrium. Assume, for the sake of simplicity, that the turbine has only four stages. Fig. 102 shows the T - ϕ diagram

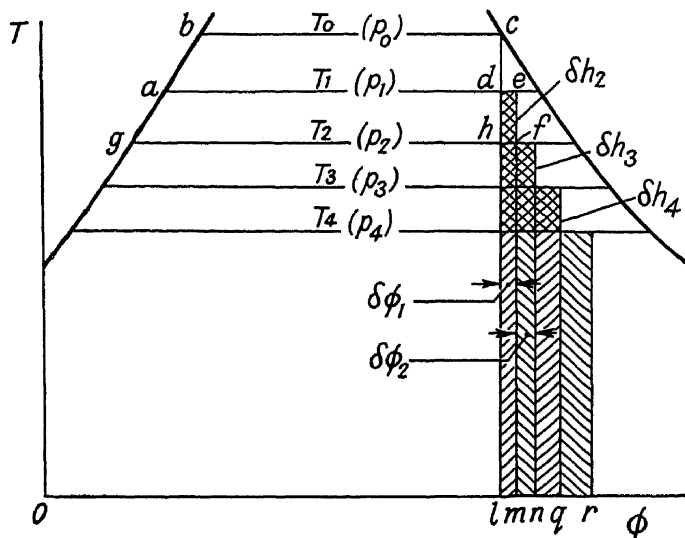


FIG. 102

and Fig. 103 the ϕ - I diagram for the expansion. These are lettered to correspond, as far as possible.

The adiabatic heat drop h_1 in the first stage of the turbine is represented by the area $abcd$ in Fig. 102 and by the length cd in Fig. 103. This is unaffected by losses. The point e represents the condition of the steam at inlet to the second stage. Assuming equal initial and final velocities in each stage, then

$$I_c - I_e = \eta_s(I_c - I_a)$$

$I_e - I_a$ is termed the "frictional reheat." This is the energy dissipated in friction, shock and leakage losses and "returned" to the steam as heat. Clearly

$$I_e - I_a = (1 - \eta_s)h_1$$

and is represented by the area $ldem$ in Fig. 102.

Therefore, the gain of entropy due to frictional reheating in the first stage—

$$\delta\phi_1 = (1 - \eta_s) \frac{h_1}{T_1}$$

Of the first-stage reheat, represented by the area $ldem$, the part that is available for the performance of mechanical work in the remaining stages of the turbine is the fraction $\frac{T_1 - T_4}{T_1}$, and is represented by the cross-hatched area. The increase in the adiabatic

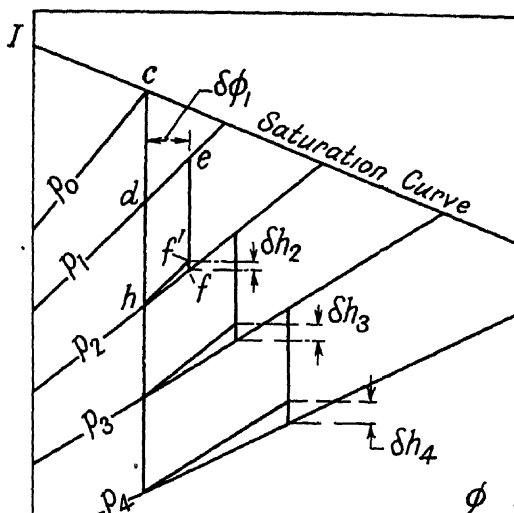


FIG 103

heat drop in the second stage due to the friction losses in the first stage is represented by the area $defh$

In the Mollier diagram, let hf' be drawn parallel to de . Then $def'h$ is a parallelogram and ef' is equal to dh . But ef is the adiabatic heat drop in the second stage. Therefore, $f'f$ represents the increase in the second stage adiabatic heat drop due to the friction losses in the first stage. Now the slope of a constant pressure line has been shown in Art 27 (eq. (24)) to be equal to the absolute temperature. Thus, for de and hf' —

$$\frac{dI}{d\phi} = T_1$$

and for hf'

$$\frac{dI}{d\phi} = T_2$$

$$\therefore I_{f'} - I_h = T_1 \delta\phi_1$$

and

$$I_f - I_h = T_2 \delta\phi_1$$

$$\begin{aligned} \therefore I_{f'} - I_f &= \delta h_2 = (T_1 - T_2) \delta\phi_1 \\ &= \text{Area } defh \text{ (Fig. 102)} \end{aligned}$$

A similar reheating of the steam takes place in the succeeding stages, but the availability of the reheat becomes smaller and smaller as the pressure at which reheating is effected falls. For this reason, it is preferable to have an inefficient stage, e.g. a two-row velocity wheel, at the H.P. end of the turbine rather than at the L.P. end. The total increase in the sum of the stage adiabatic heat drops is represented in Fig. 102 by the double-hatched area, and in Fig. 103 by the sum $\delta h_2 + \delta h_3 + \delta h_4$.

When the expansion in a stage takes place wholly in the superheat region, the conditions are as shown in Fig. 104 (a) (T- ϕ diagram) and Fig. 104 (b) (ϕ -I diagram). The gain in heat drop in the second

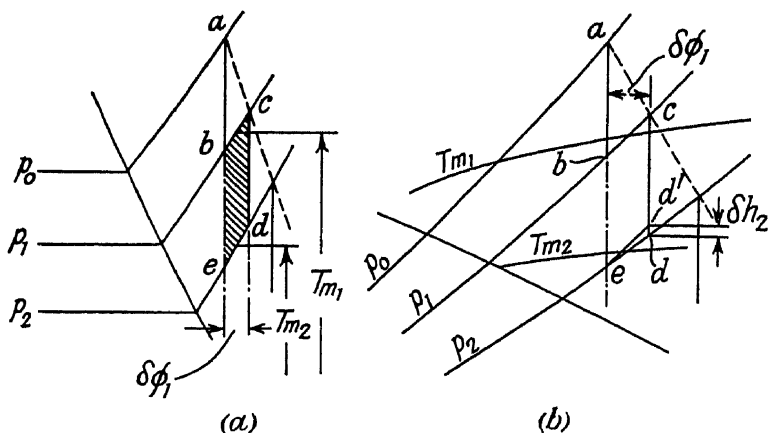


FIG. 104

stage is clearly equal to the area $ebcd$, to scale, on Fig. 104 (a). In the ϕ -I diagram, Fig. 104 (b), the constant-pressure lines bc and ed , although curved, are so short that they may be considered as straight lines. As before, draw ed' parallel to bc . Let T_{m1} and T_{m2} represent the mean absolute temperatures of the steam between b and c , and between e and d , respectively.

Then

$$I_{a'} - I_e = T_{m1} \cdot \delta\phi_1$$

and

$$I_a - I_e = T_{m2} \cdot \delta\phi_1$$

$$\therefore \delta h_2 = I_{a'} - I_a = (T_{m1} - T_{m2})\delta\phi_1 \\ = \text{Area } bcde \text{ (Fig. 104 (a))}$$

A similar argument may be applied to other stages where expansion takes place entirely in the superheat region, or partly in the superheat and partly in the saturated region.

The reheat factor is greater, the larger the number of stages for a given pressure range and the lower the stage efficiency.

93. Reheat Factor for an Expansion with a Uniform Adiabatic Index and a Constant Stage Efficiency. Consider the frictionally

resisted expansion of steam which, when expanding adiabatically, follows the law $p v^n = \text{constant}$, and let η_s be the stage efficiency. Furthermore, assume that the number of stages is very large and, therefore, that the adiabatic heat drop in each stage is extremely small. In this case the condition curve, instead of being stepped, is a continuous curve, corresponding to which we have the curve BC in the p - v diagram, Fig. 105. It will be shown that given an adiabatic relation $p v^n = \text{constant}$, there is a corresponding equation $p v^m = \text{constant}$ for the curve BC

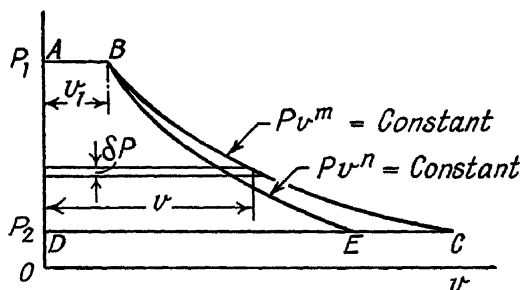


FIG. 105

In expanding from P to $P + \delta P$, the heat available is equal to $-v \delta P$ ft-lb, the minus sign being used because δP is negative. If the efficiency of the small expansion is η_s , the reheat is

$$\delta H = -(1 - \eta_s)v \cdot \delta P \text{ ft-lb}$$

By the First Law of Thermodynamics—

$$\text{Heat supplied} = \text{Work done} + \text{Gain of internal energy}$$

$$\text{or} \quad \delta H = P \cdot \delta v + \delta E$$

For an adiabatic change, since $\delta H = 0$,

$$\delta E = -P \delta v$$

$$\text{But} \quad P v^n = \text{Const} = C$$

$$\therefore \delta E = -C \cdot v^{-n} \cdot \delta v$$

$$\text{or} \quad E = -C \int v^{-n} dv$$

$$= -C \left(\frac{v^{1-n}}{1-n} \right) + \text{Const of integration } C_1$$

$$= -\frac{Pv}{1-n} + C_1$$

$$\therefore \delta E = \frac{P}{n-1} \delta v + \frac{v}{n-1} \delta P$$

$$\therefore - (1 - \eta_s) v \cdot \delta P = P \cdot \delta v + \frac{P \cdot \delta v + v \cdot \delta P}{n - 1}$$

$$\therefore \{n - \eta_s(n - 1)\} \frac{\delta P}{P} + n \cdot \frac{\delta v}{v} = 0$$

Integrating—

$$\{n - \eta_s(n - 1)\} \log_e P + n \log_e v = \text{Constant}$$

$$\therefore P v^m = \text{Constant}$$

where
$$m = \frac{n}{n - \eta_s(n - 1)} \quad . \quad . \quad . \quad (123)$$

Thus the modified expansion can be represented by the equation $P v^m = C$, where m , the index, is a function of the adiabatic index n and the stage efficiency η_s . Provided n is constant and also that η_s is constant, m must be constant.

Referring to Fig. 105, the sum of the adiabatic heat drops in the actual turbine is represented by the area ABCD, whereas the adiabatic heat drop in the frictionless turbine is ABED.

Hence
$$R = \frac{\text{Area ABCD}}{\text{Area ABED}} = \frac{H_c}{H_a}$$

$$H_c = \frac{m \cdot P_1 v_1}{m - 1} \left\{ 1 - \left(\frac{1}{r} \right)^{\frac{m-1}{m}} \right\} \text{ ft.-lb.}$$

$$H_a = \frac{n \cdot P_1 v_1}{n - 1} \left\{ 1 - \left(\frac{1}{r} \right)^{\frac{n-1}{n}} \right\} \text{ ft.-lb.}$$

where
$$r = \frac{P_1}{P_2}$$

Hence
$$R = \frac{m(n-1)}{n(m-1)} \cdot \frac{1 - \left(\frac{1}{r} \right)^{\frac{m-1}{m}}}{1 - \left(\frac{1}{r} \right)^{\frac{n-1}{n}}}$$

Substituting for m the value
$$\frac{n}{n - \eta_s(n - 1)}$$

$$R = \frac{1}{\eta_s} \cdot \frac{1 - \left(\frac{1}{r} \right)^{\eta_s \left(\frac{n-1}{n} \right)}}{1 - \left(\frac{1}{r} \right)^{\frac{n-1}{n}}} \quad . \quad . \quad . \quad (124)$$

94. **Reheat Factors for Steam Expanding wholly in the Superheat Region.** Here $n = 1.3$ and therefore—

$$R = \frac{1}{\eta_s} \cdot \frac{1 - \left(\frac{1}{r}\right)^{0.2808\eta_s}}{1 - \left(\frac{1}{r}\right)^{0.2808}} \quad . \quad . \quad . \quad (125)$$

For values of η_s lying between 0.8 and unity, the reheat fraction $R - 1$ is very nearly proportional to $(1 - \eta_s)$. By plotting these values against $\log_{10} r$, the author finds that—

(1) For values of $\frac{P_1}{P_2}$ between 5 and 15—

$$R = 1 + (1 - \eta_s) \left(0.246 \log_{10} \frac{P_1}{P_2} + 0.003 \right) \quad . \quad (126)$$

(2) For values of $\frac{P_1}{P_2}$ between 15 and 40—

$$R = 1 + (1 - \eta_s) \left(0.205 \log_{10} \frac{P_1}{P_2} + 0.051 \right) \quad . \quad (127)$$

These formulae are theoretically correct for values of η_s between 0.8 and 1.0, and for an infinite number of stages

95. **Reheat Factors for Steam Initially Dry and Saturated and Expanding in Thermal Equilibrium.** Zeuner's index $n = 1.135$ is not the same for all initial pressures, nor is it strictly constant for the whole of any expansion. Consequently, formula 124 does not yield very reliable values of the reheat factor. The author finds that R is given sufficiently nearly for practical purposes by the equation—

$$R = 1 + (1 - \eta_s) \left(0.09 \log_{10} \frac{P_1}{P_2} - 0.02 \right) \quad (128)$$

96. **Correction for Finite Number of Stages.** The expressions given above for the reheat factor apply only to a turbine having an infinite number of stages. They could be applied with little error to turbines having a relatively large number of stages. The approximate correction for a finite number of stages may be easily found provided certain simplifying assumptions are made. Figs. 106 (a) and 106 (b) show the T - ϕ diagrams for expansions in the saturated and superheat regions respectively. AB shows the adiabatic expansion and AC the actual expansion line for an infinite number of stages. In each case, let ϕ be the gain of entropy due to reheating, n the number of stages in the actual turbine, and assume that the total gain of entropy is the same for a finite as for an infinite number of stages. Let T be the temperature range AB in each case and suppose that the temperature drop in each stage is uniform.

For the purpose of the proof we shall further assume that AC is a straight line and that the constant-pressure lines in Fig. 106 (b) are straight. Then—

- (1) Increase of adiabatic heat drop when n is infinite

$$= \text{Area ABC} = \frac{T\phi}{2}$$

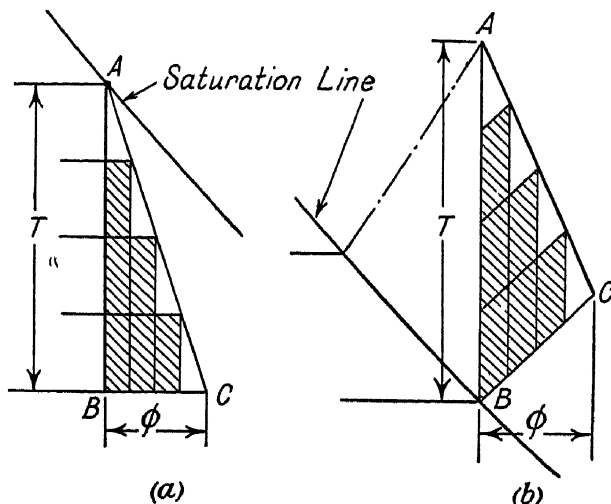


FIG. 106

- (2) Increase of adiabatic heat drop when n is finite

= Shaded area

$$= \text{Area ABC} - \frac{n}{2} \cdot \frac{T}{n} \cdot \frac{\phi}{n}$$

$$= \frac{T\phi}{2} \left(1 - \frac{1}{n} \right)$$

Let R_∞ = reheat factor for infinite number of stages.

R_n = reheat factor for n stages.

Then
$$R_\infty - 1 = \frac{T\phi}{2H_a}$$

and
$$R_n - 1 = \frac{T\phi}{2H_a} \left(\frac{n-1}{n} \right)$$

i.e.
$$\frac{R_n - 1}{R_\infty - 1} = \left(\frac{n-1}{n} \right) \quad . \quad . \quad . \quad (129)$$

97. Reheat Factors for Steam Expanding partly in the Superheated Condition and partly in the Saturated Condition. This offers a rather more complicated problem which is beyond the scope of this book. Values of R may, however, be obtained by drawing the condition curve on the Mollier chart by the step-by-step method,

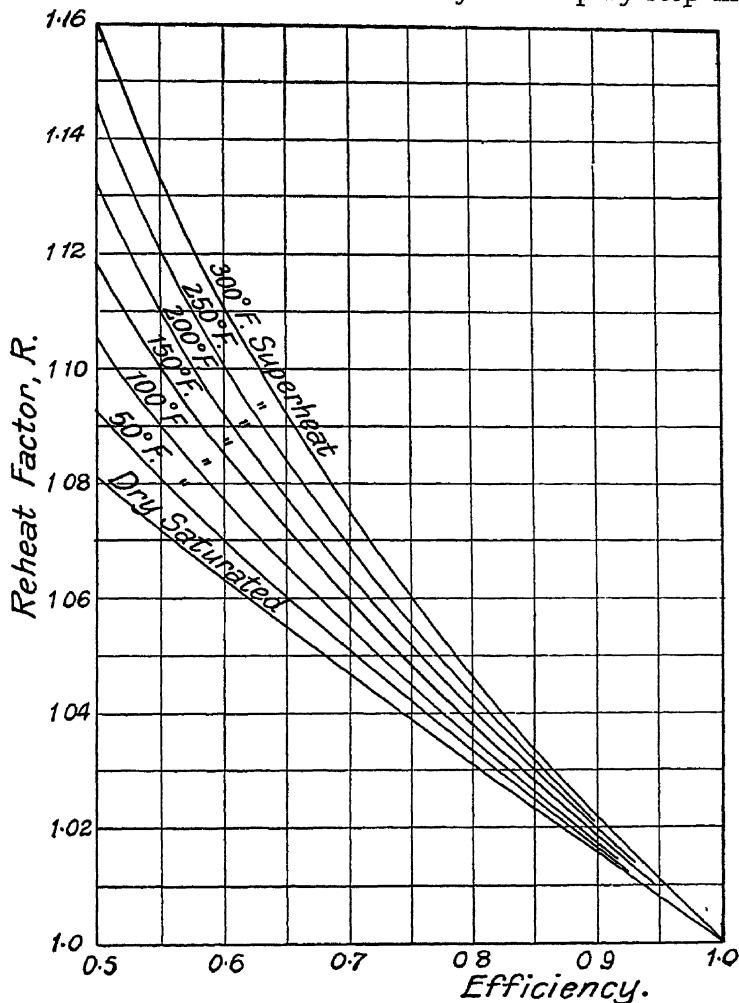


FIG. 107

using suitable heat drops. The curves given in Fig. 107 were obtained in this way for the following basic data—

Initial steam pressure	200 lb per sq. in. abs.
Exhaust steam pressure	1 " Th U " per lb."
Approximate stage heat drop	40 B Th U " per lb."

In turbines having more than ten stages, the value of R would be slightly higher than that given by the curves.

For further work on this subject the reader is referred to articles and papers by Messrs. E. L. Robinson (2), H. G. Yates (3), R. B. Smith (4), Professor C. G. Thatcher (5), and Dr. D. M. Smith (6).

98. Internal Efficiency. The internal efficiency η_i is defined as the ratio of the internal work done in the turbine, expressed in B.Th.U. per lb. of steam, to the adiabatic heat drop.

$$\text{i.e.} \quad \eta_i = \frac{H_i}{H_a} \quad . \quad . \quad . \quad . \quad (130)$$

If η_s is the mean stage efficiency, then

$$\eta_s = \frac{H_i}{H_o}$$

and since $H_o = RH_a$

$$\eta_i = R \cdot \eta_s \quad . \quad . \quad . \quad . \quad (131)$$

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EXAMPLES X

1. Steam at a pressure of 100 lb. per sq. in. abs. and superheated 100° F. expands adiabatically to a pressure of 1.0 lb. per sq. in. abs. What is the difference between the initial and final enthalpy and what is the final quality of the steam? If only 60 per cent of the adiabatic heat drop is converted into mechanical work, calculate the increase in the final dryness fraction and in the final entropy value due to reheating.

2. The initial enthalpy at inlet to the nozzles of a turbine stage is 1,150 B.Th.U. per lb. and the enthalpy at outlet from the blades is 1,118 B.Th.U. per lb. If the total internal steam flow through the turbine is 24 lb. per sec., and the power absorbed in disc friction is 10 h.p., what is the work done on the blading per lb. of steam? Neglect diaphragm gland leakage.

3. The total internal steam flow through a 20,000 kW steam turbine is 54 lb. per sec. In a given stage, the diaphragm gland leakage is 0.5 lb. per sec. The mean blade speed is 550 ft. per sec. and the ratio of blade speed to steam speed is 0.5. The nozzle angle is 18°, the nozzle efficiency 0.88, the value of

k for the blades is 0.86, and the carry-over factor for the nozzles is 0.90. Given that the blade outlet angle is 36° and that the power absorbed by disc friction is 55 h.p., calculate the stage adiabatic horse-power, the shaft horse-power, and the stage efficiency.

4. The following particulars refer to a steam turbine of 25,000 kW output—

Mean diameter of last wheel	120 in
R.p.m.	1,500
Effective height of blades on last wheel	28 in
Exhaust pressure	0.5 lb per sq in abs
Dryness fraction of exhaust steam	0.9
Internal steam flow	8.8 lb per kWh of turbine output

Assuming that the steam flows axially from the blades of the last wheel, calculate—

- (1) Terminal velocity
- (2) Terminal velocity loss
- (3) Percentage loss, given that the adiabatic heat drop in the turbine is 47.4 B.Th.U. per lb
- (4) Final enthalpy value, given that the initial enthalpy value is 1391.83 B.Th.U. per lb and that the total internal power is 25,750 kW

5. Steam which is initially dry and saturated at an absolute temperature T_1 expands in a turbine to an absolute temperature T_2 , the stage efficiency being η_s . Assuming a very large number of stages and that the condition curve on the p - v diagram is a straight line, show that the reheat factor

$$R = \frac{T_1 + T_2}{2T_2 + \eta_s(T_1 - T_2)}$$

is the actual reheat factor greater or less than this approximate value?

6. Dry and saturated steam is supplied to a four-stage steam turbine at a pressure of 200 lb per sq in abs and exhausted at a pressure of 1 lb per sq in abs. If the stage efficiency is 60 per cent and the pressures in the intermediate stages are 70, 20, and 5 lb per sq in respectively, derive the value of the reheat factor by using the Mollier chart.

7. Describe with reference to the T - ϕ and ϕ - I charts the effect of internal losses on the condition of the steam in a multi-stage turbine. State what is meant by "reheat factor," and state the influence of (a) efficiency, (b) the number of stages, on the value of the reheat factor.

8. Steam which is initially dry and saturated is expanded in a multi-stage turbine, in each stage of which 30 per cent of the heat available is reconverted into heat. If the adiabatic index be taken as 1.35, calculate the index m of the expansion curve and the ratio of the sum of the stage adiabatic heat drops to the adiabatic heat drop with frictionless expansion, if the pressure-ratio of the expansion be 180.

9. Steam at a pressure of 150 lb per sq in abs. and a temperature of 500°F expands in a five-stage turbine to a pressure of 1 lb per sq in. abs. The steam pressures in the five stages are 70, 30, 10, 3, and 1 lb per sq in. respectively. The stage efficiency is 65 per cent. Tabulate the entropy and total heat data with reference to the ϕ - I chart and derive the reheat factor.

10. Given that the adiabatic index for superheated steam is 1.3, calculate the actual index for an adiathermal expansion in which the heat drops are infinitely small and the stage efficiency is 0.75. Hence calculate the theoretical reheat factor for a pressure-ratio of expansion of 20. What would be the approximate value of R for a turbine with ten stages and the same ratio of expansion?

REGENERATIVE FEED-HEATING

99. Introductory. In the fourth chapter of this book we have considered the simplest possible form of cycle on which a steam turbine may operate, namely, a cycle in which all the heat supplied is given externally, in which the steam is expanded adiabatically and exhausted at constant pressure. Such a cycle, termed the Rankine cycle, has formed a useful basis for comparing the performance of turbines operating under different conditions.

The object of the present chapter is to consider modifications to the usual cycle of operations which increase its thermal efficiency, and which have become standard practice in steam power plants both on land and, in the more powerful installations, at sea.

It has long been the practice to preheat the feed water in a so-called "economizer" by means of the heat in the gases leaving the boiler. Since both coal and oil contain hydrogen, a certain amount of water vapour is formed during combustion. A very small amount of water vapour carried into the furnace by the combustion air goes to increase the total amount of vapour in the flue gases. Now if the gases are cooled below the dew-point by contact with relatively cool metal surfaces, such as economizer tubes, then a part of the vapour contained in the gases will be condensed and deposited on the outer surface of the tubes. For coal and oil firing, with the usual amount of excess air, the dew-points are, respectively, about 95° F. and 110° F. The actual wall temperature of the tubes will, of course, lie between the mean gas temperature and the feed temperature but, owing to the greater resistance to heat transmission on the gas side of the tubes, the wall temperature will be much nearer to the feed temperature than the gas temperature. Consequently, in order to prevent condensation or "sweating," the feed temperature at inlet to the economizer should be somewhat higher than the dew-point of the flue gases. If condensation should occur, the sulphur dioxide or sulphur trioxide formed by combustion of the sulphur in the fuel will be dissolved in the water to form sulphurous or sulphuric acid respectively. As the condensate temperature is sometimes as low as 70° F. to 75° F., some preheating of the feed water is necessary.

Such feed-heating may be done by live steam or by exhaust steam from auxiliaries. The former method is not efficient and in many cases the latter method cannot be adopted because the auxiliaries are all electrically driven. An alternative method of feed-heating is by extracting or "bleeding" a portion of the steam from a stage in the turbine and utilizing the latent heat of the bled

steam for raising the temperature of the whole of the feed water. As both the steam and the feed water are the same working substance, the process is a regenerative one

Quite apart from being a means of preventing the sweating of economizer tubes, regenerative feed-heating has an important

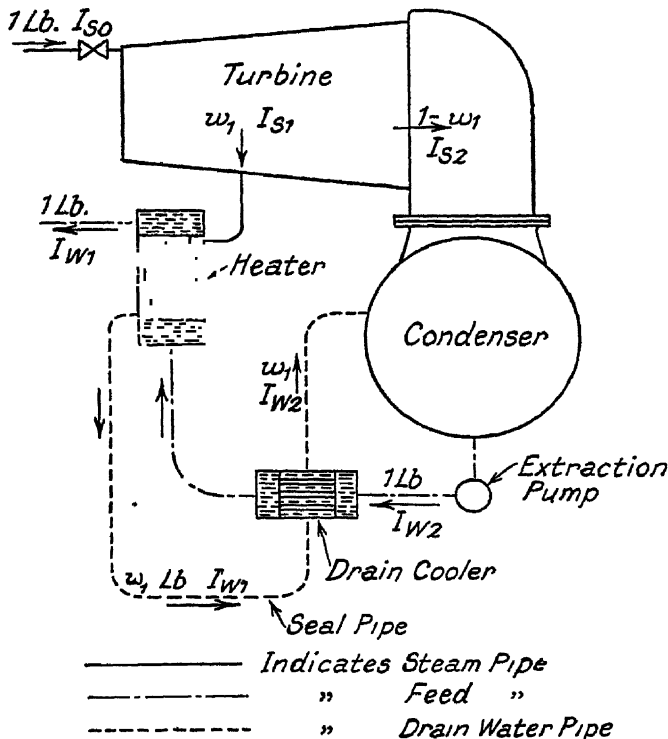


FIG. 108 —Diagram of Single-stage Regenerative Feed-heating System

thermodynamic aspect, for if the principle be extended so that the feed water is heated up to temperatures well in excess of that required to prevent sweating, the thermal efficiency of the turbine operating cycle may be appreciably increased. A brief analysis of the cycle will now be given.

100. Single-stage Heating. The arrangement of the plant for single-stage heating is shown diagrammatically in Fig. 108. Steam is bled off at some pressure p_1 , the corresponding saturation temperature being t_1 . The extracted steam is being condensed continuously in the heater and gives up its latent heat for the heating of the water. The condensed heating steam, or drain water as it is termed, at t_1 may then pass through a cooler in which its temperature is reduced to t_2 , which is the temperature of the feed water entering the drain cooler. This assumes perfect action in

the cooler in the sense that there is no resistance to heat flow from the hot drain water to the feed water. Similarly, we shall assume perfect heat transmission in the main heater, so that the feed water will be assumed to reach the same temperature as the extracted steam.

The drain cooler is sometimes omitted in practice for economic reasons, the drain water being led into the condenser. In an alternative arrangement, the drain water is extracted from the heater by a pump which discharges it into the feed pipe after the heater. An extension of this principle is illustrated later in Fig. 111.

Let w_1 = weight of steam tapped off at p_1 per lb. of steam supplied to the turbine.

I_{s0} , I_{s1} , etc., represent the enthalpy of the steam at the pressures p_0 , p_1 , etc.

I_{w1} , I_{w2} represent the enthalpy of water at the saturation temperatures t_1 , t_2 .

In deriving the following expressions it will be assumed that the feed water is in the saturated condition at every point in the system. This requires that its pressure be raised gradually as its temperature is raised. We shall ignore the very small amount of work done in feed compression and its effect on the heat supplied. In the arrangement shown in Fig. 108 we may regard the heater and drain cooler as a single unit, in which—

$$\left. \begin{array}{l} \text{Heat given up by steam} \\ \text{and water} \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat taken up by feed} \\ \text{water} \end{array} \right.$$

$$\text{or} \quad w_1(I_{s1} - I_{w2}) = (I_{w1} - I_{w2}) \times 1 \text{ lb.}$$

$$\text{or} \quad w_1 = \frac{I_{w1} - I_{w2}}{I_{s1} - I_{w2}} \quad \quad \quad (132)$$

The heat supplied from external sources to generate 1 lb. of steam = $I_{s0} - I_{w1}$.

The heat rejected to the circulating water in the condenser = $(1 - w_1)(I_{s2} - I_{w2})$.

$\therefore$ Work done by steam expanding in turbine

$$= (I_{s0} - I_{w1}) - (1 - w_1)(I_{s2} - I_{w2})$$

and the thermal efficiency of the modified cycle

$$\eta_c = 1 - \frac{(1 - w_1)(I_{s2} - I_{w2})}{I_{s0} - I_{w1}} \quad \quad \quad (133)$$

For the purpose of comparison the thermal efficiency of the Rankine cycle, expressed in the same symbols, is

$$1 - \frac{I_{s2} - I_{w2}}{I_{s0} - I_{w2}}$$

In actual practice, where the feed water reaches a temperature t_3 (I_{w3}), which is some 5° to 20° F lower than t_1 , and where the drain water leaves the drain cooler at a temperature t_4 (I_{w4}), which may be about 20° to 30° F higher than t_2 , the value of w_1 will be

$$w_1 = \frac{I_{w3} - I_{w2}}{I_{s1} - I_{w4}} \quad (134)$$

and
$$\eta_e = 1 - \frac{(1 - w_1)(I_{s2} - I_{w2})}{I_{s0} - I_{w3}} \quad (135)$$

The gain in theoretical thermal efficiency with single-stage heating naturally depends on the temperature to which the feed water is heated. If that temperature is relatively low, then the heat to be supplied in the boiler is reduced by a small amount. On the other hand, if the temperature to which the feed is heated approaches close to the steam saturation temperature, the system becomes approximately that of live steam feed-heating, which produces no gain in thermal efficiency because the reduction of sensible heat to be supplied in the boiler is just equal to the heat which has been extracted from the live steam produced in the boiler. If a temperature be chosen intermediate between the condenser and live steam saturation temperatures, then the extracted steam has already done a certain amount of mechanical work, and all its remaining latent heat and a certain amount of its sensible heat may be utilized for feed-heating purposes with a quite definite thermal advantage.

The influence of temperature may be followed from Fig. 109, which shows the results obtained for the following conditions and assumptions—

Initial steam pressure	600 lb. per sq. in. abs.
Initial steam temperature	850° F.
Initial steam saturation temperature	486.3° F.
Exhaust pressure	0.505 lb. per sq. in. abs.
Exhaust saturation temperature	80° F.
Mean value of turbine stage efficiency	80 per cent.

It is assumed—

- (1) That the feed outlet temperature from any heater is equal to the steam saturation temperature in that heater.
- (2) That the condensed steam from each heater, together with the drains from the next high-pressure heater, is passed into the next low-pressure heater.
- (3) That the combined drains from the lowest-pressure heater are cooled in a drain cooler to 80° F.

The thermal efficiency of the turbine without feed-heating is 33.36 per cent. The percentage gain in thermal efficiency due to

feed-heating in one stage reaches a maximum value of 6.45 at a temperature of 280° F., which is roughly midway between the exhaust and live steam saturation temperatures.

101. Multi-stage Heating. Greater gains in thermal efficiency may be attained by the use of several heaters arranged in series, and by raising the temperature to which the feed water is heated.

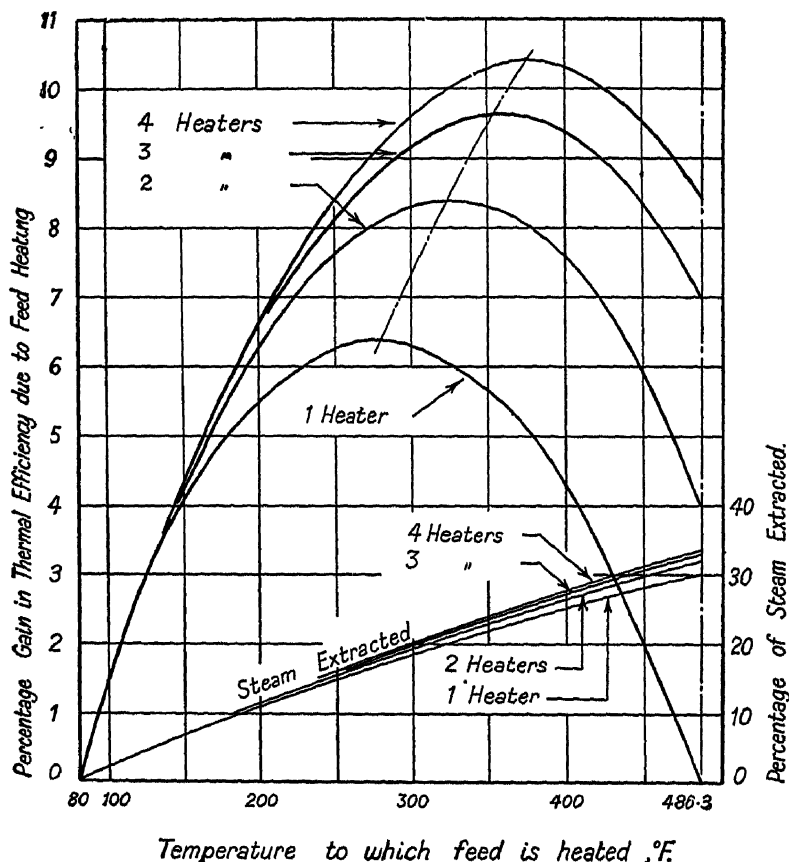


FIG. 109.—Gain in Thermal Efficiency due to Regenerative Feed-heating

A typical arrangement of the plant with three-stage heating is shown diagrammatically in Fig. 110. The water is heated in steps, and the turbine stages from which steam is extracted are selected so that the saturation temperature of the steam in each particular bleeding stage is a little higher (to allow for heat transmission) than the temperature to which it is desired to heat the feed water. Actually, of course, the feed temperatures are ultimately dependent on the steam temperature available.

In deriving the following relations, it is assumed that the hot drain water from heater No. 1 passes into the steam space of heater No. 2, that the drains from No. 2 pass into No. 3, and that the combined drains from heater No. 3 are cooled in a drain cooler to

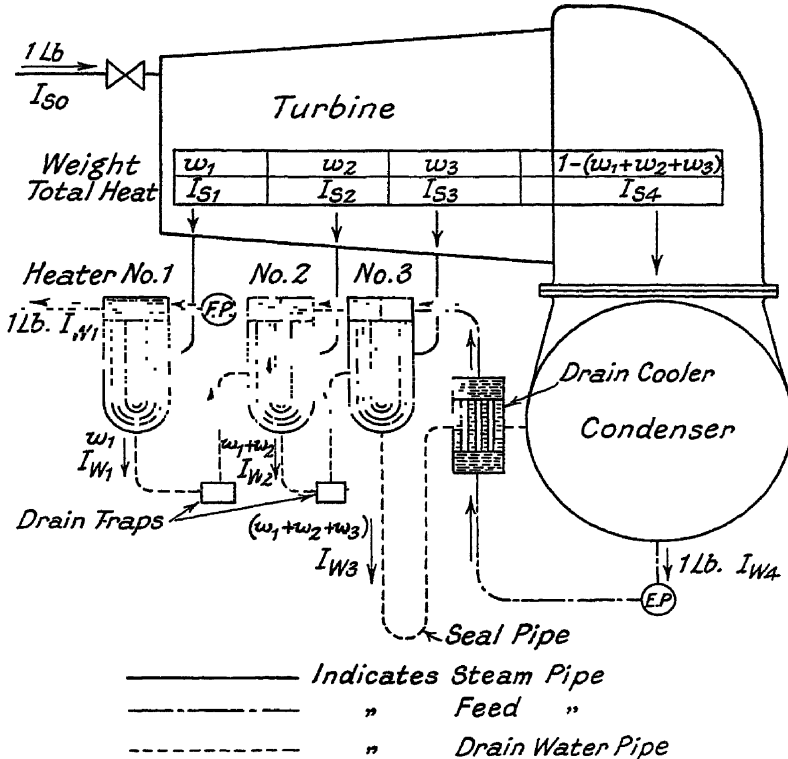


FIG. 110—Diagram of Three-stage Regenerative Feed-heating System

Note—E P represents the condensate extraction pump
F P represents the boiler feed pump

the temperature of the main condensate. Then, applying the steady flow equation to each heater in turn, it may be shown that

$$w_1 = \frac{I_{w1} - I_{w2}}{I_{s1} - I_{w1}} \quad (136a)$$

$$w_2 = \frac{(I_{w2} - I_{w3}) - w_1(I_{w1} - I_{w2})}{I_{s2} - I_{w2}} \quad (136b)$$

$$w_3 = \frac{(I_{w3} - I_{w4}) - (w_1 + w_2)(I_{w2} - I_{w4})}{I_{s3} - I_{w4}} \quad (136c)$$

Work done in the turbine per lb. of steam

$$\begin{aligned}
 H_w = & w_1(I_{s0} - I_{s1}) + w_2(I_{s0} - I_{s2}) \\
 & + w_3(I_{s0} - I_{s3}) \\
 & + [1 - (w_1 + w_2 + w_3)](I_{s0} - I_{s4}) \quad (137)
 \end{aligned}$$

$$\text{Heat supplied} \quad H = I_{s0} - I_{w1} \quad . \quad . \quad . \quad . \quad (138)$$

$$\text{Thermal efficiency} \quad \eta_c = \frac{H_w}{H}$$

The work done in the turbine per lb. of steam supplied may also be calculated from the relation

$$\begin{aligned} H_w &= \text{Heat supplied} - \text{Heat rejected} \\ &= (I_{s0} - I_{w1}) - [1 - (w_1 + w_2 + w_3)](I_{s4} - I_{w4}) \end{aligned}$$

Fig. 109 shows the gain in thermal efficiency due to the use of two, three, or four heaters for varying feed temperatures. A study of this graph shows the advantage of multi-stage heating as compared with single-stage heating, for not only is the maximum thermal gain greater, but the temperature at which the maximum gain is obtained approaches nearer the steam saturation temperature the larger the number of heaters used.

The above relations may be easily modified for any actual case in which temperature differences must be allowed for heat transmission.

The drainage of the feed heaters is sometimes effected by means of centrifugal pumps which extract the condensed heating steam and deliver it into the feed pipe immediately after the heater in question. This system is illustrated diagrammatically in Fig. 111 for three-stage feed-heating. It is assumed that steam is bled off at three points where the steam pressures are p_1 , p_2 , and p_3 , the corresponding saturation temperatures being t_1 , t_2 , and t_3 , and the water enthalpies I_{w1} , I_{w2} , and I_{w3} respectively. It is reasonable to assume that the drain water temperatures from the three heaters are also t_1 , t_2 , and t_3 .

Consider the drain pump for heater No. 1, and suppose that the pressure rise produced by that pump is P_1 lb. per sq. ft., the specific volume of the water being v_1 cub. ft. per lb. Then, assuming the compression of the water in the drain pump to be adiabatic, the increase in enthalpy, given by eq. (25), Art. 28, must be

$$I'_{w1} - I_{w1} = \frac{P_1 v_1}{J} \text{ B.Th.U. per lb.}$$

$$\text{Also} \quad I'_{w2} - I_{w2} = \frac{P_2 v_2}{J} \text{ B.Th.U. per lb.}$$

$$\text{and} \quad I'_{w3} - I_{w3} = \frac{P_3 v_3}{J} \text{ B.Th.U. per lb.}$$

Allowing for the temperature differences necessary for heat transmission in the feed heaters, let I''_{w1} , I''_{w2} , and I''_{w3} be the water enthalpies immediately after heater Nos. 1, 2, and 3 respectively, and I'''_{w1} , I'''_{w2} , and I'''_{w3} the water enthalpies after mixing the

feed and drain pump discharge after heaters Nos. 1, 2, and 3 respectively. Then $I''_{w1} < I_{w1} < I'_{w1}$, and so on.

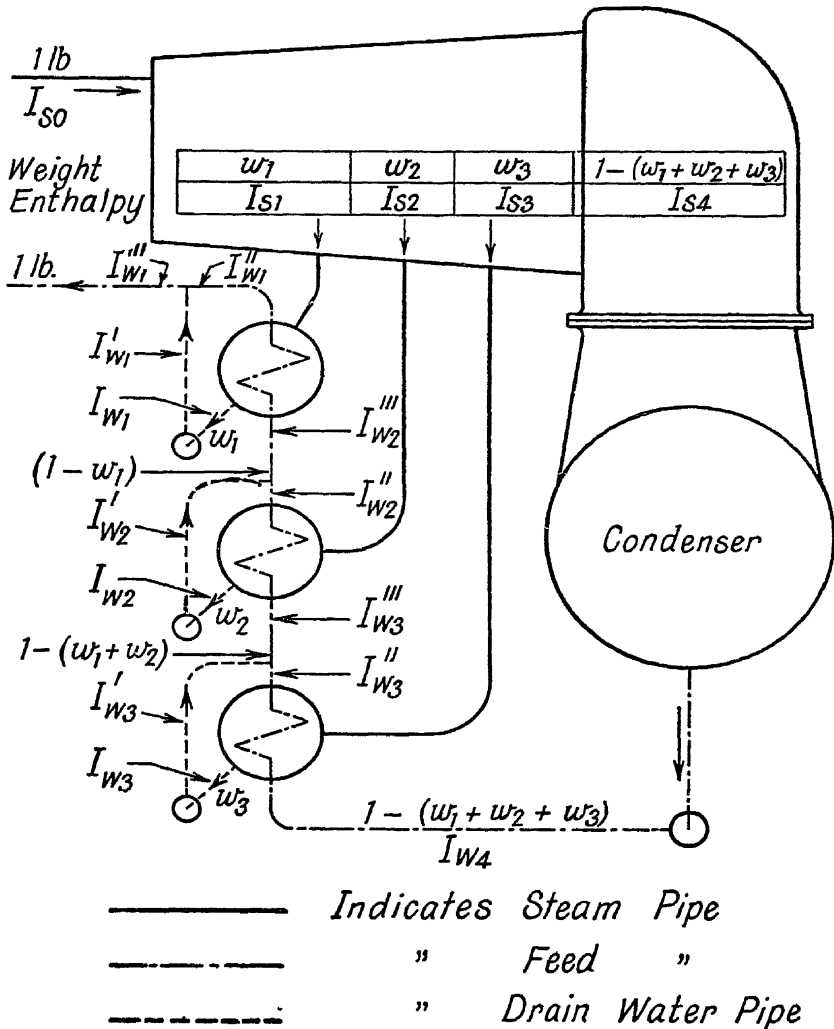


FIG. 111.—Diagrammatic Arrangement of Three-stage Regenerative Feed-heating System in which Heater Condensate is Extracted by Drain Pumps

Then considering heater No 1, ignoring heat losses and kinetic energies, and applying the steady flow equation, we have

(Mass-enthalpies entering system) + (Drain pump work)

= (Mass-enthalpies leaving system)

$$w_1 I_{s1} + (1 - w_1) \cdot I'''_{w2} + w_1 (I'_{w1} - I_{w1})$$

$$= (1 - w_1) I''_{w1} + w_1 \cdot I'_{w1}$$

from which

$$w_1 = (I''_{w1} - I'''_{w2}) \div (I_{s1} + I''_{w1} - I'''_{w2} - I_{w1}) \quad . \quad (139a)$$

In the same way it may be shown that

$$w_2 = \{(I''_{w2} - I'''_{w3}) - w_1(I''_{w2} - I'''_{w3})\} \div (I_{s2} + I''_{w2} - I'''_{w3} - I_{w2}) \quad . \quad (139b)$$

$$w_3 = \{(I''_{w3} - I_{w4}) - (w_1 + w_2)(I''_{w3} - I_{w4})\} \div (I_{s3} + I''_{w3} - I_{w4} - I_{w3}) \quad . \quad (139c)$$

In order to obtain the final condition of the feed water and to use the above expressions it is necessary to know the enthalpy values I'''_{w1} , I'''_{w2} , and I'''_{w3} .

$$\begin{aligned} I'''_{w1} &= (1 - w_1) \cdot I''_{w1} + w_1 \cdot I'_{w1} \\ &= I''_{w1} + w_1(I'_{w1} - I''_{w1}) \end{aligned}$$

Similarly—

$$\begin{aligned} I'''_{w2} &= \{I''_{w2} - w_1 \cdot I''_{w2} + w_2(I'_{w2} - I''_{w2})\} \div (1 - w_1) \\ I'''_{w3} &= \{I''_{w3} - (w_1 + w_2)I''_{w3} + w_3(I'_{w3} - I''_{w3})\} \\ &\quad \div \{1 - (w_1 + w_2)\} \end{aligned}$$

For practical purposes the effect of the drain pump could safely be neglected and left to balance the heat loss. In practice it is not usual to employ a drain pump on each heater. The drains may be cascaded from one heater to the next, as shown in Fig. 110, and extracted from the L.P. heater by a drain pump instead of being passed through the drain cooler. For details of heaters, drainage devices, etc., and for typical practical arrangements of heaters, the reader is referred to the author's book, *Steam Turbine Operation*.

102. General Notes on Feed-heating. We have seen that the thermal efficiency of a steam turbine plant may be increased by the use of bled steam feed heaters, the percentage gain at the optimum feed temperature increasing as the number of heaters is increased. In practice the appreciable cost of the heaters, pipes, valves, and accessories puts a limit to the number of heaters employed. In large units the temperature rise of the feed water per heater is normally about 50° to 60° F., three to five heaters being installed.

Up to the present we have only considered the influence of regenerative feed-heating so far as it affects the thermal efficiency of the turbine. It is essential, however, to remember that the turbine is only a part of an organic whole, and that if by the adoption of this method of feed-heating we render the use of an economizer unnecessary, then the flue gases will leave the boiler plant at an appreciably higher temperature, and the reduction in boiler efficiency will partly offset the increase in turbine cycle efficiency.

The solution of the problem lies in the use of air heaters wherein

the flue gases are cooled by the combustion air prior to its admission to the furnace. This is, of course, but a further application of the regenerator principle. In practice, air heating possesses certain advantages and disadvantages. The mean temperature in the boiler furnace is higher when the air temperature is higher, and, in consequence, the percentage of fuel heat transmitted by direct radiation is appreciably greater. Trouble is sometimes experienced with mechanical stokers operating with preheated combustion air, and, in this respect, pulverized fuel firing would appear to be at an advantage. Some authorities state that the air temperature should not exceed about 350° F. with chain-grate stokers, but the limiting temperature depends to a large extent on the type of fuel and stoker. A somewhat recent development is to supply to the stoker itself moderately heated air at about 250° F., and to supply a quantity of secondary air above the fire at a temperature from 350° to 600° F.

Because of the fact that a certain proportion of the steam supplied to the turbine is extracted before it reaches the exhaust end, the quantity of exhaust steam may be reduced by as much as 12 per cent per unit generated. This reduction is a considerable help to the designer who may be faced with the problem of designing a very high capacity machine. There is a reduction in the cooling surface to be provided in the main condenser, but this is actually more than offset by the necessity of providing condensing surface (of an expensive type) in the feed heaters.

EXAMPLES XI

1. A steam turbine plant equipped with a single regenerative feed heater operates under the following conditions—

Initial steam pressure	250
Initial superheat	200° F
Extraction pressure	30
Exhaust pressure	0.9

Compare the regenerative and non-regenerative cycles with respect to the following: (a) thermal efficiency, (b) steam consumption in lb per kWh, and (c) condenser duty (steam condensed per kWh).

It may be assumed that the expansion is adiabatic, that the feed water is heated to the steam saturation temperature in the heater, and that the drain water from the heater is cooled in a drain cooler to the condenser temperature.

2. Steam is supplied to a turbine at a pressure of 400 lb per sq in abs and a temperature of 712° F., and is expanded adiabatically to a pressure of 0.5 lb per sq in abs. At a stage of the turbine where the pressure is 40 lb per sq in abs a connection is made to a surface feed heater in which the feed water is heated by bled steam to a temperature of 260.9° F. The condensed steam from the feed heater is cooled in a drain cooler to 79.5° F., the feed water passing through the drain cooler before entering the feed heater. The cooled drain water combines with the condensate in the well of the condenser.

Assuming no heat losses in the system, calculate the following—

- (a) Weight of steam used for feed-heating per lb of steam entering the turbine
- (b) Thermal efficiency of cycle.

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3. During a test on a steam turbine equipped with a single stage of regenerative feed-heating, the following results were obtained—

Steam pressure at stop valve	300
Steam temperature at stop valve	600° F.
Steam pressure in nozzle box	250
Bled steam pressure	40
Exhaust pressure	1.0
Temperature of condensate	100° F.
Temperature of drain water leaving drain cooler	110.7° F.
Temperature of feed water leaving feed heater	257.6° F.
Steam entering turbine	50,000 lb. per hour

Calculate the coupling power of the turbine, assuming (1) that the drain water leaving the drain cooler enters the steam space of the condenser, (2) that the internal work done by the steam which completes its expansion is 300 B.Th.U. per lb., (3) that the condition curve on the ϕ -I diagram is a straight line, and (4) that the bearing friction and other external losses amount to 2 per cent of the internal power.

4. Steam is supplied to a turbine at 450 lb. per sq. in. abs. and superheated 200° F. The turbine exhaust pressure is 1.0 lb. per sq. in. abs.

The main condensate is heated regeneratively in two stages by steam bled from the turbine at 70 lb. and 12 lb. per sq. in. abs. respectively. Calculate the weights of steam bled off at each pressure per lb. of steam entering the turbine and the theoretical thermal efficiency of the cycle.

The following assumptions are to be made—

(a) That the expansion of the steam in the turbine is adiabatic and frictionless.

(b) That the condensate is heated to the saturation temperature in each of the heaters.

(c) That the drain water from the H.P. heater passes through a trap without loss of heat into the steam space of the L.P. heater.

(d) That the combined drains from the L.P. heater are cooled in a drain cooler to the condenser temperature.

5. A steam turbine operates under the following conditions—

Initial pressure	300
Initial superheat	200° F.
Final pressure	0.5

Steam is bled off from turbine stages at pressures of 70, 20, and 4 lb. per sq. in. abs. for the purpose of heating the main condensate. The expansion in the turbine is adiabatic and frictionless. It may be assumed that the condensate is heated in each heater up to the saturation temperature of the steam in that heater, and also that the drain water from each heater is cascaded through a trap into the next heater on the low-pressure side of it. The combined drains from the heater operating at 4 lb. pressure are cooled in a drain cooler to condenser temperature.

Calculate the following—

(a) Weight of steam extracted for each heater per lb. of steam entering the turbine.

(b) Thermal efficiency of cycle.

(c) Thermal efficiency of the Rankine cycle.

(d) Theoretical gain due to regenerative feed-heating.

(e) Steam consumption of turbine in lb. per kWh with and without regenerative feed-heating.

(f) Quantity of steam passing through the last-stage nozzles of a 50,000 kW turbine with and without regenerative feed-heating.

(g) Amount of heat carried away by the cooling water in the condenser per kWh with and without regenerative feed-heating.

6 A steam turbine plant is equipped with a single regenerative feed heater of the surface type. The condensed heating steam is withdrawn from the heater by a drain pump which delivers the hot drain water into the feed pipe immediately after the heater. Calculate the enthalpy of the feed water at outlet from the system, assuming adiabatic compression in the drain pump and no heat losses. Calculate also the weight of drain water to be pumped per hour when the steam flow into the turbine is 55,000 lb per hr, given the following information—

Steam pressure in nozzle box	250 lb per sq. in. abs
Steam temperature in nozzle box	650° F
Bled steam pressure . . .	40 lb per sq. in. abs
Exhaust steam pressure . .	0.7 lb. per sq. in. abs.
Internal efficiency of turbine	78 per cent
Pressure in feed pipe . . .	120 lb per sq in. abs.
Condensate temperature . .	87.7° F
Feed temperature at heater outlet	258° F
Specific volume of water at 40 lb per sq in.	0.017 cub ft. per lb

Assume that the condition curve on the Mollier diagram is a straight line

REHEATING AND WATER-EXTRACTION CYCLES

103. **Flow of Wet Steam in Nozzles and Blades.** In the foregoing chapters of this book dealing with the flow of steam through nozzles and blades it has been tacitly assumed that the steam is a homogeneous fluid and that at any point in the flow of wet steam, the

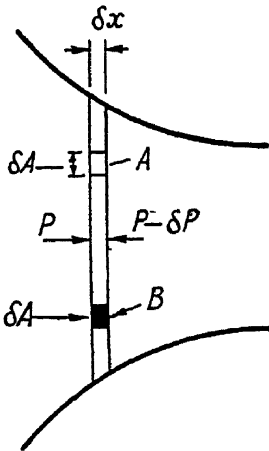


FIG. 112

steam and the suspended water particles are moving in the same direction and with identical velocities. Whilst this is a very convenient assumption, in that it simplifies the theoretical work and calculations based thereon, it is not in accord with the facts.

Consider the expansion of wet steam in a nozzle. The flowing steam will carry along with it a large number of suspended water droplets. These are of varying size. There are the exceedingly minute particles consisting of a few hundred molecules and formed by condensation an extremely short time before the instant under consideration. There will also be much larger droplets due to the joining up of smaller particles. There is, however, an upper limit to the size of such droplets.

Fig. 112 shows a section through a steam nozzle. Consider two transverse sections a distance δx apart and suppose that these sections are also sections of equal pressure P and $P - \delta P$ respectively. Consider the motion of two elementary masses, one — A, of steam, and the other — B, of water, assuming for the sake of simplicity that B represents a water particle of prismatic shape instead of the natural spherical shape. It is assumed also that at the section considered the particles are, at the given instant, moving with the same velocity so that the only accelerating force is that due to the pressure drop.

Let ρ_s = density of steam, lb. per cub. ft.

ρ_w = density of water, lb. per cub. ft.

The accelerating force on the two masses is the same, namely, $g \cdot \delta P \cdot \delta A$ pounds, and since the masses are $\rho_s \cdot \delta A \cdot \delta x$ and $\rho_w \cdot \delta A \cdot \delta x$ respectively—

Acceleration of steam

$$= f_s = g \cdot \frac{\delta P}{\delta x} \cdot \frac{1}{\rho_s}$$

Acceleration of water

$$= f_w = g \frac{\delta P}{\delta x} \cdot \frac{1}{\rho_w}$$

$$\therefore \frac{f_w}{f_s} = \frac{\rho_s}{\rho_w} = \frac{v_w}{v_s}$$

At 10 lb per sq in

$$\frac{f_w}{f_s} = \frac{0.0166}{38.37} = 0.000434$$

At 1 lb per sq in.

$$\frac{f_w}{f_s} = \frac{0.0161}{333} = 0.0000484$$

In other words, the rate of acceleration of the steam *due to pressure difference only* is 2,300 times that of the water particle at 10 lb per sq in, and 20,700 times at 1 lb per sq in abs

The result is that the water particle is accelerated at a much smaller rate than the steam, and soon lags behind. Under the new conditions there is a certain relative velocity of the steam past the lagging particle which introduces a new accelerating force, for if at any point the velocity of the steam is V_s and of the moisture V_m , the relative velocity is $V_s - V_m$ and the accelerating force on the water particle *due to the relative velocity only* is

$$F = a \rho_s (V_s - V_m)^2 \delta S \text{ poundals}$$

where a = a number which is a function of the Reynolds number, but may for the purpose of this simple analysis be assumed as a constant

δS = external surface of droplet

Assuming a spherical droplet of diameter d , $S = \pi d^2$, and since the volume is $\frac{\pi d^3}{6}$, the mass is $\rho_w \frac{\pi d^3}{6}$

$$\begin{aligned} \therefore \text{Acceleration } f_w &= \frac{\text{Force}}{\text{Mass}} \\ &= \frac{6a \left(\frac{\rho_s}{\rho_w} \right) (V_s - V_m)^2}{d} \end{aligned}$$

This expression shows that the acceleration of the water particles will be least for the larger droplets and also least in the low-pressure stages where ρ_s is so very small.

The foregoing analysis of the problem is very incomplete, but it suffices to show that in the stages of a turbine where the steam is wet the water droplets do not acquire the same velocities as the expanding steam, and, moreover, that the velocity difference is

likely to be greatest for the larger particles and in the stages where the steam pressure is lowest. Theoretical calculations show that in the L.P. stages of a turbine the ratio of the moisture velocity to

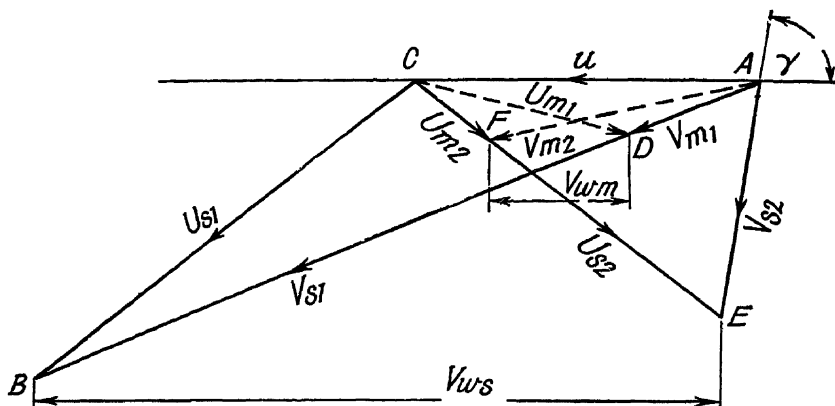


FIG. 113.—Velocity Diagram for Dry Steam and Water Particles

steam jet velocity might vary from 0.1 to 0.3, according to the jet velocity, being higher at the higher jet velocities.

Consider now the velocity diagram for an impulse turbine, Fig. 113.

Let V_{s1} = steam velocity at exit from nozzle.

V_{m1} = moisture velocity at exit from nozzle (assumed to be uniform for all particles).

$$m_1 = \frac{V_{m1}}{V_{s1}}.$$

U_{s1} = relative velocity of steam at inlet to blades.

U_{m1} = relative velocity of moisture at inlet to blades.

U_{s2} = relative velocity of steam at outlet from blades.

U_{m2} = relative velocity of moisture at outlet from blades.

$$m_2 = \frac{U_{m2}}{U_{s2}}.$$

V_{ws} = velocity of whirl for dry steam.

V_{wm} = velocity of whirl for moisture particles.

In drawing the velocity diagram it has been assumed that the entrained water issues from the nozzle in the same direction as the steam. The inlet velocity diagrams for the steam and water are ABC and ADC respectively. From the latter it is seen that the water particles entering the blades have a relative velocity $U_{m1} = CD$ and thus strike the leading face of the blades, as shown in Fig. 114. The subsequent action in the blade channels must be

somewhat conjectural. In all probability, the spherical water droplets are flattened out and more or less broken up into smaller droplets which will be swept forward by the rapidly flowing steam with the result that the water droplets leave the blade channels with a certain relative velocity U_{m2} , which is less than U_{s2} . U_{m2} is not, by any means, uniform, but will be assumed so. It will also be assumed that the relative directions of the dry steam and entrained moisture are the same at the blade outlet.

It will be clear that the interactions of the steam and water in the blade channels will affect the velocities U_{s2} and U_{m2} . The

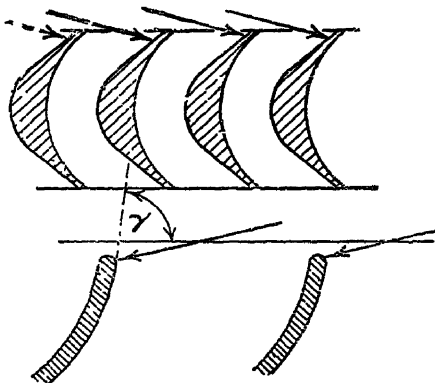


FIG. 114 — Showing Oblique Impact of Water Drops on Moving Blades and Nozzle Guide Vanes

momentum of the incoming moisture particles is practically destroyed at inlet and the available momentum is, therefore, that of the dry steam. If x is the dryness fraction of the steam, then the momentum per lb. of stuff, i.e. of steam and water mixture, is $x \cdot U_{s1}$. With homogeneous flow, the momentum at outlet would be $k \cdot x \cdot U_{s1}$, where k has the usual meaning. Equating the momentum of the steam and water at outlet from the blade channel, we have

$$\begin{aligned} x \cdot U_{s2} + (1-x)U_{m2} &= x \cdot k \cdot U_{s1} \\ U_{s2}(x + (1-x)m_2) &= x \cdot k \cdot U_{s1} \\ \therefore U_{s2} &= \frac{x \cdot k \cdot U_{s1}}{x + (1-x)m_2} \end{aligned} \quad (140)$$

$$U_{m2} = \frac{m_2 \cdot x \cdot k \cdot U_{s1}}{x + (1-x)m_2} \quad (141)$$

This enables us to complete the velocity diagram. Note that the water particles leave the wheel in the direction of AF and strike the nozzle guide vanes of the next stage obliquely, as shown in Fig. 114

The incoming energy per lb. of stuff is

$$\frac{x \cdot V_{s1}^2}{2g} + \frac{(1-x)V_{m1}^2}{2g} \text{ ft.-lb.}$$

The force on the blades per lb. of stuff per second is

$$\frac{1}{g} (x \cdot V_{ws} + (1-x)V_{wm}) \text{ lb. wt.}$$

where correct account is taken of the sign of V_{wm} . Note that in Fig. 113 the component of V_{m2} in the direction of motion is greater than that of V_{m1} and that, accordingly, the sign of V_{wm} is negative.

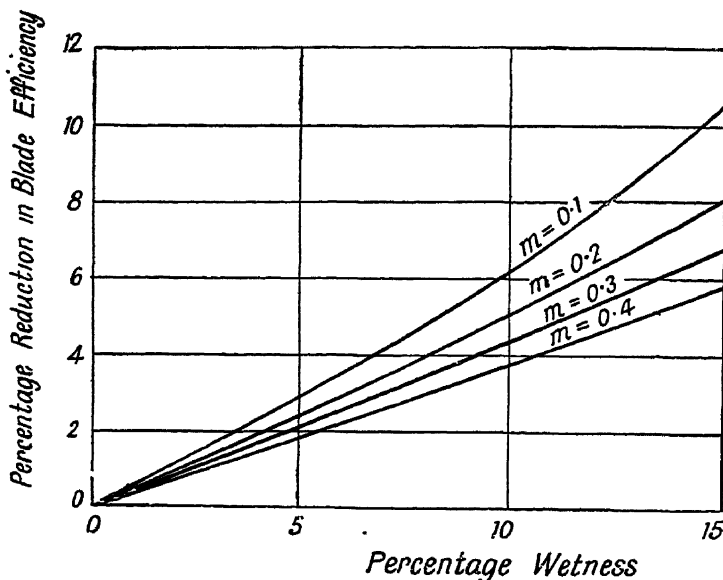


FIG. 115.—Reduction in Blade Efficiency owing to Moisture in Steam

In this way, the braking action of the water is correctly accounted for, provided the assumptions which have been stated above are reasonably correct.

Hence the work done per lb. of stuff is

$$W = \frac{u}{g} (x \cdot V_{ws} + (1-x)V_{wm}) \text{ ft.-lb.} \quad (142)$$

and the blade efficiency is given by the expression

$$\eta_b = \frac{2u(x \cdot V_{ws} + (1-x)V_{wm})}{V_{s1}^2(x + (1-x)m_1^2)} \quad (143)$$

The effects of moisture based on the above tentative theory are shown in Fig. 115 for the following basic data—

Nozzle angle = 24° ; $\rho = 0.5$.

Equiangular blades, $\beta_1 = \beta_2 = 44.6^\circ$; $k = 0.85$; and $m_2 = m_1$.

Taking the values at 10 per cent wetness as a basis and assuming $m = 0.1$ only, the percentage reduction in blading efficiency is 0.62 per 1 per cent of wetness. Actually, this represents only a part of the losses in a turbine stage operating with wet steam. Owing to the interactions between the steam and suspended moisture in

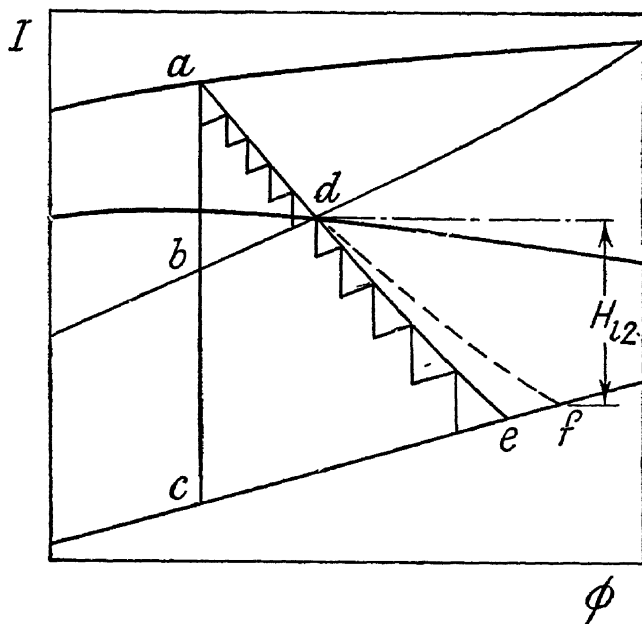


FIG. 116—Correction of Condition Curve to allow for Effects of Wetness

the nozzles, certain losses of energy occur. Moreover, after impact on the leading face of the blade, it is very probable that the water droplets resulting from the disintegration of the larger droplets will be flung outwards to a greater radius and will thus have a certain amount of kinetic energy imparted to them by the moving blades. This reduces the useful work done. It has been stated by the well-known authority, Mr K. Baumann, (1) that the stage efficiency η_s' of a turbine stage working with wet steam is equal to the dry stage efficiency η_s multiplied by the dryness fraction.

i.e.

$$\eta_s' = x \cdot \eta_s \quad . \quad . \quad (144)$$

This simple rule is apparently based on test figures and is very easily applied in practical calculations.

Considerations similar to those outlined above apply to the axial-flow impulse-reaction (Parsons) turbine. Water drops strike both fixed and moving blades obliquely.

104. Correction to Condition Curve for Wetness. If the efficiency of a stage operating with dry steam—we shall term this the dry

stage efficiency—is known, the condition curve for a constant stage efficiency may be corrected in the following manner. Referring to Fig. 116, let H_{c1} be the cumulative heat drop for the superheat part of the expansion, and H_{c2} that for the wet part.

Then $I_a - I_d = \eta_s \cdot H_{c1}$ and $I_d - I_e = \eta_s \cdot H_{c2}$

Considering the wet stages, if h is the adiabatic heat drop in any one stage and x is the dryness fraction in that stage, then, by

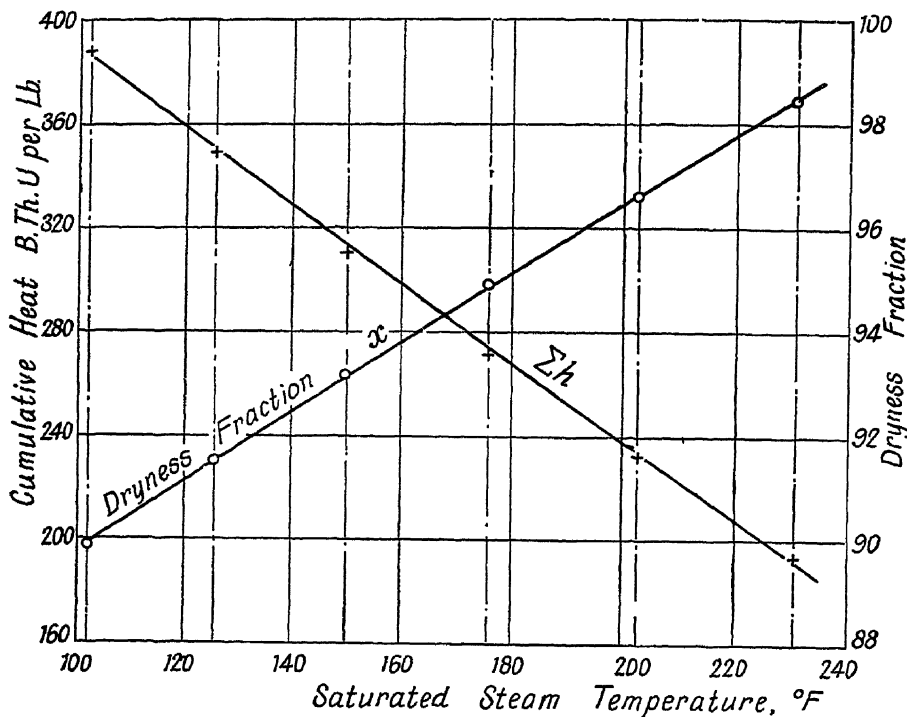


FIG. 117

eq. (144), the work done in the stage is $x \cdot \eta_s \cdot h$. If values of Σh and x are plotted against saturation temperature for the wet stages, it is found that the relations are linear. This is shown in Fig. 117, the data being taken from Table VII, page 190. It is obvious, without further proof, that the internal work done in the turbine is equal to

$$H_{t1} + H_{t2} = \eta_s (H_{c1} + x_m \cdot H_{c2})$$

where x_m is the mean dryness fraction in the wet stages.

EXAMPLE 15. Using the data given in Table VII, page 190, and given that the stage heat drop is 38.8 B.Th.U. per lb., and that the dry stage efficiency is 75 per cent, calculate—

(a) Internal work done per lb. of steam, neglecting the effect of wetness.

(b) Internal work done per lb. of steam, taking wetness into account.

(c) Percentage loss due to wetness

(d) Final enthalpy and dryness fraction given that the initial enthalpy is 1288.8 B.Th U per lb. and the final pressure 1.0 lb per sq in

From the data given in the table, it is clear that the first five stages may be assumed to operate in dry steam, and the last five in wet steam

$$\therefore H_{c1} = H_{c2} = 5 \times 38.8 = 194$$

(a) Neglecting wetness, $H_i = 0.75 \times 388 = 291$ B.Th.U

(b) Mean wetness in L.P stages = 0.95

$$\therefore H_{i1} + H_{i2} = 0.75\{194 + (0.95 \times 194)\} \\ = 283.7$$

(c) Loss due to wetness = $\frac{7.3}{291} \times 100 = 2.5$ per cent

(d) Final enthalpy = $1288.8 - 283.7$
 $= 1005.1$ B.Th U per lb.

Final dryness fraction read from Mollier chart = 0.9063
 Neglecting wetness the final dryness fraction is 0.8989

It will be clear that the modification of the condition curve will give, for a constant dry stage efficiency, slightly greater specific volumes in the L.P stages and also a slightly higher reheat factor.

105. Erosion of Blades. Quite apart from causing a reduction in nozzle and blade efficiency, the presence of water in the L.P stages sometimes brings about serious erosion of the moving blades. Reference to Figs 113 and 114 shows that the water particles strike the leading surface of the blades with a velocity CD which, with the small ratio m_1 at present considered to obtain, is commensurate with the blade velocity. Such impact, if sufficiently heavy, produces severe local stresses in the blade material, causing the surface metal to fail and flake off.

The erosion, if any, is more likely to occur in the region where the steam is wettest, i.e. in the last one or two stages of the turbine. Moreover, as has been shown, the water droplets are concentrated in the outer parts of the flow annulus where the peripheral velocity and therefore the velocity of impact are highest. Experiments and operating experience have shown that the factors which most affect the erosion are the following—

(a) *The final wetness of the steam* The consensus of opinion at the present time (1939) is that a nominal final wetness fraction exceeding about 10 per cent, combined with other factors which encourage erosion, would tend to produce severe erosion of the

blades. The final wetness obviously depends upon the initial pressure and temperature of the steam, the exhaust pressure and stage efficiency. Fig. 118 shows the final dryness fraction for various initial steam pressures and temperatures, assuming an internal stage efficiency ratio of 0.85. From these curves it is apparent that if the final wetness is to be kept below 10 per cent, then the employ-

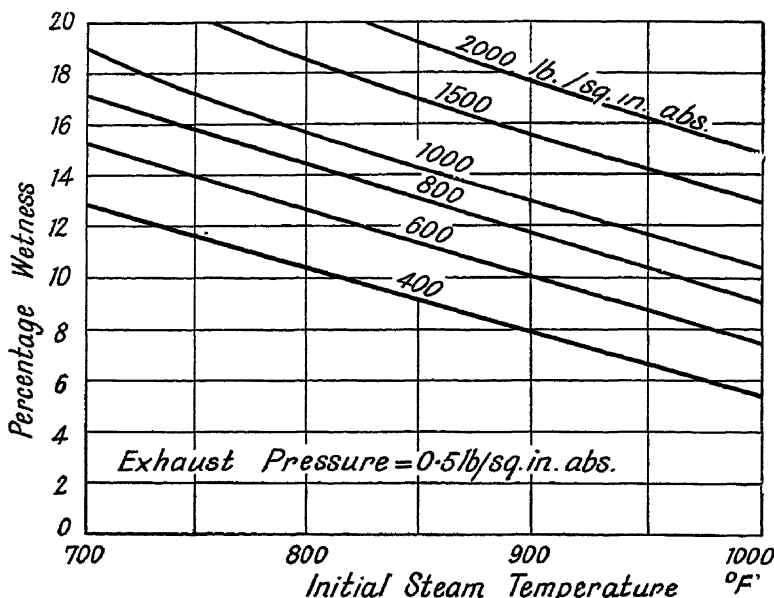


FIG. 118.—Exhaust Wetness as a Function of Initial Steam Pressure and Temperature (Internal Efficiency = 0.85)

ment of high steam pressures and a simple non-reheating cycle must imply the use of very high initial temperatures.

(b) *Peripheral velocity.* In an important series of articles by Mr. F. W. Gardner (2), it is shown that the intensity of pressure produced by the impact of water drops is directly proportional to the velocity of impact, and that it does not depend on the size of the drops. It is estimated that the pressure resulting from the direct impact at 1,000 ft. per sec. amounts to about 40 tons per sq. in. When it is realized that in some cases the blade tip velocity reaches 1,200 ft. per sec., the great stresses set up by the impact of the water drops become a very serious matter. For further information on the subject of blade erosion, tests, etc., the reader is referred to the author's book, *Steam Turbine Operation*.

106. Reheating. Erosion difficulties due to moisture in the steam may be avoided by reheating. The whole of the steam is taken from the turbine at a suitable point and a further supply of heat is

given to it, after which the steam is readmitted to the turbine and expanded to condenser pressure

Consider, first, the theoretical reheating cycle in which the steam is assumed to expand at constant entropy. Figs. 119 and 120 show,

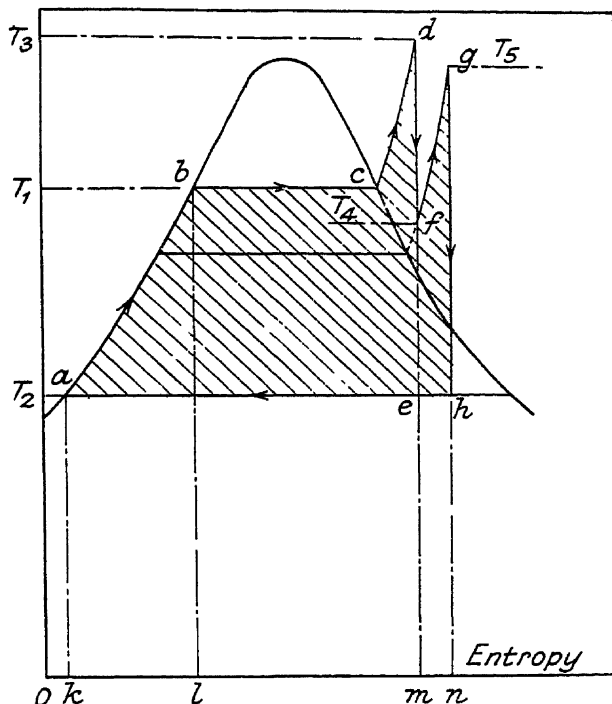


FIG 119 — T- ϕ Chart for Simple Reheating Cycle

respectively, the T- ϕ and ϕ -I diagrams and are lettered to correspond with each other. Steam enters the turbine at p_1 and T_3 and is expanded adiabatically to a certain intermediate pressure where the state point is f and the absolute temperature T_4 . Here the steam receives a further supply of heat, at constant pressure, which raises the temperature to T_5 . The steam is then expanded to p_2 and T_2 . This is an example of single-stage reheating

Referring to Fig. 120—

$$\text{Heat supplied} = (I_a - I_u) + (I_g - I_f)$$

$$\text{Heat rejected} = I_h - I_a$$

$$\begin{aligned} \therefore \text{Work done} &= \text{Heat supplied} - \text{Heat rejected} \\ &= (I_a - I_f) + (I_g - I_h) \end{aligned}$$

i.e. the sum of the adiabatic heat drops df and gh .

∴ Theoretical thermal efficiency of cycle

$$\eta_c = \frac{(I_d - I_f) + (I_g - I_h)}{(I_d - I_a) + (I_g - I_f)} \quad . \quad . \quad . \quad (145)$$

Provided the last stage of the expansion terminates at the lower limit of temperature T_2 , reheating generally leads to an increase in

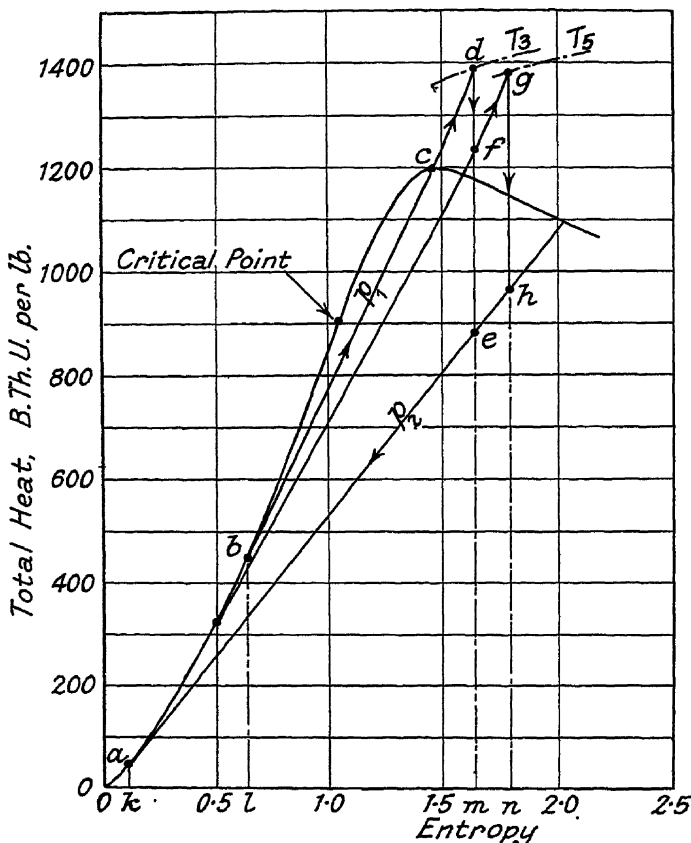


FIG. 120.— ϕ -I Chart for Simple Reheating Cycle

the theoretical cycle efficiency because the additional heat is supplied at a higher mean temperature. If the process is extended in its application, we may imagine a cycle such as that illustrated diagrammatically in Fig. 121 in which the steam is initially superheated to T_3 and repeatedly reheated at various pressures in such a way that the additional heat supplied to the steam is absorbed at a high mean temperature T_4 , nearly equal to T_3 . Such a cycle is theoretically desirable but is difficult to realize in practice. Up

to the present, practical considerations have limited the number of reheating stages to one.

Consider now the practical case of single-stage reheating. Referring to the ϕ -I chart, Fig. 122, let p_0 be the stop-valve pressure, p_1 the nozzle-box pressure, and p_3 the exhaust pressure.

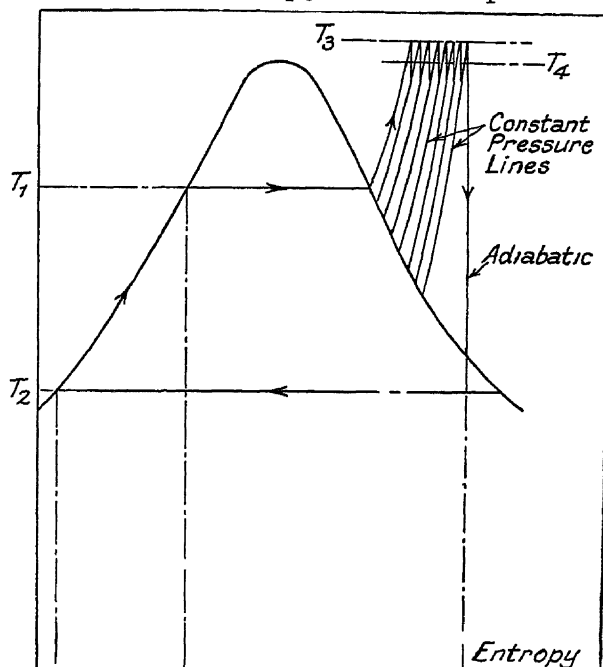


FIG. 121 — T- ϕ Chart for Multi-stage Reheating Cycle

Non-reheating Cycle This is shown by the adiabatic and actual expansion lines bc and bd . Ignoring the effect of wetness in the L.P. stages, the internal efficiency of the turbine cycle is

$$\eta_{ci} = \frac{I_b - I_d}{I_a - I_{w3}}$$

where I_{w3} is the enthalpy of water at p_3 .

Let x_3 = actual dryness fraction at p_3 .

H_{c1} = cumulative heat down to saturation line.

H_{c2} = cumulative heat from saturation line down to exhaust pressure

Then, allowing for the effects of wetness, internal work done

$$H_i = \eta_s \left\{ H_{c1} + \left(\frac{1 + x_3}{2} \right) H_{c2} \right\}$$

and

$$\eta_{ci} = \frac{H_i}{I_a - I_{w3}}$$

Reheating Cycle. Suppose that the steam is extracted at some pressure p_2 and reheated to a temperature t_2 . Assume that due to resistance in the steam pipes and reheater tubes the steam re-enters the turbine at a pressure p'_2 . The drop in pressure may be assumed to take place at constant enthalpy and is shown, purely for convenience in illustration, by the constant enthalpy line ef . fg shows

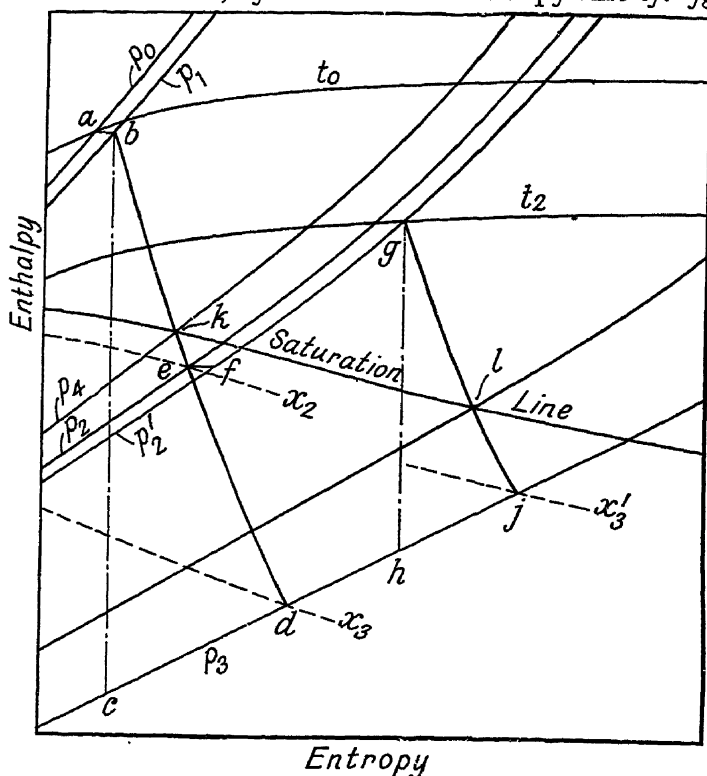


FIG. 122.— ϕ - I Chart for Single-stage Reheating Cycle

the reheating and gj the subsequent expansion to the exhaust pressure p_3 , the new final dryness fraction being x'_3 .

The heat supplied in the reheater may be deduced from the steady flow equation. Neglecting the relatively small kinetic energies of the steam at inlet and outlet, we have—

$$\text{Heat supplied per lb. of steam} = I_g - I_e.$$

$$\therefore \text{Heat supplied in cycle} = (I_a - I_{w3}) + (I_g - I_e).$$

Ignoring first the effect of wetness, the internal work done in the cycle is

$$H_i = (I_b - I_a) + (I_g - I_j)$$

$$\eta_{ci} = \frac{H_i}{(I_a - I_{w3}) + (I_g - I_e)}$$

In order to allow for the effects of wetness, suppose that—
Before reheating—

H_{c1} = cumulative heat drop down to saturation line

H_{c2} = cumulative heat drop below saturation line.

x_2 = final dryness fraction.

After reheating—

H_{c3} = cumulative heat drop down to saturation line

H_{c4} = cumulative heat drop below saturation line.

Suppose that the dry stage efficiency η_s is uniform for all stages
Then the total internal work

$$H_i = \eta_s \left\{ H_{c1} + \left(\frac{1 + x_2}{2} \right) H_{c2} + H_{c3} + \left(\frac{1 + x'_3}{2} \right) H_{c4} \right\}$$

The heat supplied is the same as before

EXAMPLE 16 Steam is supplied to a H P steam turbine at a nozzle-box pressure of 1,200 lb per sq in abs, and a temperature of 800° F. The exhaust pressure is 0.5 lb per sq in abs. What is the internal cycle efficiency (a) neglecting the effects of wetness, (b) allowing a standard wetness correction of 1 per cent per 1 per cent wetness? The dry stage efficiency is 80 per cent.

In an alternative cycle, steam is exhausted from the H P turbine at 200 lb per sq in. abs, reheated, and returned to the I.P. turbine at 180 lb per sq in abs and 800° F. It is then expanded to the same pressure as before and with the same dry stage efficiency. What is the internal thermal efficiency of the reheating cycle (a) neglecting the effects of wetness, (b) allowing the same wetness correction as before?

(1) *Non-reheat Cycle* The condition curve *bd* is first drawn by a step-by-step process, Fig 123, yielding the *I* values shown.

(a) *Neglecting effects of wetness—*

$$H_i = 1382.7 - 916.9 = 465.8$$

$$\text{Heat supplied} = 1382.7 - 47.4 = 1335.3$$

$$\therefore \eta_{ci} = \frac{465.8}{1335.3} = 34.88 \text{ per cent}$$

(b) *Allowing for wetness—*

$$H_{i1} = 1382.7 - 1193.0 = 189.7$$

$$\text{Mean } x \text{ in lower stages} = \frac{1 + 0.8325}{2} = 0.9162$$

$$\therefore H_{i2} = 0.9162 (1193.0 - 916.9) = 253.0$$

$$\therefore \text{Internal work done} = 442.7$$

$$\eta_{ci} = \frac{442.7}{1335.3} = 33.15 \text{ per cent}$$

(2) *Reheating Cycle.* The expansion line gj is first drawn by a step-by-step process and the relevant I values obtained.

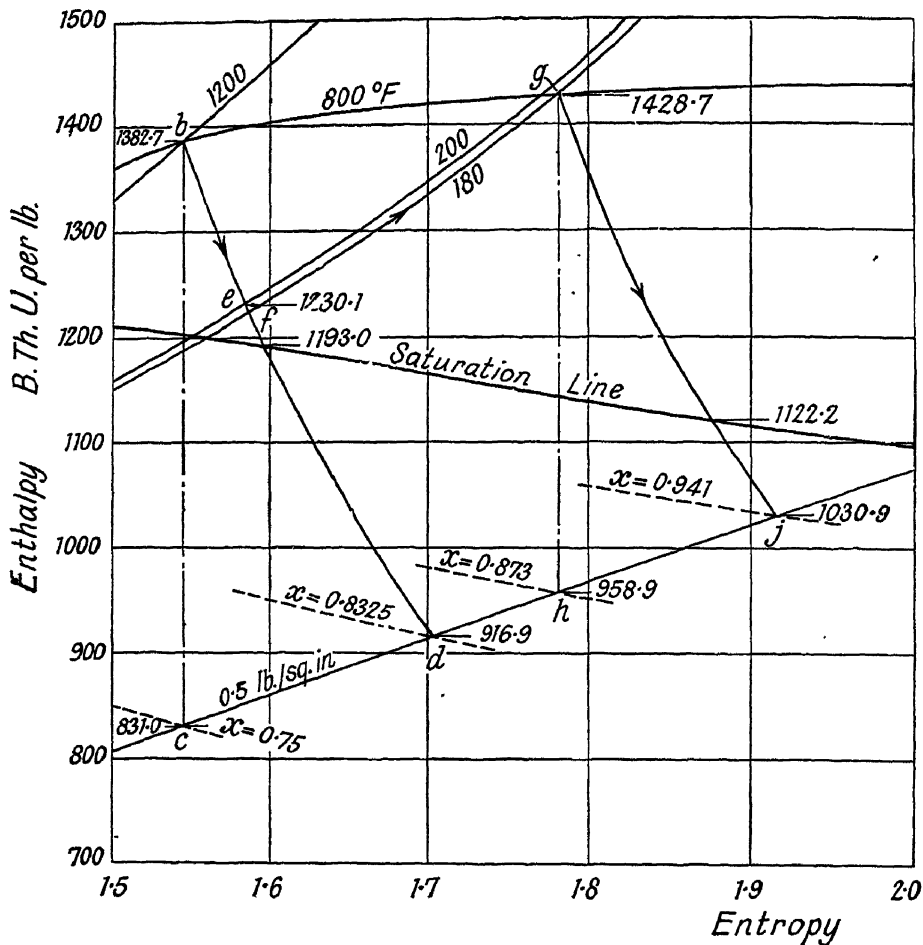


FIG. 123.— ϕ - I Chart Illustrating Example 16

(a) *Neglecting effects of wetness—*

For H.P. cylinder, $H_i = 1382.7 - 1230.1 = 152.6$.

For L.P. cylinder, $H_i = 1428.7 - 1030.9 = 397.8$.

Total internal work = 550.4.

Heat supplied = $(1382.7 - 47.4) + (1428.7 - 1230.1)$
 $= 1533.9$

$\therefore \eta_{ci} = \frac{550.4}{1533.9} = 35.88 \text{ per cent}$

(b) *Allowing for wetness—*

Mean x in L.P. stages = 0.9705

$\therefore$ Internal work done in wet stages

$$= 0.9705 (1122.2 - 1030.9) = 88.6$$

$\therefore$ Total internal work done

$$= (1382.7 - 1230.1) + (1428.7 - 1122.2) + 88.6$$

$$= 547.7$$

$$\therefore \eta_{es} = \frac{547.7}{1533.9} = 35.72 \text{ per cent}$$

The percentage gain due to reheating is only 2.87 per cent if the effects of wetness are ignored, but 7.75 per cent if the effect of

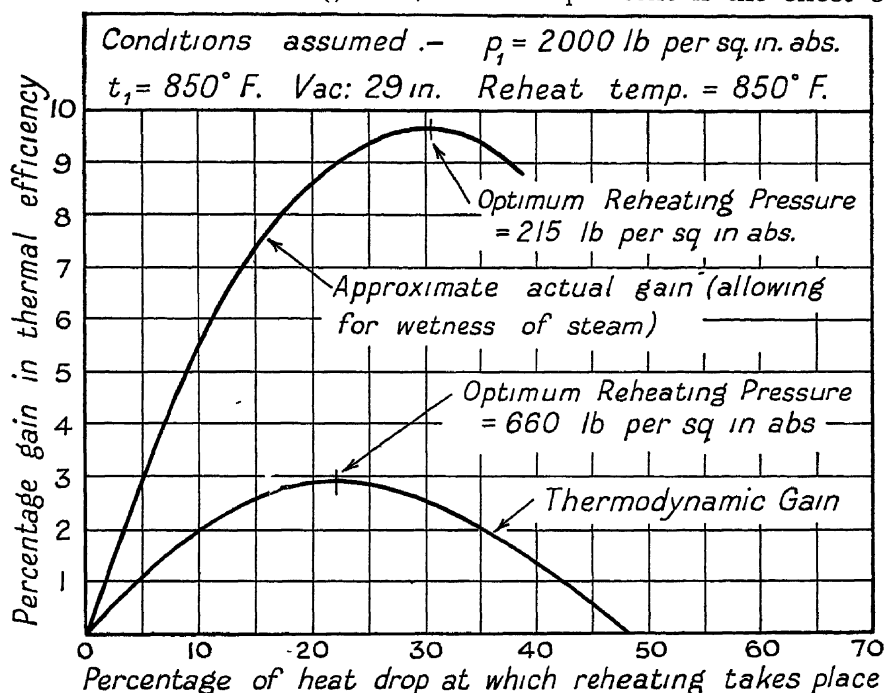


FIG 124 —Curves Showing the Gain in Thermal Efficiency due to Single-stage Reheating at Various Pressures

wetness is taken into account. A calculation for the simple reheating cycle with isentropic expansion in both turbines and ignoring the wetness correction shows a thermal gain of a mere 1.23 per cent.

There is, for any assumed temperature conditions, as, for example, reheating of the steam to its initial temperature, a particular pressure at which it is most economical to reheat the steam. A little thought will show that if the steam is reheated early in its expansion, then the additional quantity of heat supplied will be small,

with a consequently small gain, whereas, if the reheating is done at fairly low pressures, then, although a large amount of additional heat is supplied, the steam will still have a high degree of superheat at the exhaust, with the result that a large proportion of the heat supplied in the reheating process will be thrown to waste in the condenser. This point is well shown by Fig. 124, based on calculations by the author. The following assumptions were made: (a) no feed-heating; (b) turbine stage efficiency 82 per cent for dry stages; (c) no allowance for pressure drop in reheater.

It should be pointed out that when regenerative feed-heating is adopted, part of the thermal gain due to reheating disappears owing to the reduction in mass flow through the L.P. stages.

107. Reheating in Practice. The following are possible ways of reheating turbine steam—

(1) By means of a special reheater boiler, the greater part of whose surface is super-heater surface, or by the use of reheaters incorporated in each of the boilers serving the turbine. Such flue-gas reheaters enable high reheat temperatures to be employed and the maximum economic gain to be realized. There are, however, certain disadvantages. The pipes conveying the steam to and from the boilers are large and costly and add to the radiation and pipe friction losses.

(2) By live steam reheating. The reheaters are placed as close to the turbine as possible and are supplied either with high-pressure saturated or superheated steam. The condensed steam may either be extracted from the heater by a pump and discharged into the feed line or it may be flashed into steam in a special feed heater. In either case the sensible heat of the condensate is conserved. The maximum reheat temperature is very limited and the thermal gain is appreciably lower than that which is possible with flue-gas reheating.

(3) By using very highly superheated steam which is on its way to the H.P. turbine. In this way the heat for reheating is abstracted from the whole of the live steam with the result that the steam enters the H.P. turbine at a more moderate temperature. The method has the practical advantage that it protects the H.P. turbine from the effects of high temperature, and the L.P. turbine from the effects of excessive wetness.

(4) By a combination of live steam and flue-gas reheating. The steam first passes through the live steam reheater and then through the flue-gas reheater. This is said to result in a better control of the reheat temperature. At light loads most of the reheating is done by the steam, whereas at heavier loads most of the reheating is done by the flue gases.

(5) By using the vapour (produced in a special boiler) of a substance such as mercury, diphenyl, diphenyloxide, etc., having a high boiling-point. The vapour gives up heat by condensing. The heating substance is thus really a heat-conveying medium from the

products of combustion to the steam. This method has advantages, provided a cheap and stable substance is available, because a high and easily controlled temperature is attainable at quite moderate pressures.

Whatever system of reheating is adopted, the problem of governing is made more difficult. In the normal case there is always a certain amount of steam in the turbine over which the governor has no further control and, apart from the time taken for the governor to close the throttle in the event of a sudden drop in load, this steam goes on expanding and so accelerating the rotor. In the case of a reheater turbine, however, there is a very large volume of steam beyond the throttle capable of causing very serious increases of speed should the load suddenly drop, and in order to avoid dangerous overspeeding more or less complicated governor gear must be provided.

To sum up, reheating would seem to be a necessary evil accompanying the employment of very high steam pressures. It appears to be used mainly to avoid blade erosion in the L P stages rather than to realize the thermal advantages which result from its use.

108. Regenerative Water-extraction Cycle. Another method of combating the erosion problem in the L P stages of H P turbines is by the partial or complete extraction of suspended water particles from the steam during its flow through the wet stages.

As already stated in Art 105, owing to inertia, the water droplets move radially outwards during their passage through the moving blades with the result that the larger droplets are concentrated in the outer half of the blades. The centrifugal force on a water droplet moving through a blade which has a peripheral velocity of 1,000 ft per sec. at 1,500 r p m is almost 5,000 times its own weight, but as the time of passage through the blade is exceedingly short—perhaps 0.0005 to 0.002 sec—and as there is friction at the blade surface the actual outward movement is not so great as might be imagined. However, a certain quantity of moisture is flung off the blade, probably at the outlet side. Various water-catching devices have been installed in turbines for the purpose of trapping this water and leading it away to the condenser. Such devices are described in the author's book, *Steam Turbine Operation*. Mr. L S Robson (3) has stated that up to 40 per cent of the moisture in any given stage may be extracted by such traps. This implies a reduction in the total moisture at exhaust of something like 60 per cent.

In these drainage systems the water is, in some cases, drained direct to the condenser through holes drilled in the diaphragms and cylinder. The sensible heat in the extracted water is thus lost. In other cases, the water is cascaded from stage to stage; under these conditions there is a partial recovery of the sensible heat, for the water is partially evaporated due to the reduction of pressure.

Attention has recently been directed to the possibility of effecting complete extraction of the suspended water at certain definite points in the expansion and utilizing the heat in the extracted water for feed heating purposes. Such a system with two-point extraction is shown diagrammatically in Fig. 125. The extracted

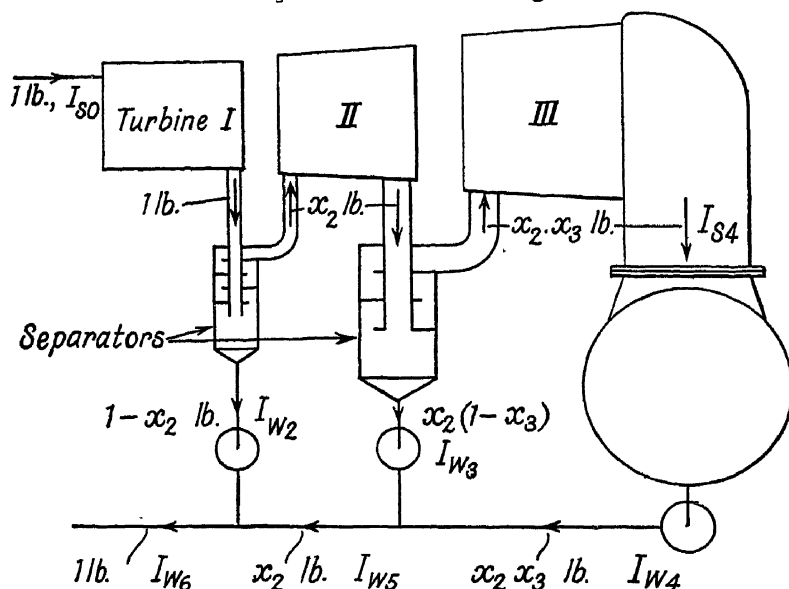


FIG. 125.—Diagrammatic Arrangement of Regenerative Water-extraction Cycle

water is driven by pumps into the feed pipe, arrangements being made so that water only is extracted.

The ϕ - I diagram for the cycle is shown in Fig. 126, in which bd represents the condition curve for the unmodified cycle. Complete extraction is assumed to take place at pressures p_2 and p_3 . Suppose that the dryness fraction at c is x_2 ; then the water extracted is $1 - x_2$ lb. per lb. of steam entering the turbine. Also, the weight of steam passing through turbine II is x_2 lb. This expands along ef and reaches a dryness x_3 . The water extracted at the second point is $(1 - x_3)x_2$ and the steam flow through turbine III is $x_2 x_3$. This is also the flow into the condenser.

Neglecting the extremely small increase in enthalpy due to the compression in the water extraction pumps—

$$\begin{aligned} x_2 I_{w5} &= x_2 x_3 \cdot I_{w4} + x_2(1 - x_3)I_{w3} \\ \therefore I_{w5} &= x_3 \cdot I_{w4} + (1 - x_3)I_{w3} \\ \text{Also } I_{w6} &= x_2 \cdot I_{w5} + (1 - x_2)I_{w2} \\ &= I_{w2} - x_2(I_{w2} - I_{w5}) \\ \text{Heat supplied} &= I_{s0} - I_{w6}. \end{aligned}$$

Internal work done

$$= (I_b - I_c) + x_2(I_e - I_f) + x_2x_3(I_g - I_h)$$

From these expressions the internal thermal efficiency may be calculated. Similar expressions may be readily deduced for any

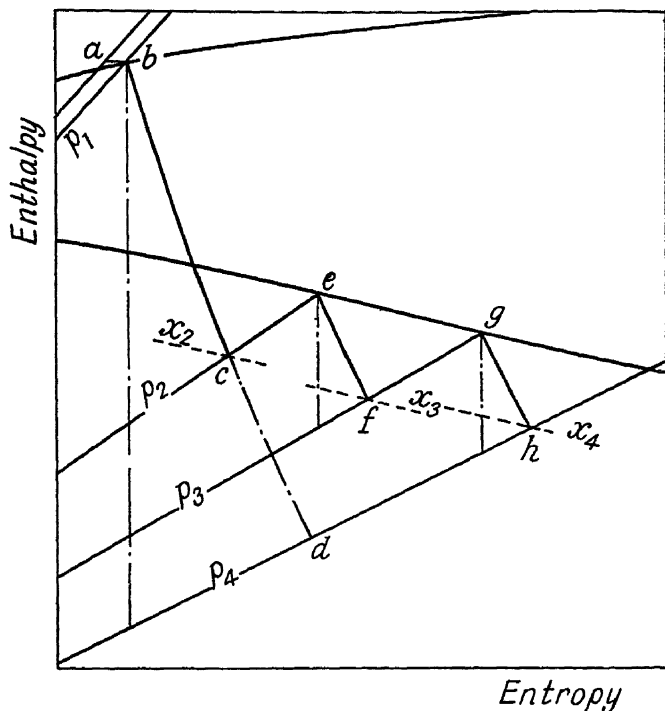


FIG 126 — ϕ -I Diagram for Water-extraction Cycle

number of extraction points. The theoretical possibilities may be gauged from the following example.

EXAMPLE 17. Steam expands in a H P turbine from $p_1 = 1,200$, $t_1 = 800^\circ \text{F}$ to $p = 0.5$, the dry stage efficiency being 80 per cent. Calculate the theoretical increase in cycle efficiency with one-point and two-point regenerative water extraction over the simple non-extraction cycle. The extraction pressures should be chosen so as to give approximately equal dryness fractions at exhaust from each section of the turbine, and a uniform wetness correction of 1 per cent per 1 per cent of wetness should be used.

The data for the non-extraction cycle are the same as for example 16, page 227. The internal thermal efficiency, allowing for wetness, is 33.15 per cent.

The extraction pressures to give approximately equal dryness fractions may be found quickly by trial and error and, together

with other relevant data, are given in Table VIII. The student should verify the results given by working them out on the Mollier chart.

TABLE VIII
CALCULATIONS FOR TYPICAL REGENERATIVE WATER-
EXTRACTION CYCLE

Note. The steam flows given for each turbine are per lb. of steam entering turbine I (see Fig. 125). In any turbine, x_2 is the dryness fraction at the end of expansion in that turbine, x_m is the mean dryness fraction in the wet stages, and I_1 and I_2 are the initial and final enthalpies respectively.

	One-point Extraction	Two-point Extraction
<i>Turbine I</i>		
Steam flow	1 lb.	1 lb.
Outlet pressure	10	26
x_2	0.9065	0.9371
x_m	0.953	0.968
I_1	1382.7	1382.7
I_2	1051.2	1102.6
Internal work done	324.85	277.2
<i>Turbine II</i>		
Steam flow	0.9065	0.9371
Outlet pressure	0.5	4
x_2	0.908	0.9353
x_m	0.954	0.967
I_1	1143.0	1162.5
I_2	995.6	1060.6
Internal work done	127.44	92.32
<i>Turbine III</i>		
Steam flow	—	0.8762
Outlet pressure	—	0.5
x_2	—	0.9355
x_m	—	0.968
I_1	—	1125.7
I_2	—	1024.6
Internal work done	—	85.76
<i>Enthalpies of Water</i>		
After extraction pump	47.4	47.4
After 1st drain pump	58.0	52.16
After 2nd drain pump	—	62.13
Heat supplied in cycle	1324.7	1320.6
Total internal work done	452.3	455.3
Internal thermal efficiency per cent	34.14	34.47
Increase above thermal efficiency of non-extraction cycle	0.99	1.32
Percentage increase	2.98	3.98

It would be a comparatively easy matter to dry the steam by means of a separator as it passes from one turbine cylinder to another.

Unfortunately, even when there are three cylinders in series, the wetness fraction at inlet to the L.P. cylinder is comparatively small and little benefit would be obtained by extracting the moisture there. For two-point extraction, at least one of the extraction points should be located somewhere in the L.P. cylinder. Whether it will be possible to effect approximately complete extraction at such a point remains to be seen, but there is no doubt that the cycle is a promising alternative to the reheat cycle.

Readers interested in the matter should refer to a recent paper by Dr. Ir. W. M. Meijer (4).

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3. ROBSON "The Development of Large Turbines for Land Power Installations" *Trans Liverpool Eng Soc*, 1936, 57, 73.
4. MEIJER "The Extraction of Condensate from Expanding Steam, its Effect upon the Efficiency of the Ideal Heat Cycle and its Influence upon the Selection of Initial Steam Conditions" *Trans Inst of Naval Architects*, 1939, 81, 36

EXAMPLES XII

1. The following particulars relate to a low-pressure stage of an impulse steam turbine—

Nozzle angle	24°
Dryness fraction at nozzle outlet	0.90
Steam velocity at nozzle outlet	1,400 ft per sec
Mean velocity of water particles at nozzle outlet	200 „ „
Mean blade velocity	650 „ „
Ratio of relative velocity at outlet to relative velocity at inlet with homogeneous flow	0.87
Steam flow (total) lb per sec	140

If the blades are equiangular, calculate the horse-power developed in the blades and the blade efficiency ignoring carry-over effects—

- (a) assuming homogeneous flow,
- (b) assuming non-homogeneous flow

Assume that there is no rebound at the inlet edge of the blades, that the momentum of the water particles referred to the relative velocity is reduced to zero at impact, and that the relative velocity of the water particles at outlet is 0.15 times that of the dry steam.

2. Steam is supplied to a turbine at a pressure of 500 lb. per sq. in. abs. and a temperature of 750° F. It expands at constant entropy to a pressure of 1.2 lb. per sq. in. abs. What is the dryness fraction at the end of expansion and the thermal efficiency of the cycle?

Calculate the modified exhaust condition and thermal efficiency if the steam is reheated at 80 lb. per sq. in. to a temperature of 730° F. and then expanded adiabatically to a pressure of 1.2 lb. per sq. in.

3. Steam is supplied to a turbine at a pressure of 1,000 lb. per sq. in. abs. and a temperature of 800° F., the exhaust pressure being 0.5 lb. per sq. in. abs. If the internal efficiency ratio of the turbine is 0.8, what is the internal thermal efficiency of the turbine and the final wetness fraction of the exhaust steam?

In a modified arrangement, the steam is first expanded from the initial

condition given above to a pressure of 120, the internal efficiency ratio being 0.8. The steam is then reheated in a surface heater by the live steam at 1,000 lb. pressure and 800° F., to a temperature of 520° F. It is then expanded in a low-pressure turbine to 0.5 lb. per sq. in. abs., the internal efficiency ratio being 0.8. Calculate the internal thermal efficiency of the modified turbine and the final wetness fraction of the exhaust steam.

4. Steam is generated at the critical pressure ($p = 3,200$; $t = 706^\circ \text{F.}$; $I_w = I_s = 902 \text{ B.Th.U. per lb.}$) and is superheated to a temperature of 806° F. The mean specific heat of the superheated steam at the critical pressure for this range is 3.48.

The steam is then throttled to a pressure of 1,000 lb. per sq. in. abs., and after throttling heat is supplied to the steam so that it becomes superheated 220° F. It is then expanded adiabatically until it is dry and saturated, when it is further heated so as to superheat it by 300° F. The steam is then further expanded adiabatically to a final pressure of 0.5 lb. per sq. in., at which pressure it is condensed.

Derive the theoretical thermal efficiency of the cycle.

5. Steam is supplied to a turbine at a pressure of 200 lb. per sq. in. abs. ($t = 382^\circ \text{F.}$, $L = 843.1$) and is expanded isothermally to a pressure of 0.5 lb. per sq. in. abs. ($t = 80^\circ \text{F.}$, $L = 1047.0$). Assuming that the specific heat of steam at 0.5 lb. pressure is 0.46, calculate the heat supplied, the heat rejected, the work done, and the thermal efficiency of the cycle.

6. Steam is supplied to a turbine at a pressure of 800 lb. per sq. in. abs. and at 840° F. It is expanded in a H.P. turbine to 80 lb. per sq. in., the internal efficiency ratio of the H.P. turbine being 0.8. The steam is reheated at constant pressure by saturated live steam and is returned to the L.P. turbine at 80 lb. pressure and 500° F. It is then expanded to 0.5 lb. per sq. in. abs. with the same internal efficiency ratio. If the mechanical efficiency of the turbine is 99 per cent and the alternator efficiency is 96 per cent, calculate the weight of steam generated by the boiler per kWh output and also the heat consumption of the turbine in B.Th.U. per kWh. Assume that the heating steam gives up heat by condensing only.

7. Calculate, using the Mollier chart, the gross and net theoretical thermal efficiencies of the following cycle—

Feed water at 80° F. ($v_w = 0.016 \text{ cub. ft. per lb.}$) is compressed to a pressure of 1,000 lb. per sq. in. abs., and is then heated to the saturation temperature. It is evaporated at a pressure of 1,000 lb. and superheated to a total temperature of 800° F. The steam is then expanded adiabatically to a pressure of 100 lb. per sq. in. abs., at which pressure it is reheated to 700° F. It is then expanded adiabatically to 0.5 lb. per sq. in. abs., at which pressure it is condensed.

8. Steam is supplied to a turbine at a pressure of 500 lb. per sq. in. abs. and superheated 300° F. It is expanded adiabatically to 1 lb. per sq. in. abs. What is the thermal efficiency of the cycle?

In a modification of the cycle, the steam is extracted from the turbine at the point where it becomes just dry and saturated and is reheated so that its superheat is again 300° F. After further adiabatic expansion the steam is extracted at the point where it becomes just dry and saturated, and is reheated to such an extent that it becomes dry and saturated after expansion to 1 lb. per sq. in. abs. It is then exhausted and condensed.

Derive from the ϕ -I chart the reheating pressures and the final superheat of the steam as it leaves the second reheater. Calculate the thermal efficiency of the modified cycle.

9. Steam at a pressure of 400 lb. per sq. in. abs. and at a temperature of 950° F. ($I_s = 1496.2$) is supplied to a steam turbine plant consisting of H.P. and L.P. turbines in tandem.

The live steam does not go direct to the H P. turbine, but first passes through a steam reheater in which it gives up heat to reheat the steam passing from the H P. to the L P. turbine. The live steam enters the H P. turbine at 400 lb. pressure and 700°F ($I_s = 1366.2$). The pressure of the steam exhausted from the H P. turbine is 100 lb. per sq. in. abs., and the L P. turbine exhaust pressure is 1 lb. per sq. in. abs.

What is the condition of the steam between the H P. and L P. turbines before and after reheating, and what is the thermal efficiency of the cycle? Assume expansion at constant entropy

The diagram illustrates a hydraulic press system. A vertical cylinder contains a piston (A) and a plunger (B). Oil is pumped into the cylinder from a pump (D) through a valve (C). The pressure in the cylinder is p_{sv} , and the pressure in the pump is p'_{sv} . The output pressure is p_0 .

on the particular method employed for controlling the supply of steam to the turbine so that the speed of rotation will remain sensibly constant irrespective of the load. The principal methods of governing may be enumerated as follows—

- (1) Throttle governing.
- (2) Nozzle control governing.
- (3) By-pass governing.
- (4) Combinations of (1) and (2), and (1) and (3).

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pressure and dynamic action are balanced. Except in small turbines, the opening of the valve C is effected by an oil servo motor D which, in turn, is controlled by the centrifugal speed governor

Suppose that $p_{s.v.}$ is the steam pressure before the stop valve. This is the pressure normally used as the datum in turbine testing. There will be a slightly lower pressure $p'_{s.v.}$ after the stop valve. In

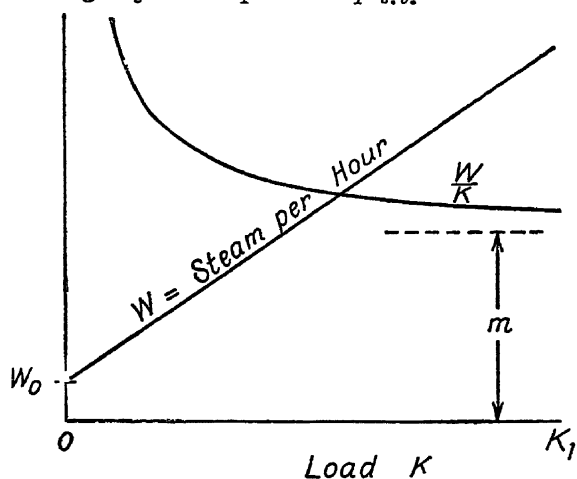


FIG 128 —Willans Line for Steam Turbine

order to effect control, the pressure p_0 in the nozzle box must be lower than $p'_{s.v.}$. The drop from $p_{s.v.}$ to p_0 appears to vary from 2 to 5 per cent of $p_{s.v.}$ at full load

The relationship between load and steam consumption for a turbine governed by throttling is given by the well-known Willans line, Fig. 128. Numerous tests show that when a turbine is governed by throttling, the Willans line is straight

If W_1 = steam per hour at full load.

W_0 = steam per hour at no load

K_1 = full load (say in kW).

K = any other load.

W = steam consumption at load K .

Then $W = m \cdot K + W_0$ (146)

where m is a constant.

The full-load steam consumption can be estimated fairly accurately from the heat drop and expected efficiency ratio. If the quantity W_0 could be expressed as a definite fraction of the full-load consumption, then the quantity W could be calculated for any intermediate power. For condensing turbines, the no-load steam consumption varies from about 0.10 to 0.14 times the full-load consumption.

The steam per kWh is equal to

$$\frac{W}{K} = m + \frac{W_0}{K}$$

and is shown by the curve in Fig. 128. It will be seen that the steam per kWh curve is asymptotic to the value m .

Another experimental result of considerable importance is shown in generalized form in Fig. 129. If measured steam flows W lb.

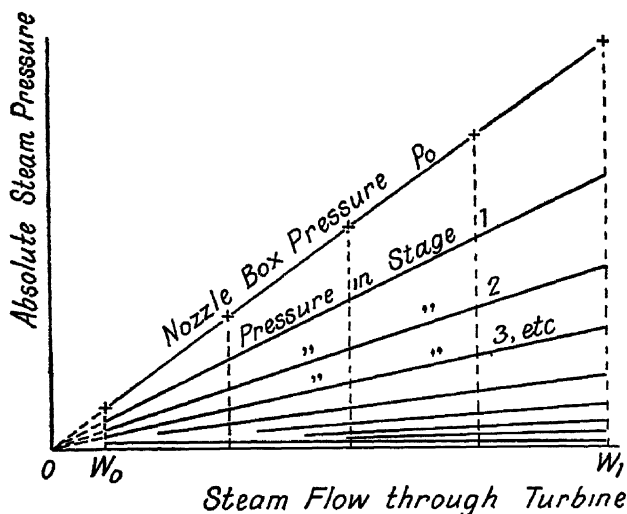


FIG. 129

per hour for the range from W_0 to W_1 be plotted as abscissae and values of the nozzle-box pressure be plotted as ordinates, then the points will lie on a straight line. In many cases, the line when produced passes through the origin, showing that the steam flow is directly proportional to the nozzle-box pressure, and vice versa. In some cases, when the nozzle-box pressure is plotted against the condensate, the line when produced passes a little to the right of the origin; this is probably due to the inflow of steam supplied for sealing the low-pressure shaft gland. It is also an experimental fact that when stage pressures are plotted against steam flow, the plotted points lie on straight lines which, excepting those for the last few low-pressure stages, also pass through the origin when produced.

The two graphs may be combined, as shown in Fig. 130, to give the nozzle-box pressure at any load K . The diagram is self-explanatory and illustrates yet another experimental result, namely, that with simple throttle governing the relation between nozzle-box pressure and load is given by a straight line.

The effect of throttle governing may now be illustrated with reference to the Mollier chart, Fig. 131. It will be assumed (1) that

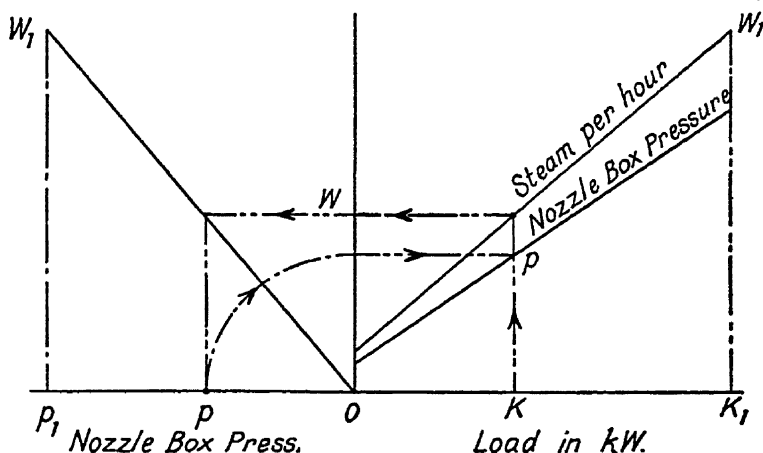


FIG. 130

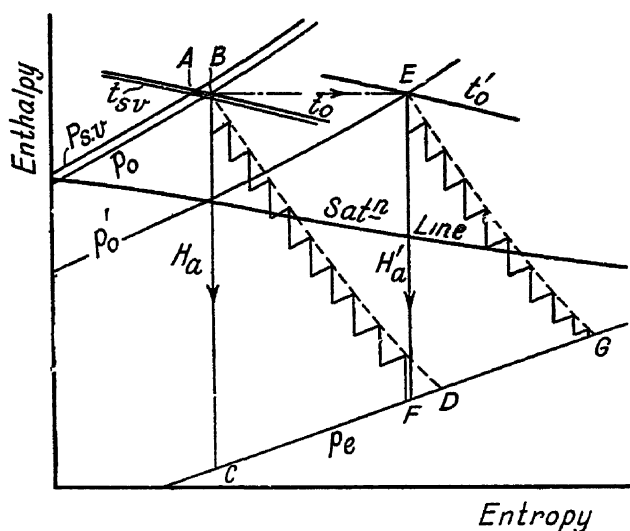


FIG. 131

the steam passes through the throttle valve with a negligible loss of heat and (2) that the vacuum in the condenser remains constant. Actually, unless the quantity of cooling water is reduced as the steam quantity entering the condenser is diminished, the steam

temperature in the condenser will fall, with a corresponding fall in the exhaust pressure.

Suppose that $p_{s.v.}$ is the steam pressure before the turbine stop valve and that p_0 is the pressure in the nozzle chest of the first stage at full load. A represents the condition before the stop valve as specified by the pressure $p_{s.v.}$ and the superheat $t_{s.v.}$. B is the condition at full load after throttling, the superheat being t_0 . BC represents the adiabatic expansion to the exhaust pressure p_e , and BD the locus of actual state points.

At some fractional load where the steam flow is W' , the first-stage nozzle-box pressure will be p'_0 , such that

$$p'_0 = \frac{W'}{W} \cdot p_0$$

The throttling process is conveniently represented by the line AE. As before, EF and EG show the adiabatic and actual expansion curves. It will be seen that the main effect of throttling, apart from reducing the steam flow, is to reduce the adiabatic heat drop. It will be shown later that the internal efficiency ratio η_i is not likely to be affected to any appreciable extent by the changes brought about by throttle governing. Consequently, if K_i = internal power at full load and K'_i = internal power at fractional load,

$$K_i = \frac{\eta_i \cdot W \cdot H_a}{3413}$$

and

$$K'_i = \frac{\eta_i \cdot W' \cdot H'_a}{3413}$$

A secondary effect of throttling is to increase the initial superheat and to delay the dew point. As a result, the amount of wetness in the low-pressure stages will be reduced, with beneficial results.

III. Pressure Distribution at Varying Loads with Throttle Governing. The distribution of pressure may be found approximately by the following method. Suppose that for any given fractional load the internal steam flow derived from the Willans line is w lb. per second. The corresponding initial pressure is calculated and the condition curve is plotted, as shown in Fig. 131, using the method outlined in Chapter X. The corresponding Σh and $\log_{10} v_n$ curves are drawn to a suitably large scale (Fig. 132).

Two conditions must be satisfied in each stage of the turbine. In the first place, since the nozzles are assumed to be running full, the equation of continuity must be satisfied.

i.e.

$$w \cdot v_n = A \cdot V_1$$

where V_1 is the actual efflux velocity and A is the nozzle area. If the nozzle is convergent and the pressure drop in the nozzle is greater than the critical, then, in applying the above equation

v_n must be the specific volume at the throat, A the throat area, and V_1 the actual velocity at the throat.

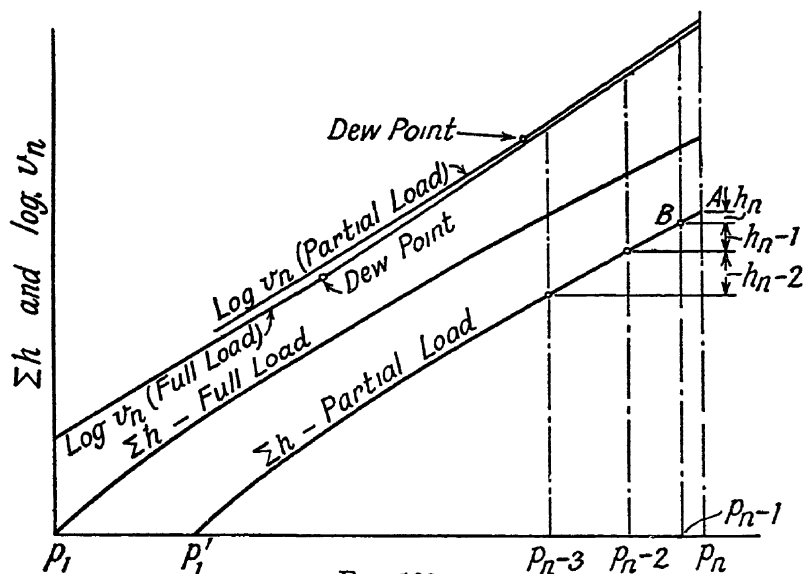


FIG. 132

In the second place, the adiabatic heat drop in a given stage equals

$$h = \left(\frac{V_t}{223.8} \right)^2 \text{ B.Th.U./lb.}$$

and since $V_t = \frac{V_1}{\eta_v}$ and $V_1 = \frac{w \cdot v_n}{A}$

$$h = \left(\frac{w \cdot v_n}{223.8 \eta_v A} \right)^2 \text{ B Th U./lb.} \quad . \quad . \quad (147)$$

Starting from the exhaust end and again assuming a constant exhaust pressure, h is calculated for the last or n th stage of the turbine, the value of v_n being taken from Fig 132 as the exhaust specific volume and the area A being that of the nozzles in the last stage. Owing to the comparatively small value of w , the velocity V_1 is comparatively small and the heat drop h_n is smaller still. This heat drop is now set down from A and gives the point B on the Σh line. Since the Σh curve gives the sum of the adiabatic heat drops as a function of the stage pressures, it follows that if the pressure at inlet to the nozzles of the last stage is p_{n-1} , then the available adiabatic heat drop will be sufficient to generate the velocity V_1 .

By projecting upwards on to the $\log_{10} v_n$ curve, the specific volume v_n at outlet from the nozzles in the preceding stage may be obtained. Using this value in eq. (147) and the appropriate nozzle area, the necessary adiabatic heat drop h_{n-1} in the penultimate stage is obtained. Setting this down as shown in Fig. 132, the pressure p_{n-2} is obtained. The calculation is thus carried out stage by stage, the information yielded being the stage pressures, specific volumes, heat drops, jet velocities, and blade speed ratios.

The above method, given in a paper by the author (1), cannot perhaps claim to be rigorously accurate, but it certainly gives results in good accord with experiment. The application of the method will now be illustrated by means of a worked example.

EXAMPLE 18. The following particulars relate to an impulse turbine of 3,000 kW, governed by simple throttling—

Initial pressure at full load	. . .	200 lb. per sq. in. abs.
Initial superheat at full load	. . .	150° F.
Exhaust pressure, assumed constant.	. . .	1 lb. per sq. in. abs.
Stage efficiency		75 per cent
Number of stages		10
η_v for nozzles (assumed constant)		0.94
Full-load steam consumption		10.12 lb. per sec.
No-load steam consumption		1.012 lb. per sec.
Mean blade velocity in each stage		524 ft. per sec.

Nozzle angles and areas—

Stage No.	Nozzle Angle	Nozzle Area Sq. Ft.
1	16°	0.0295
2	16°	0.0416
3	16°	0.0605
4	16°	0.0902
5	16°	0.1436
6	16°	0.2358
7	16°	0.395
8	16°	0.690
9	18°	1.228
10	24°	2.298

It is required to determine the steam consumption, nozzle-box pressure, stage pressures, jet velocities, and blade speed ratios at 0, $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ load.

The Willans line is first drawn and then, from the proportionality of steam consumption and nozzle-box pressure, the latter is calculated. These are given in Table IX.

The expansion lines and condition curves are next drawn on the Mollier chart, as shown in Fig. 133, thus yielding data on which the curves of Fig. 134 are drawn. These curves are then used for finding the pressures and steam velocities in the various stages.

TABLE IX
PERFORMANCE OF 3,000 kW TURBINE AT VARYING LOADS

	Load on Turbine				
	Full	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
Steam per hour . . .	36,480	28,272	20,064	11,856	3,648
Nozzle } Pressure . . .	200	155	110	65	20
Box } Superheat °F . . .	150	162 9	180	207 3	267
Heat } Adiabatic . . .	369 8	354 6	333 9	302 5	229 8
Drop } Cumulative . . .	388	370 1	349 9	316 8	242 4

Calculated Steam Velocities at Outlet from Nozzles

	Load on Turbine				
Stage No	Full	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0
1	1,310	1,297	1,300	1,297	1,301
2	„	1,305	1,318	1,304	1,313
3	„	1,299	1,308	1,324	1,318
4	„	1,320	1,326	1,340	1,350
5	„	1,303	1,302	1,293	1,284
6	„	1,294	1,288	1,267	1,088
7	„	1,308	1,301	1,290	773
8	„	1,298	1,292	1,134	475
9	„	1,321	1,148	796	272
10	„	1,032	745	452	146

The jet velocity at outlet from the nozzles is given in Table IX. The calculations were made with a slide rule, and the slight irregularities of the figures are due to this and to small errors in reading from the graph.

The most important facts emerging from these calculations may be enumerated as follows—

(1) The greatest variations in steam velocity occur in the stages nearest the exhaust.

(2) Even at no-load, when the steam consumption is only one-tenth of the full-load consumption, the steam velocities in the first five stages are practically unaffected

Consider now the flow through the blades of the last three stages. The inlet velocity triangles and the blade sections for stages 8, 9, and 10 are shown in Fig. 135. It will be seen that in stage 10 there must be serious impact losses at quarter-load and at no-load, and a rather less serious loss at half-load, in stage 9 the losses are not appreciable except at quarter-load and no load, whereas in stage 8 the impact losses are inappreciable except at no load.

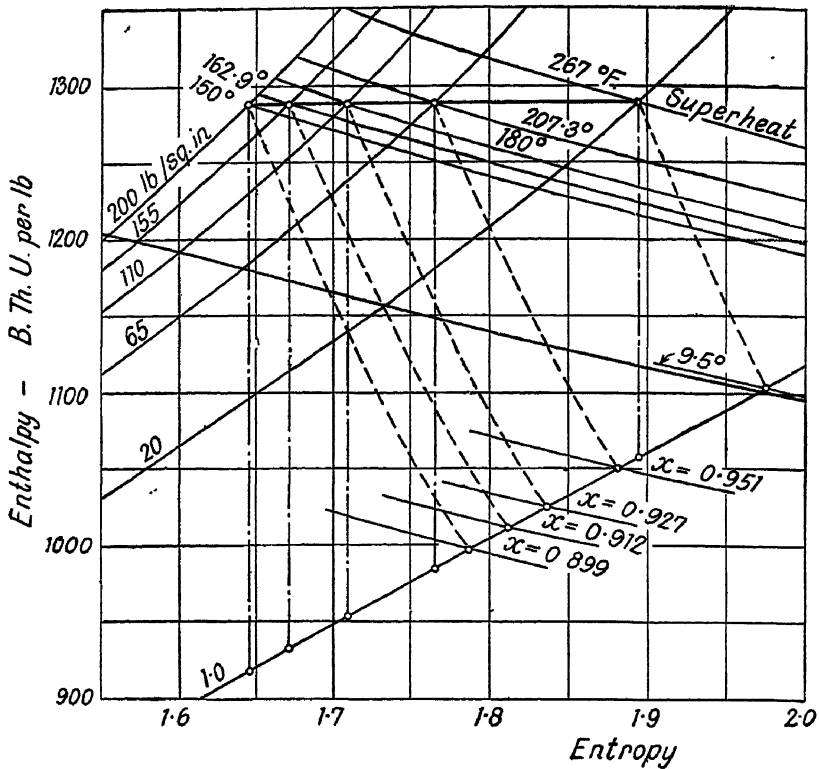


FIG. 133

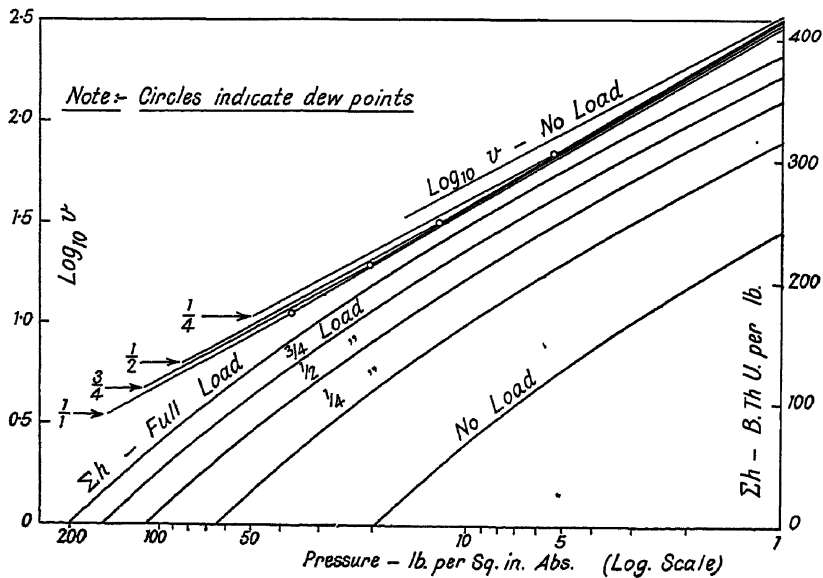


FIG. 134

In the majority of the stages, therefore, the impact losses at all loads above about quarter-load are very small. When the loss is appreciable, as in the final stage, the adiabatic heat drop in that stage is then very small and the effect on the internal efficiency of the

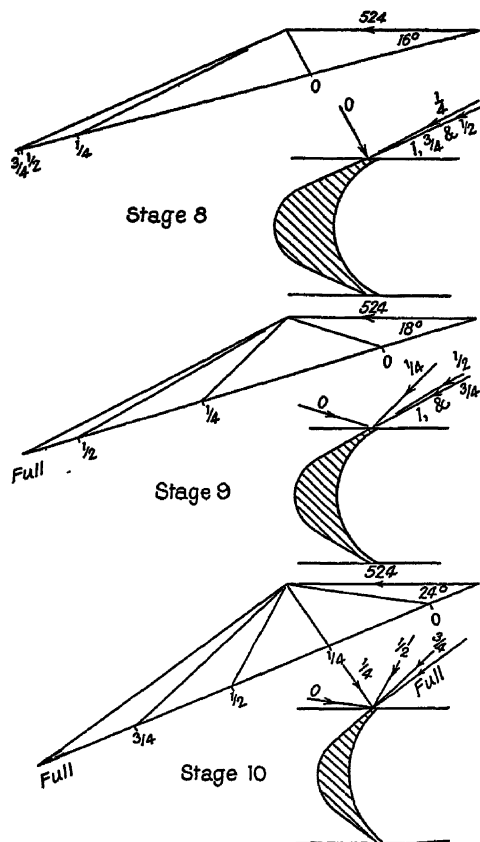


FIG. 135 —Velocity Diagrams and Directions of Relative Velocities for a Steam Turbine Governed by Throttling

stages, as a whole, is inappreciable. A factor which tends to increase the stage efficiency at partial loads is the reduction in wetness in the later stages due to throttling.

If the quantity of cooling water is kept constant and, in consequence, the exhaust pressure falls as the load is reduced, the volumes of steam passing through the L.P. nozzles are not reduced to the same extent as when the exhaust pressure is kept constant. Consequently, the changes in jet velocity, etc., are not so great as those based on the assumption of a constant exhaust pressure.

112. Throttle Governing in Reaction Turbines. The methods of analysis set out in the foregoing articles are not suitable for the impulse-reaction (Parsons) turbine, owing to the large number of stages in this type. The following alternative method is especially suitable for a series of relatively small expansions such as occur in the Parsons turbine.

The two equations which must be satisfied are again the equation of continuity and the energy equation. Considering any point in the turbine, let V be the velocity at outlet from either a fixed or a moving row of blades, let p be the pressure in lb. per sq. in. abs., let P be the pressure in lb. per sq. ft. abs., let v be the specific volume of the steam in cub. ft. per lb., let A be the effective area at the outlet side of the blades, in sq. ft., and w the flow of steam *through* the blades in lb. per sec.

$$\text{Then} \quad w = \frac{AV}{v} = \frac{V}{\frac{1}{Av}}$$

Using suffixes to represent the number of the blade rows, and assuming there are n rows of blades ($n/2$ fixed and $n/2$ moving) in the turbine—

$$w^2 = \frac{\frac{V_1^2}{v_1^2}}{\frac{1}{A_1^2}} = \frac{\frac{V_2^2}{v_2^2}}{\frac{1}{A_2^2}} = \dots = \frac{\frac{V_n^2}{v_n^2}}{\frac{1}{A_n^2}} = \frac{\sum_1^n \left(\frac{V^2}{v^2} \right)}{\sum_1^n \left(\frac{1}{A^2} \right)}$$

Suppose that the pressure drop in any given row is δP , then by the equations given in Art. 76, page 161—

$$\frac{V^2 - \phi \cdot U^2}{2g} = -\eta_n \cdot v \cdot \delta P$$

If it be assumed that the variation in blade speed ratio with variation in load is comparatively small—and judging by the performance of impulse turbines we are justified in making this assumption for all stages except the last few—then $\frac{U}{V}$ will be a constant, and assuming that ϕ is constant, we may write

$$\frac{V^2 - r \cdot V^2}{2g} = -\eta_n \cdot v \cdot \delta P$$

where $r = \frac{\phi U^2}{V^2}$, and is a constant.

$$\therefore \quad \frac{V^2}{v^2} = -\frac{2g \cdot \eta_n}{1-r} \cdot \frac{\delta P}{v}$$

The differential of $\frac{P}{v}$ is $d\left(\frac{P}{v}\right) = \frac{\delta P}{v} - \frac{P \cdot \delta v}{v^2}$

$$\therefore \quad \frac{\delta P}{v} = \delta\left(\frac{P}{v}\right) + \frac{P \cdot \delta v}{v^2}$$

We shall assume, for the sake of simplicity, that for the whole expansion $Pv^m = C$, where C is a constant. The index m is given by eq. (123), page 196.

$$\text{Then} \quad \frac{\delta P}{P} + m \cdot \frac{\delta v}{v} = 0$$

$$\therefore \quad \frac{P \cdot \delta v}{v^2} = -\frac{1}{m} \cdot \frac{\delta P}{v}$$

$$\therefore \quad \frac{\delta P}{v} = \delta\left(\frac{P}{v}\right) - \frac{1}{m} \cdot \frac{\delta P}{v}$$

$$\text{or} \quad \frac{\delta P}{v} = \frac{m}{m+1} \delta\left(\frac{P}{v}\right)$$

$$\therefore \quad \frac{V^2}{v^2} = -\frac{2g \cdot \eta_n}{1-r} \cdot \frac{m}{m+1} \cdot \delta\left(\frac{P}{v}\right)$$

$$\therefore \quad w^2 = \frac{2g \cdot \eta_n}{1-r} \left(\frac{m}{m+1}\right) \frac{\frac{P_1}{v_1} - \frac{P_n}{v_n}}{\sum_1^n \left(\frac{1}{A}\right)^2} \quad (148)$$

This equation is applicable to the whole, or to any group or groups of stages in the turbine. Thus, provided η_n , m , and r are constant—

$$w^2 = \text{Constant} \times \left(\frac{P_1}{v_1} - \frac{P_n}{v_n}\right)$$

$$\text{or} \quad w = C_1 \sqrt{\left(\frac{P_1}{v_1} - \frac{P_n}{v_n}\right)} \quad (149)$$

$$\text{where} \quad C_1 = \sqrt{\frac{2g \cdot \eta_n}{1-r} \left(\frac{m}{m+1}\right) \div \sum_1^n \left(\frac{1}{A}\right)^2} \quad (150)$$

In a condensing turbine, the ratio $\frac{P_n}{v_n}$ is very small compared with $\frac{P_1}{v_1}$ and may be ignored. Hence we may write

$$w = C_1 \sqrt{\frac{P_1}{v_1}}$$

If the turbine is governed by throttling, then the product $P_1 v_1$ is constant, because I is constant. This follows from Callendar's equation for the specific volume of superheated steam, namely—

$$v_1 = \frac{1.2464(I_1 - 835.2)}{p_1} - 0.00212$$

Neglecting the very small term, we have

$$w = C_1 \sqrt{\frac{P_1 p_1}{1.2464(I_1 - 835.2)}}$$

and since $P_1 = 144 p_1$

$$w = \frac{12 \cdot C_1 \cdot p_1}{\sqrt{1.2464(I_1 - 835.2)}} \quad (151)$$

This expression shows that the rate of steam flow w is directly proportional to p_1 , and vice versa. In the same way it may be shown that the pressure in any other row, sufficiently far removed from the exhaust end, is directly proportional to the steam flow. Consequently, in any such stage, the ratio $\frac{\delta P}{P}$ is constant.

$$\begin{aligned} \text{But} \quad V^2 &= -\frac{2g \cdot \eta_n}{1 - r} \cdot v \cdot \delta P \\ &= -\frac{2g \cdot \eta_n}{1 - r} \cdot K \cdot Pv \end{aligned}$$

where K is a constant.

Since $P_1 v_1$ is constant and $Pv^m = \text{Constant}$

$$Pv^m = P_1 v_1^m$$

$$\therefore Pv = P_1 v_1 \left(\frac{P}{P_1} \right)^{\frac{m-1}{m}} = \text{Constant}$$

Therefore, in all stages except the last one or two, the steam velocity V is a constant, and the heat drop, blade speed ratio, etc., must also be constant. It will be seen that conditions in a Parsons turbine are affected by throttle governing in very much the same way as in an impulse turbine.

113. Nozzle Control Governing. The principle of nozzle control governing has already been outlined in Art. 7, and illustrated diagrammatically in Fig. 5. In applying the principle to land turbines,

which are almost entirely governed by automatic devices, various arrangements of valves and groups of nozzles are employed. Three arrangements are shown diagrammatically in Fig 136. An arrangement often adopted with large steam turbines and with turbines using high-pressure steam is shown in Fig 136 (a). The nozzles are divided into groups N_1 , N_2 , and N_3 , under the control of valves V_1 ,

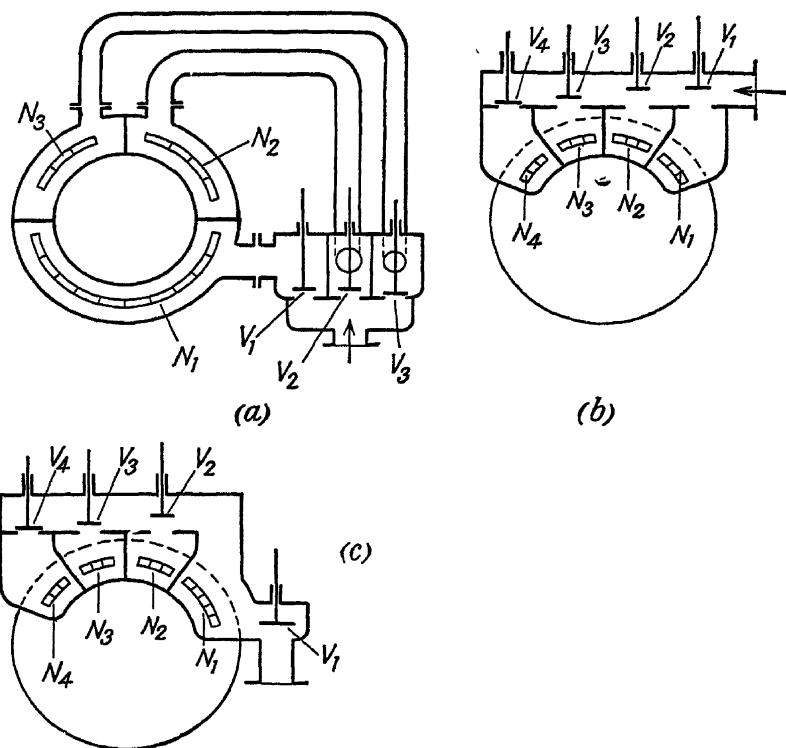


FIG. 136 —Diagrammatic Arrangements of Nozzle Control Governing

V_2 , and V_3 respectively. The number of nozzle groups may vary from three to five, or more. Fig 136 (b) differs from 136 (a) only in that the nozzle control valves are arranged in a casting forming part of the cylinder, or bolted thereto, and containing passages leading to the individual nozzle groups. Although this arrangement is compact, the nozzles are confined to the upper half of the cylinder and the arc of admission is usually limited to 180° , or less. The number of nozzle groups varies from four to twelve. Fig 136 (c) shows an arrangement frequently employed. For the sake of illustration only four groups of nozzles are shown. The group of nozzles N_1 is under the control of the valve V_1 , through which all the steam entering the turbine passes. Further admission of steam is through the

valves V_2 , V_3 , etc., in turn. In some instances, the nozzle group N_1 has been arranged in the lower half of the turbine and supplied with steam through a throttle V_1 up to, say, half-load. For loads greater than half-load, a further supply of steam is admitted through the valves V_2 , V_3 , etc.

Whatever method of construction is adopted, the nozzle control is necessarily restricted to the first stage of the turbine, the nozzle areas in the other stages remaining constant. It follows that, provided the condition of the steam at inlet to the second stage is not materially affected by the changed conditions in the first stage, the absolute pressure of the steam in front of the second-stage nozzles will be directly proportional to the rate of steam flow through the turbine.

114. Nozzle-box Pressures with Nozzle Control Governing. We shall now consider the variations in nozzle-box pressures as these affect the flow conditions in the first-stage nozzle groups. The problem will be illustrated with reference to the ten-stage impulse turbine for which the condition curve has been worked out in Art. 90 and Table VII, page 190. Further particulars are given in Example 18 and Table IX of the present chapter. Assume that the first-stage nozzles are divided into four groups, each group having a net area of 0.007375 sq. ft.

The pressure in the first-stage wheel chamber is represented by the straight line OA, Fig. 137, the pressure at A (full load) being 134.9 lb. per sq. in. abs. At full load, the ratio $\frac{p_2}{p_1}$ for the first-stage

nozzles is 0.6745, i.e. greater than the critical pressure ratio (0.5456) for superheated steam. When steam is passing through three groups of nozzles only, the pressure in the first stage is lower than the critical. Consequently, the quantity of steam passed through each group of nozzles, i.e. 9,480 lb. per hr., is slightly greater than one-quarter of the full-load steam flow.

Consider now the operation with only one nozzle valve open. The no-load steam consumption is 3,648 lb. per hr. Consequently, valve No. 1 controls the steam supply between the limits 3,648 and 9,480 lb. per hr. Over this range the pressure in the first stage varies from 13.5 to 35, and is always lower than the critical. Consequently, with fair accuracy, we may say that the steam flow through the nozzles will be directly proportional to the absolute pressure in the nozzle box. Hence if the line OBC be drawn through the origin, then for any steam flow W_1 , the pressure in the nozzle box will be p_{n1} and in the wheel chamber p_1 . The valve V_1 will throttle the steam from 200 to p_{n1} .

Consider next the conditions when the load is gradually increased and valve V_2 is brought into action. At the point of opening, the steam flow through the nozzles N_2 is very small and the pressure in nozzle box 2 will be, to all intents and purposes, 35 lb. per sq. in. abs.

As the valve V_2 is opened an increasing amount, the pressure in the second nozzle box will rise from 35 to 200 lb per sq in. At first, the ratio $\frac{p_2}{p_1}$ for the nozzles will be close to unity and the pressure in the nozzle box will no longer be given by any straight line

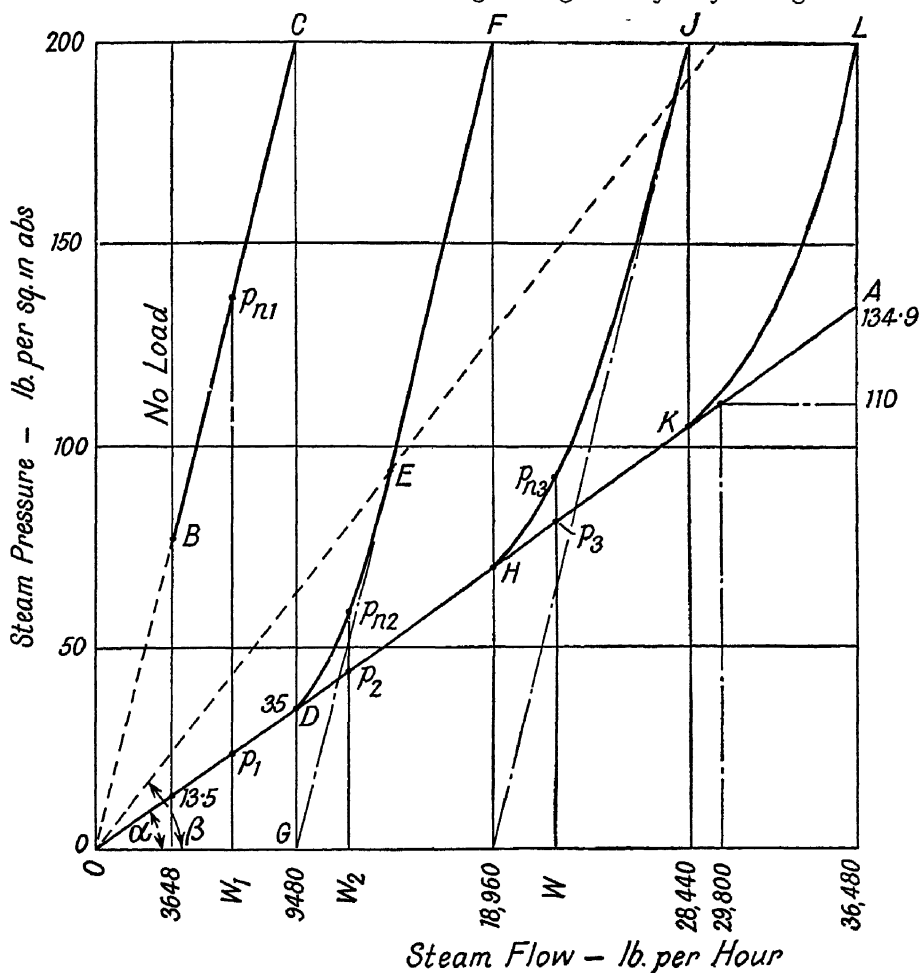


FIG 137.—Steam Pressures in Nozzle Boxes with Nozzle Control Governing

relationship. To obtain the pressure for any given total steam flow W_2 , we proceed as follows. Since the steam flow through group No. 1 is 9,480 lb. per hr, that through group No. 2 must be $W_2' = W_2 - 9480$. The pressure at outlet from the nozzles

$$p_2 = W_2 \times \frac{134.9}{36,480}$$

It remains to find, by trial and error, the initial pressure p_{n2} which, combined with the back pressure p_2 and the known area of the nozzles, will give a discharge W_2' lb. per hr. Such points are shown in the figure, curve DE. OE is a straight line drawn through the origin such that

$$\frac{\tan \alpha}{\tan \beta} = 0.5457$$

the critical pressure ratio for superheated steam. Then for any steam flow, if the pressure in the nozzle box is higher than the pressure given by the line OE, the pressure drop is greater than the critical. Hence, if the points G and F are joined by a straight line, the nozzle-box pressure for a total steam flow greater than about 13,900 lb. per hr. is given by the straight line EF. The nozzle-box pressure for the full range of group 2 is given by the line DEF. Similarly, that for group 3 is given by the line HJ.

Considering next the operation when valves 1, 2, and 3 are full open, and valve 4 is partly open, up to a steam flow of 29,800 lb. per hr., the pressure drop in the first-stage nozzles is greater than the critical and the total flow through groups 1, 2, and 3 is 28,440 lb. per hr. For steam flows greater than 29,800 lb. per hr. the pressure in the first stage exceeds 110, and the discharge through the first three groups falls off slightly. This must be taken into account when calculating the nozzle-box pressure for group 4, given by the curve KL.

115. Condition Curve with Nozzle Control Governing. Consider the flow W lb. per hr., Fig. 137, and let p_{n3} be the nozzle-box pressure for group No. 3 and p_3 the pressure in the first stage. Further, let W_1 , W_2 , and W_3 be the steam flow through groups 1, 2, and 3 respectively. The quantity $(W_1 + W_2)$ expands from 200 down to p_3 following the locus ABC, Fig. 138. On the other hand, the quantity W_3 is throttled by the nozzle valve V_3 down to the pressure p_{n3} , and therefore its initial condition at inlet to the nozzles is given by point D. The subsequent flow in the nozzles is given by the curve DE and reheating in the blades by EF. Hence some of the steam issuing from the first-stage blades has the condition represented by the point C, whereas the small quantity has the condition F. It is improbable that these steam masses are perfectly mixed before entering the second-stage nozzles, but assuming that perfect mixing could occur, then the condition of the steam at inlet to the second-stage nozzles would be represented by point G, such that

$$(W_1 + W_2 + W_3)I_G = (W_1 + W_2)I_C + W_3 \cdot I_F$$

from which I_G may be found.

The subsequent expansion of the steam proceeds from G instead of from C. The increase in specific volume from C to G is usually fairly small, and the change in pressure distribution in the remaining

stages is so small that the previous assumption that p varies directly as W is sufficiently accurate for most purposes

116. Comparison of Throttle and Nozzle Control Governing. The chief advantage claimed for nozzle control governing is the elimination of throttling losses. Actually, as we have seen, this is only partially true, because for steam flows intermediate between those corresponding to full-open positions of the consecutive nozzle control valves there is always more or less throttling at the valve which

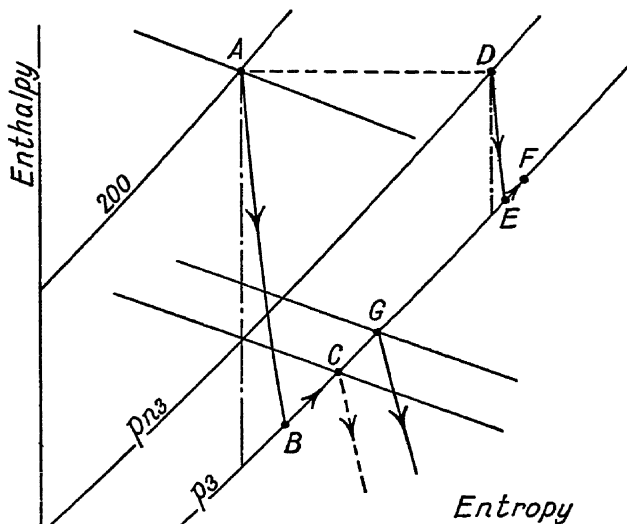


FIG 138

is partially open. However, if the number of valves is fairly large the loss of available energy is not appreciable. The next question is the efficiency with which the extra heat drop is utilized. Since the initial steam condition at inlet to the second-stage nozzles is very nearly the same as with simple throttle governing, at light loads the distribution of pressure and of stage heat drop in the following stages is also nearly the same, i.e. the greatest changes in blade speed ratio, etc., take place in the last few stages. On the other hand, as the load is reduced the heat drop in the first stage is increased, the steam velocity V_1 is increased, and the blade speed ratio ρ is diminished. The change in ρ from full-load to no-load is quite appreciable, as shown by the figures in Table X. The effect of this, however, is most marked when the designed heat drop in the first stage is comparatively small. In many cases, when nozzle control governing is used, either a large-diameter single-row wheel is used, or a two-row wheel is used in the first stage.

In addition to the great increase in jet velocity there will also be at light loads a considerable amount of jet deflection of the type

TABLE X

VARIATION OF HEAT DROP IN FIRST STAGE OF 3,000 kW TURBINE
GOVERNED BY NOZZLE CONTROL GOVERNING

Pressure in Nozzle Boxes Constant at 200 lb. per sq. in. abs. Initial
Superheat 150° F.

Blade Velocity 524 ft. per sec. $\eta_n = 0.9$. For other data, see
Art. 114.

Steam Flow Lb. per Hour	Pressure in First Stage	Adiabatic Heat Drop	V_1	$\rho = \frac{u}{V_1}$
36,480	134.9	38.8	1,321	0.396
28,440	105.1	62.8	1,680	0.311
18,960	70.2	97.1	2,095	0.250
9,480	35.1	151.4	2,615	0.202
3,648	13.5	218.3	3,140	0.166

described in Art. 51. This will certainly lead to greater losses in the first stage.

Summing up the matter then, we may say that there is a greater heat drop available when nozzle control is employed, but that the efficiency with which the greater heat drop is utilized becomes more and more impaired as the load diminishes. Comparative tests show that when there is a fairly large heat drop in the first stage, nozzle control leads to a reduction in steam consumption.

117. By-pass Governing. In modern impulse turbines, and especially those operating at very high pressures, the H.P. turbine comprises a number of stages of comparatively small mean diameter. All such turbines are usually designed for a definite load, termed the economic load, at which the thermodynamic efficiency ratio of the turbine is a maximum. In the case of power station turbines, the economic load (E.L.) is made about 80 per cent of the maximum continuous rating (M.C.R.), which is the maximum output the generator is capable of giving continuously.

Owing to the very small heat drop in the first stage it is not possible, for the reasons given in the preceding article, to employ nozzle control governing efficiently. Furthermore, it is desirable to have full admission in the high-pressure stages at the economic load so as to eliminate the partial admission losses. Hence it is not possible to admit through additional nozzles in the first stage the 25 to 27 per cent extra steam necessary to generate the full output. In fact, in most cases it would be quite impossible to do so because the steam pressure in the first-stage wheel chamber would increase so much, due to the restriction at the second-stage nozzles, that it would require to be greater than the initial pressure.

These difficulties of regulation are overcome by the employment of by-pass governing, the principle of which is illustrated diagrammatically in Fig 139. All the steam entering the turbine passes through a valve A (which is under the control of the speed governor) and enters the nozzle box or steam chest B. In certain cases, for example, this would suffice for all loads up to the economical load, the governing being effected by throttling. For loads

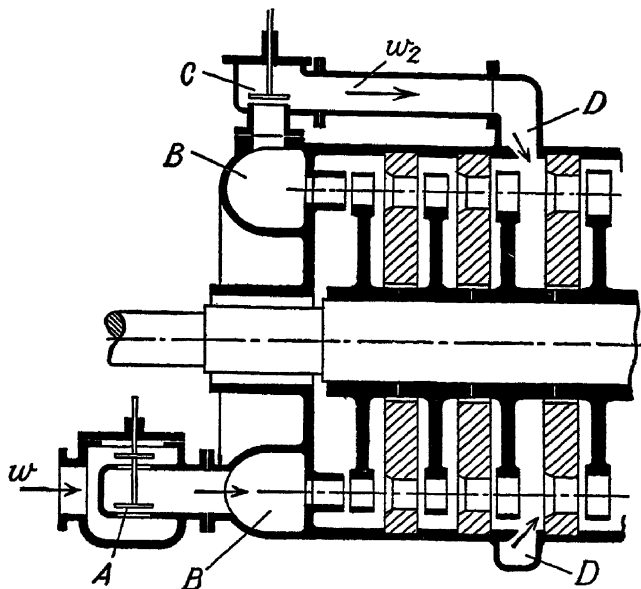


FIG 139 —Diagrammatic Arrangement of By-pass Governing

greater than the economic load a by-pass valve C is opened, allowing steam to pass from the first-stage nozzle box into the steam chest D and so into the nozzles of the fourth stage. The valve C is not opened until the lift of the valve A exceeds a certain amount, also as the load is diminishing the valve C closes first. The by-pass valve is under the control of the speed governor for all loads within its range.

The number of by-pass valves is not always restricted to one. Fig 140 shows the outline of the 105,000 kW steam turbine constructed by the Metropolitan-Vickers Electrical Co., Ltd., and installed in the Battersea Power Station. There are three cylinders arranged in tandem, and containing the following numbers of stages: H.P., 1-2-row wheel + 16 (single-row) stages; I.P., 18 stages; L.P., a double-flow turbine each half with 7 single-row wheels and a duplex multi-exhaust stage. For loads up to 60 per cent M.C.R., or 63,000 kW, steam is admitted before the velocity stage; for loads up to 80 per cent M.C.R., or 84,000 kW, steam is

admitted at B, after the velocity stage, while for the full output of 105,000 kW, steam is admitted at C, the first nine stages being by-passed.

118. Theory of By-pass Governing.

Let w_e = steam consumption in lb. per sec. at the economical load with no by-pass.

w = steam consumption at any load above the economical load; by-pass open.

w_1 = quantity of steam passing through the stages which are by-passed.

w_2 = quantity of steam which enters the by-pass stage.

p_n = pressure in by-pass stage at the economical load.

p'_n = pressure in by-pass stage at the higher load.

To a first approximation, i.e. neglecting the effect of by-passing

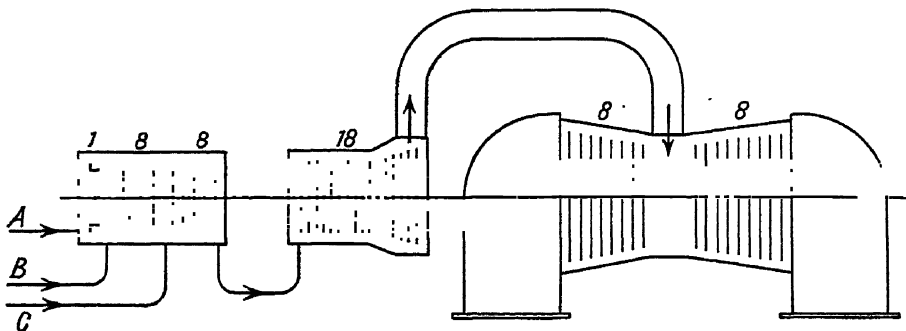


FIG. 140.—Diagrammatic Arrangement of 105,000 kW Turbine showing By-pass Governing Arrangements

on the steam condition at inlet to the nozzles following the supplementary admission, we may write

$$p'_n = p_n \cdot \frac{w}{w_e}$$

The higher value of p'_n will modify the mass flow through the nozzles of the stages that are by-passed, causing it to be less than it was at the economical load. In order to find the mass flow w_1 , we may proceed by a process of trial and error. Trial values of w_1 are taken, and from the known pressure p'_n and the Σh and v_n curves, the pressure distribution is calculated for the by-passed stages by the method given in Art. 111. If the trial value of w_1 is too high, the calculated nozzle-box pressure will be higher than the actual pressure, and vice versa. Having thus found the correct flow w_1 , we obtain the weight of by-pass steam w_2 .

Suppose now that ABCDEFG, Fig. 141, represents the course of the state point in the three by-passed stages of the turbine in Fig.

139. The steam passing through the by-pass throttle has a final state point H where, neglecting heat losses, $I_H = I_A$.

w_1 lb. of steam having the enthalpy value I_G is mixed with w_2 lb. of steam having the enthalpy I_H . The state point after mixing is J, and

$$w \cdot I_J = w_1 \cdot I_G + w_2 \cdot I_H$$

from which I_j may be found. Since I_j is greater than I_G , the condition of the steam in the succeeding stages is somewhat different from that corresponding to the economical load. A second calculation of the pressure distribution in the remaining stages will lead to a somewhat higher pressure p_n' , necessitating a recalculation of w_1 and w_2 . The second approximation is sufficiently accurate for all practical purposes.

119. By-pass Governing of Reaction Turbines. Owing

to the pressure drop across the moving blades, it is necessary to employ full admission in reaction turbines. Consequently, it is not possible to employ any form of nozzle-control governing. The remaining alternatives are simple throttle governing and by-pass governing. The latter has been employed by Messrs C. A. Parsons & Co., Ltd., almost from the beginning.

In the majority of Parsons turbines a single by-pass valve is employed. In the 50,000 kW two-cylinder reheat turbine for the Dunston power station, however, two such valves were fitted. Up to 30,000 kW, all the steam enters the first stage of the turbine through a throttle valve, up to 40,000 kW (the economic load) the first by-pass valve takes control, between 40,000 and 50,000 kW the control is effected by the second by-pass valve. The use of the additional by-pass valve results in a flatter steam consumption curve.

The pressures and steam flows in the by-pass stages may be found approximately by using eqs (149) and (150) of Art. 112, page 249.

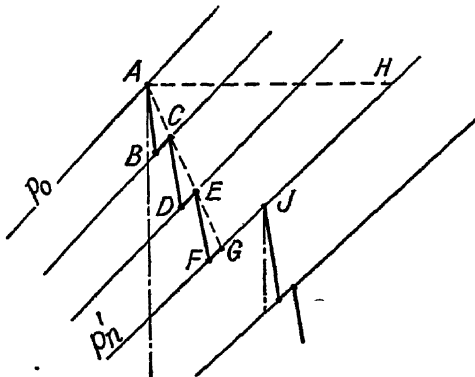


FIG 141

BIBLIOGRAPHY

- 1 KEARTON "The Distribution of Pressure in Impulse Steam Turbines at Varying Loads" (Brit. Ass, Oxford, 1926.) *Engineering*, 1926, 122; 365.

EXAMPLES XIII

1. The following data were obtained on a test of a steam turbine operating at a fractional load—

Steam pressure before governor valve	400
Steam temperature before governor valve	700° F.
Steam pressure after governor valve	120
Steam pressure in exhaust branch	1.0
Steam temperature in exhaust branch	120° F.

What is the internal efficiency ratio of the turbine?

If the steam flow through the turbine is 4,000 lb. per hr., and the power absorbed in bearing friction and governor drive is 10 h.p., what is the approximate power at the turbine coupling?

2. The full load output of a steam turbine, measured at the coupling, is 2,000 kW and the losses due to bearing friction, to the governor and oil pump drive, etc., amount to 60 kW. Steam is supplied at a pressure of 200 lb. per sq. in. abs. and superheated to 540° F. total temperature; the exhaust pressure is 1 lb. per sq. in. abs. The internal efficiency ratio at full load is 74 per cent.

What would be the coupling power of the turbine when the steam flow through the turbine is 20 per cent of that at full load, assuming (1) that the turbine is governed by throttling, (2) that the external losses are the same as at full load, (3) that the exhaust pressure remains unaffected, and (4) that the internal efficiency ratio is reduced to 68 per cent at this load?

3. A steam turbine is governed by throttling and the no-load steam consumption is found to be 12 per cent of the full-load consumption. The nozzle-box pressure at full load is 250 lb. per sq. in. abs., the initial superheat 150° F., and the exhaust pressure 0.5 lb. per sq. in. abs. If the steam consumption at full load is 10.45 lb. per kWh, what is the efficiency ratio of the turbine at quarter, half, and three-quarter load?

4. The following particulars refer to three successive "expansions" in the I.P. section of an impulse-reaction turbine—

Group No.	Mean Diameter Inches	Blade Height Inch	Blade Outlet Angle	Total Number of Rows (fixed and moving)
1	34.125	$\frac{1}{8}$	20°	8
2	34.25	$\frac{1}{8}$	20°	8
3	34.625	1	20°	8

The blade speed ratio is 0.5, $\phi = 0.9$, $\eta_n = 0.9$, the index of v for adiabatic expansion is 1.135, and the stage efficiency is 0.72. Calculate the steam flow through the turbine blades when the pressure at inlet to the first group is 72 lb. per sq. in. abs. (steam dry and saturated) and the pressure at outlet from the third group is 23.5 lb. per sq. in. abs.

5. Steam is supplied to the first-stage nozzle box of a turbine at a pressure of 200 lb. per sq. in. abs. and superheated 150° F. Under maximum load conditions, the weight of steam entering the first-stage nozzles is 9.45 lb. per sec., and the by-pass steam entering the second-stage wheel chamber is 3.58 lb. per sec., the pressure in that stage being then 120 lb. per sq. in. abs.

If the enthalpy of the steam at outlet from the second-stage blades is 1251.4 B.Th.U. per lb., calculate the enthalpy at inlet to the third-stage nozzles.

MIXED-PRESSURE TURBINES

120. **Low-pressure Turbines.** The exhaust steam from a non-condensing engine is normally at a temperature about 212° F, or slightly more, and is therefore well above the temperature of the

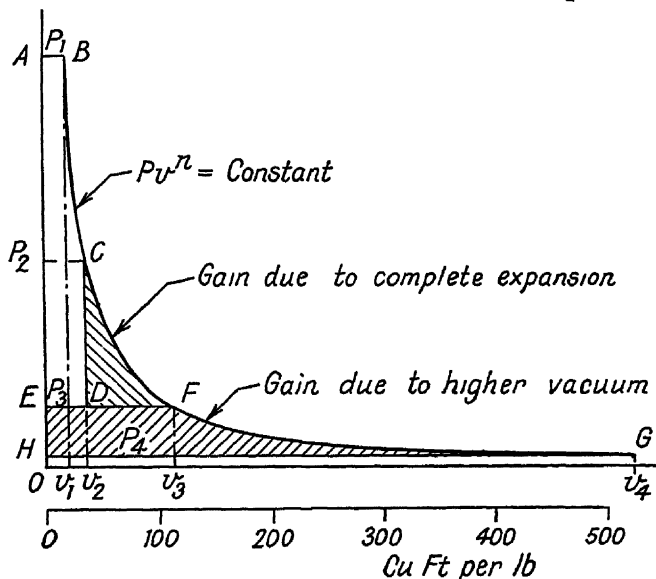


FIG 142

available condensing water. Consequently, such steam is capable of doing work by further expansion. Owing to the complete expansion which is possible in the steam turbine it is more economical to expand the available exhaust steam in a turbine than in an engine operating with a L P. cylinder. This may be seen from the $p-v$ diagram, Fig. 142, which is drawn to scale for a pressure $p_1 = 20$ lb per sq in. abs. The hypothetical indicator diagram for a reciprocator is $ABCDEA$. Owing to the limitation of size of the cylinder it is necessary to release the steam at some pressure P_2 after which the steam expands in the exhaust space to the condenser pressure P_3 .

Assuming that expansion in both reciprocator and turbine follows the law $p_v^n = \text{constant}$, and using P to represent pressures in lb. per sq. ft. abs., and v to represent specific volume, then

Work done per lb. of steam in engine

$$= P_1 v_1 + \frac{P_1 v_1 - P_2 v_2}{n-1} - P_3 v_2$$

This is represented by the work area $ABCDEA$.

In the turbine, however, the expansion is necessarily complete; the steam continues to increase in volume until the exhaust pressure P_3 is reached. The work area is ABFEA.

Work done per lb. of steam in turbine

$$\begin{aligned} &= P_1 v_1 + \frac{P_1 v_1 - P_3 v_3}{n - 1} - P_3 v_3 \\ &= \frac{n}{n - 1} (P_1 v_1 - P_3 v_3) \\ &= \frac{n \cdot P_1 v_1}{n - 1} \left\{ 1 - \left(\frac{P_3}{P_1} \right)^{\frac{n-1}{n}} \right\} \text{ ft.-lb.} \end{aligned}$$

In considering the most economical vacuum for a reciprocating engine, we have to balance the greater amount of sensible heat to be supplied when a higher vacuum is employed, against the relatively small increase of work done. In consequence, the most economical vacuum is comparatively low, being normally about 25 in. or 26 in. Hg. In the turbine, however, there is no limit, except that imposed by the temperature of the available cooling water and those practical considerations connected with the cost of pumping, size of water pipes, etc. Consequently, the L.P. turbine has a great advantage over the L.P. reciprocator on these grounds, which are ultimately due to the limit of size of the engine cylinder.

When the reciprocator is operating continuously it is simply necessary to install a L.P. turbine and suitable condensing plant. The L.P. turbine would contain a relatively small number of stages. This represents the simplest possible arrangement of exhaust turbine.

121. The Larderello Natural Steam Power Plant. An interesting adaptation of the low-pressure steam turbine hails from Larderello in the midst of a volcanic region in the Province of Tuscany (Italy), where there is a constant emission of steam and hot water from crevices and fissures in the ground. Over a century ago, it was discovered that the hot water was highly charged with boric acid, and works have long been in existence for the extraction of boric acid on a commercial scale. Helium is another constituent of the hot water.

In order to increase the output, boreholes were put down to depths ranging from 200 to 500 ft. The pressure of the steam with no delivery was found to be as much as 45 lb. gauge, and with maximum delivery was still about 15 lb. gauge, while the temperature was about 360° F.

It was decided to put down a steam turbine plant, but the use of the natural steam in the turbine was avoided for two reasons.

In the first place, there was some doubt as to the manner in which the blades of the turbine would be affected by the gases contained in the natural steam, and, secondly, there would have been some difficulty in obtaining a sufficient vacuum in the condenser, on account of the incondensable gases in the steam. It was, therefore, decided to use the heat of the natural steam for the purpose of

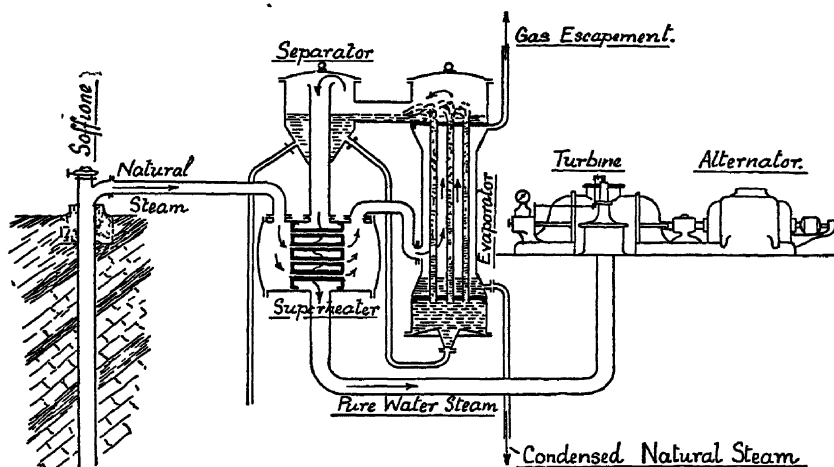


FIG. 143 —Diagrammatic Arrangement of Natural Steam Plant

evaporating pure water. The arrangement of the evaporators is illustrated diagrammatically in Fig 143 *

The evaporators consist of a number of aluminium tubes contained in a shell of sheet iron, aluminium being used in preference to iron or steel, because the sulphuric acid contained in the natural steam would form a layer of sulphide of iron on the outside of the tubes and so lower the rate of heat transmission. The vapour passing to the turbine is superheated by the incoming natural steam. In this manner steam is generated at about 7 lb. per sq. in. gauge pressure, and supplied to the turbine governor valve at a pressure of 2.5 lb gauge

The first machine of 250 kW was put down in 1912, while in 1916 three units of 2,500 kW capacity were installed. The latter turbines, which were built by Franco Tosi, of Legnano, Italy, and which run at 3,000 r.p.m., are impulse-reaction (Parsons) turbines of the double-flow type. It is stated that the installation has been successful in every way

122. Mixed-pressure Turbine. In many installations where a supply of exhaust steam from a non-condensing reciprocator is available, e.g. where the engine is a colliery winding engine, a

* Reproduced by kind permission from *Engineering*, 24th May, 1918, in which details of the plant are published.

haulage engine, rolling mill engine, etc., the supply of exhaust steam is intermittent. In such a case it is necessary to provide in the installation a low-pressure heat accumulator wherein the surplus exhaust steam (or its latent heat) may be stored during the time that the engine is operating, and from which the turbine may draw a supply of steam during the time that the engine is standing. Such a plant is shown diagrammatically in Fig. 144, which is self-explanatory.

In cases where the quantity of exhaust steam is not quite sufficient to produce the power required, or where it may be necessary to run

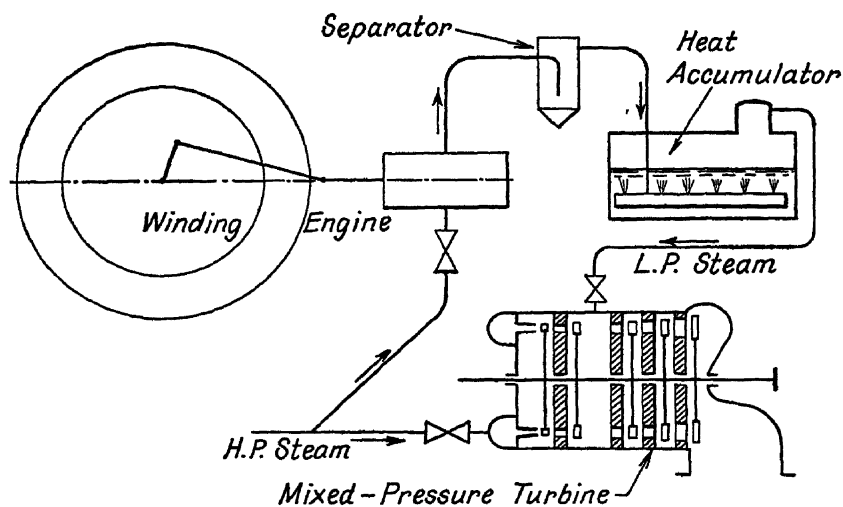


FIG. 144.—Diagrammatic Arrangement of Steam Engine, Heat Accumulator, and Mixed-pressure Turbine

the turbine during long periods when the engine is idle, the turbine is so designed that live steam may be admitted to the turbine at an earlier stage than the first L.P. stage. The turbine then consists virtually of two turbines which may be, and usually are, in the same cylinder (Fig. 144). Steam flowing from the last H.P. wheel flows directly into the first nozzles of the L.P. turbine. Governor gear is provided to utilize the whole of the exhaust steam available without allowing the exhaust steam pressure to fall below a certain predetermined value, and to supply H.P. steam, as and when required, in order to maintain the turbine speed sensibly constant irrespective of load variations. There are thus three possible conditions of operation (assuming that at certain times there is sufficient L.P. steam to develop the whole of the power)—

- (1) Power developed entirely by L.P. steam.
- (2) Power developed entirely by H.P. steam.

(3) Power developed partly by H.P. steam and partly by L.P. steam.

Hence the term "mixed-pressure" turbine.

123. Conditions of Operation of Mixed-pressure Turbine. It is beyond the scope of this book to analyse fully the performance

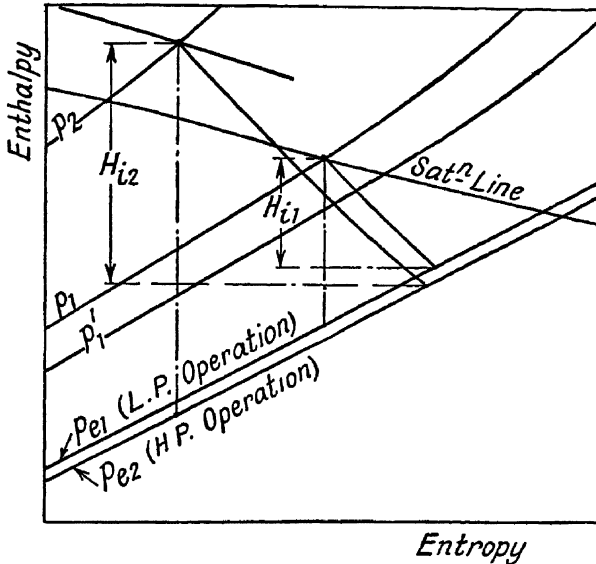


FIG 145

of this type of turbine, but the following brief analysis may be of interest.

(1) *Power Developed Entirely by L.P. Steam.*

Let K = internal power of turbine in kW (i.e. coupling power plus mechanical losses).

p_1 = pressure of L.P. steam (assumed dry and saturated).

p_{e1} = exhaust pressure when operating with L.P. steam.

H_{11} = internal work done per lb. of steam between the pressures p_1 and p_{e1} (Fig. 145)

W_1 = steam flow through turbine, lb. per hour.

Then

$$W_1 \times H_{11} = 3413K$$

i.e.

$$W_1 = \frac{3413K}{H_{11}}$$

Owing to the fact that H_{11} is comparatively small, the steam consumption W_1 for a constant load K is the maximum possible, and the L.P. nozzles and blades must be made sufficiently large to pass this quantity of steam.

(2) *Power Developed Entirely by H.P. Steam.* Suppose that the

Assuming that the L.P. steam is again supplied dry and saturated at p_1 , it is now throttled to p_1'' , the final state point being C.

Suppose that the admission of H.P. steam is controlled by a throttle valve, and that in the present case the steam is throttled from p_2 to p_2' . The pressure p_2' may be obtained from a $\log_{10} p$, $-\Sigma h$, $\log_{10} v$ diagram by the method given in Art. 111, starting from the pressure p_1'' and considering the H.P. stages only. This is a somewhat tedious process, for a tentative diagram would have to be drawn for an assumed value of p_2' in order to obtain a more accurate value of that pressure. Eq (149), page 249, although derived for the reaction turbine, may be used in approximate calculations on impulse turbines. The constant C_1 may be calculated from the relation

$$W_2 = C_1 \sqrt{\frac{p_2}{v_2} - \frac{p_1'}{v_1'}}$$

which applies to operation with H.P. steam only and, of course, refers only to the H.P. stages. Using this derived value of C_1 in the corresponding relation

$$W_4 = C_1 \sqrt{\frac{p_2'}{v_2'} - \frac{p_1''}{v_1''}}$$

and approximating, step by step, to the values of v_1'' and v_2' , the specific volumes at p_1'' and p_2' respectively, the pressure p_2' is obtained.

Having found the pressure p_2' , a constant I line BD is now drawn giving the condition D in the first-stage nozzle box. The condition curve DE is drawn next.

At inlet to the L.P. stages, W_3 lb. of steam with the enthalpy value I_C are mixed with W_4 lb. having the enthalpy I_E , resulting in the enthalpy I_F where

$$I_F = \frac{W_3 \cdot I_C + W_4 \cdot I_E}{W_3 + W_4}$$

The condition curve FG is now drawn for the expansion of the mixed H.P. and L.P. steam in the L.P. stages.

$\therefore$ Total internal power in kW

$$= [W_4(I_D - I_E) + (W_3 + W_4)(I_F - I_G)] \div 3413$$

If the calculated power is greater than K, the steam quantity W must be reduced, and vice versa.

124. Theory of Heat Accumulator. A typical steam consumption-time curve for a colliery winding engine is shown in Fig. 147. Commencing the wind at 0, steam is admitted to the cylinders at full pressure and with the link gear set to give steam for a large part of the stroke. At A the gear is linked up to cut off steam fairly early, and at B steam is shut off entirely, allowing the engine to

coast until the wind is stopped at C. The next wind is commenced at D, when the curve would be more or less repeated. It will be obvious that the area of the diagram represents the weight of steam consumed by the engine and the mean height the mean steam consumption. If the exhaust turbine is operating at a steady load and is using the whole of the steam available, then the mean height of the curve would also represent the turbine steam consumption. It will be apparent that between a and b the quantity of exhaust steam is greater than that required by the turbine, whereas between b and a' the turbine is using steam at a steady rate and there is little or none coming from the engine.

The usual L.P. heat accumulator, invented by Rateau, comprises one or more large horizontal cylindrical vessels, about 60 per cent full of water. The surplus exhaust steam is led through a pipe into the tank and allowed to pass through a large number of submerged orifices or nozzles. It is thus condensed by direct contact with the water. The latent heat is absorbed by the whole water mass as sensible heat, the temperature of the water rises, and the pressure of the steam in the space above the water also rises. This occurs from a to b , Fig. 147. As soon as the turbine begins to take more steam than is coming from the engine there is a slight reduction of pressure in the steam space of the accumulator which causes the water to evaporate. The whole water mass loses sensible heat, its temperature falls, and the steam pressure gradually falls. This regeneration period goes on from b to a' where, once again, the engine exhaust steam quantity exceeds the needs of the turbine. The quantity of steam to be condensed in the accumulator during the storage period is shown by the vertically-hatched area in Fig. 147; the quantity of steam re-evaporated is shown by the cross-hatched area, to scale. The variation of steam pressure which must be permitted depends on the variation of steam saturation temperature, and this in turn will depend on the quantity of water stored in the accumulator.

Consider the regeneration period and let

Q_1 = minimum quantity of water in accumulator, lb.

Q_2 = maximum quantity of water in accumulator, lb.

t_1 = lowest saturation temperature, ° F.

t_2 = highest saturation temperature, ° F.

L_1 = latent heat corresponding to t_1 .

L_2 = latent heat corresponding to t_2 .

Let Q , t , and L be values at any instant, and let the whole water mass cool by a very small amount δt , thus producing the small quantity of dry and saturated steam δQ . Then, assuming the specific heat of water to be unity

$$Q \cdot \delta t = L \cdot \delta Q$$

Writing $L = a - bt$, where a and b are positive constants

$$Q \cdot \delta t = (a - bt) \delta Q$$

$$\text{or } \frac{\delta Q}{Q} = \frac{\delta t}{a - bt}$$

$$\therefore \int_{Q_1}^{Q_2} \frac{1}{Q} \cdot dQ = \int_{t_1}^{t_2} \frac{dt}{a - bt}$$

$$\therefore \log_e \frac{Q_2}{Q_1} = -\frac{1}{b} \log_e \frac{a - bt_2}{a - bt_1}$$

$$\text{i.e. } \log_{10} \frac{Q_2}{Q_1} = \frac{1}{b} \log_{10} \frac{L_1}{L_2} \quad . \quad . \quad . \quad (152)$$

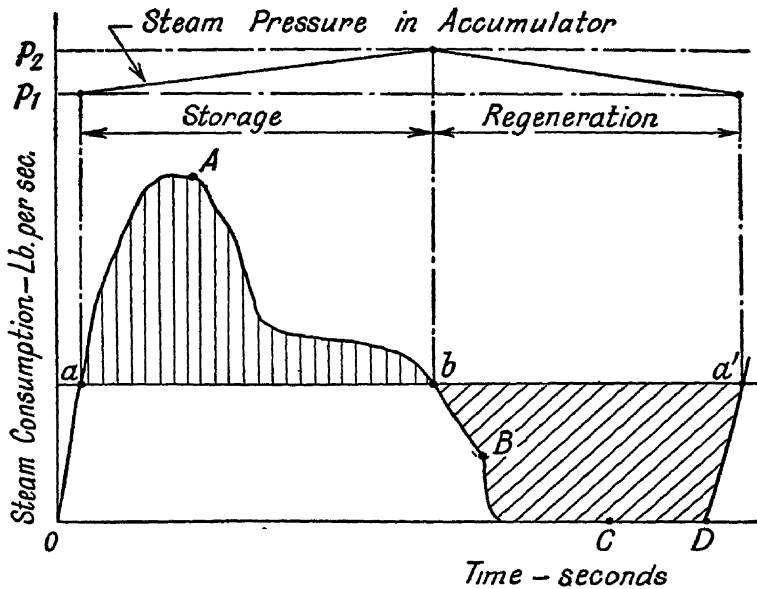


FIG. 147 —Time-Steam Consumption Diagram for Colliery Winding Engine

The weight of steam to be stored in the accumulator is $Q_2 - Q_1$. Let this be represented by W .

$$\text{Then } \frac{Q_2}{Q_1} = 1 + \frac{Q_2 - Q_1}{Q_1} = 1 + \frac{W}{Q_1}$$

$$\therefore \log_{10} \left(1 + \frac{W}{Q_1} \right) = \frac{1}{b} \log_{10} \frac{L_1}{L_2} \quad . \quad . \quad . \quad (153)$$

W is found from the steam consumption-time curve. Hence the weight of water $Q_2 = Q_1 + W$ which the accumulator must hold

may be found for any given range of pressure and temperature in the accumulator.

EXAMPLE 19. Calculate the minimum water capacity of a low-pressure heat accumulator when the weight of steam to be stored is 1,000 lb. per cycle. The maximum and minimum steam pressures in the accumulator are 22 and 18 lb. per sq. in. abs. respectively.

When $p = 22$, $t = 233.1^\circ \text{ F.}$; $L = 957.8$.

When $p = 18$, $t = 222.4^\circ \text{ F.}$; $L = 964.4$.

Assuming $L = a - bt$

$$a - 222.4b = 964.4$$

$$a - 233.1b = 957.8$$

$$\therefore 10.7b = 6.6 \text{ and } b = 0.617$$

$$\therefore \log_{10} \left(1 + \frac{1000}{Q_1} \right) = \frac{1}{0.617} \log_{10} \frac{964.4}{957.8}$$

$$= 0.004833$$

$$\therefore 1 + \frac{1000}{Q_1} = 1.01119$$

from which $Q_1 = 89,370 \text{ lb.}$

$\therefore$ Least capacity $= 90,370 \text{ lb.}$

$$= 40.4 \text{ tons}$$

If the circulation were imperfect and the whole of the water did not undergo the same temperature change, then a larger quantity of water would have to be stored.

EXAMPLES XIV

1. A reciprocating steam engine exhausting at 16 lb. per sq. in. abs. develops 1,500 b.h.p. with a steam consumption of 23.5 lb. per b.h.p.-hr. Calculate the power available from a steam turbine utilizing all the exhaust from the above engine, assuming that the steam enters the turbine at 15 lb. per sq. in. abs., dry and saturated, that a separator in the engine exhaust pipe removes 1,300 lb. of water per hour, that the vacuum at the turbine exhaust branch is 29 in. (30 in. bar), and that the turbine thermal efficiency ratio is 70 per cent.

2. Calculate the theoretical steam consumptions in lb. per h.p.-hr. of a L.P. reciprocating engine and a L.P. turbine operating with initially dry and saturated steam at 20 lb. per sq. in. abs. when the exhaust pressure is 2.5 lb. per sq. in. abs. and the release pressure in the engine is 8 lb. per sq. in. abs. Assume $n = 1.135$ in both cases. Calculate also the steam consumption of the turbine when the exhaust pressure is reduced to 0.5 lb. per sq. in. abs.

3. A mixed-pressure turbine operates under the following conditions—

Output at turbine coupling.	800 kW
Press of L.P. steam (dry and saturated)	18 lb. per sq. in. abs.
Press of H.P. steam (160° F. superheat)	180 " " "
Exhaust pressure	0.7 " " "

Assuming that the power absorbed by bearing friction, governor drive, etc., amounts to 15 kW, and that the internal efficiency ratio is 65 per cent for both H.P. and L.P. sections, calculate the steam consumptions in lb per kW-hr for both sections during operation. What is the steam pressure in the L.P. steam belt during operation with H.P. steam?

4 Calculate the power at the turbine coupling in Question 3 when the turbine is consuming 15,000 lb of L.P. steam per hr, supplied at 18 lb per sq in. abs, dry and saturated, and 2,000 lb of H.P. steam, supplied at 180 lb per sq in. abs. and 160° F. superheat. Assume—

- (1) that the internal efficiency ratio of both the H.P. and L.P. sections of the turbine remains constant at 0.65;
- (2) that the exhaust pressure is 0.7,
- (3) that the losses due to bearing friction, etc., amount to 15 kW,
- (4) that the pressure of the H.P. steam after throttling is 37 lb. per sq in. abs., and
- (5) that the supply of H.P. and L.P. steam is controlled by throttle governing.

5. The steam consumption of a rolling-mill engine increases uniformly from zero to 13.9 lb per sec in the first 5 sec of rolling, and then diminishes at a uniform rate to 4.45 lb per sec at the end of 30 sec from the commencement of rolling. The engine then comes to rest. The interval between successive rolls is one minute.

The rolling engine supplies dry and saturated exhaust steam to a L.P. turbine at an average pressure of 20 lb per sq in. abs. The turbine exhausts at 1.0 lb abs., and its efficiency ratio is 0.64. What is the power of the turbine?

Calculate the total weight of water to be stored in a heat accumulator to be used in conjunction with the exhaust turbine, and operating under the following conditions—

Maximum steam pressure 22 lb. abs., minimum steam pressure 18 lb. abs. Assume that the whole of the water in the accumulator is at the same temperature at any instant.

6. The steam consumption of a reciprocating steam engine varies cyclically, the rate of consumption in lb per sec being given by the expression

$$w = 4 + 3 \sin pt$$

where t represents time in seconds. The total time of the cycle is 120 sec. The exhaust steam is utilized in a L.P. turbine which uses all the available steam at a steady rate.

The surplus exhaust steam is stored in a heat accumulator. If the lower pressure is 15.29 lb. per sq. in. abs. ($t = 214^\circ \text{F}$, $L = 969.1$) and the higher pressure is 20.77 ($t = 230^\circ \text{F}$, $L = 958.7$), calculate the weight of steam to be stored and regenerated each cycle and the total weight of water to be stored in the accumulator.

7. A L.P. heat accumulator contains 25 tons of water at the beginning of the storage period. If the initial and final temperatures of the water are 214°F . ($L = 969.6 \text{ B Th.U. per lb.}$) and 228°F . ($L = 961.0$) respectively, calculate the weight of steam condensed in the accumulator during the storage period. The accumulator delivers steam to a L.P. turbine and the total time of regeneration is 2 min.

Assuming a mean steam supply temperature of 221°F ($p = 17.5 \text{ lb abs.}$), that the steam is supplied dry and saturated, that the exhaust pressure is 0.75 lb per sq in. abs., and that the efficiency ratio of the turbine is 62 per cent, calculate the power available.

8. The time-rate of steam consumption of a colliery winding engine increases uniformly from zero to a maximum during the first 20 sec. of winding

from rest and then remains constant for a further 20 sec., when steam is shut off and the engine is allowed to coast. The total time of winding and stoppage is 2 min. and during each wind the engine consumes 600 lb. of steam. The exhaust steam from the winding engine passes to a L.P. turbine and a heat accumulator. The turbine may be assumed to utilize the whole of the exhaust steam at a steady rate.

If the maximum and minimum engine exhaust pressures are 20 lb. abs. ($t = 228^{\circ}\text{ F.}$, $L = 961$) and 16 lb. abs. ($t = 216.3^{\circ}\text{ F.}$, $L = 968.1$) respectively, calculate the minimum weight of water which must be stored in the heat accumulator.

BACK-PRESSURE AND PASS-OUT TURBINES

125. Introductory. There are several industries, such as paper making, textile, chemical, dyeing, sugar refining, carpet making, brewing, and so forth, in which there is a dual demand for power and steam for heating and "process" work. It was formerly the practice to generate steam for power purposes at a moderate pressure and to generate saturated steam for process work at a pressure which gave the desired heating temperature. Such a system is obviously wasteful, for of the total quantity of heat supplied to the steam which is used for power purposes, quite 70 per cent will normally be carried away by the cooling water. Admittedly, if the engine or turbine is operated with a normal exhaust pressure, then the temperature of the exhaust steam is too low to be of any use for heating purposes; but by a suitable modification of the initial steam pressure and of the exhaust pressure, it would be possible to generate the required power and still have available for process work a large quantity of heat in the exhaust steam.

Suppose, for the sake of illustration, that in a paper mill 50,000 lb of dry and saturated steam are required per hour at a pressure of 50 lb gauge. Then, assuming that the condensed steam is returned to the system, the heat to be supplied for process work is 50,000 times the latent heat of dry steam at 65 lb per sq. in. abs., i.e. 45,720,000 B.Th.U. per hour. If steam is generated at 250 lb per sq. in. gauge, then with an initial superheat of 100° F. and a turbine efficiency ratio of 70 per cent, the steam is approximately dry and saturated after expanding to 65 lb. per sq. in. abs. The useful work done per lb. of steam in expanding from 250 to 50 lb gauge is 86.3 B.Th.U. Hence, the power available (neglecting the small mechanical losses) is

$$\frac{50,000 \times 86.3}{3413} = 1263 \text{ kW}$$

The enthalpy of steam at 250 lb. 100° F superheat is 1,268 B.Th.U. per lb., and that of the water returned from the plant is 267.3. Therefore the heat supplied per hour is

$$50,000 (1268.0 - 267.3) = 50,035,000 \text{ B.Th.U.}$$

This is 4,315,000 B.Th.U. more than in the simple heating system, i.e. about 9.5 per cent greater. The difference is, in fact, exactly equal to the heat equivalent of the power generated. It follows, therefore, that from the practical standpoint, provided no steam is

condensed in a separate condenser, the thermal efficiency of such a combined power and heating plant may approach unity.

Reciprocating engines and turbines are both available for use in such plants, but the latter are now almost exclusively employed, even for comparatively small outputs. An outstanding advantage of the turbine is the complete absence of internal lubrication, resulting in clean exhaust steam. Other obvious advantages are

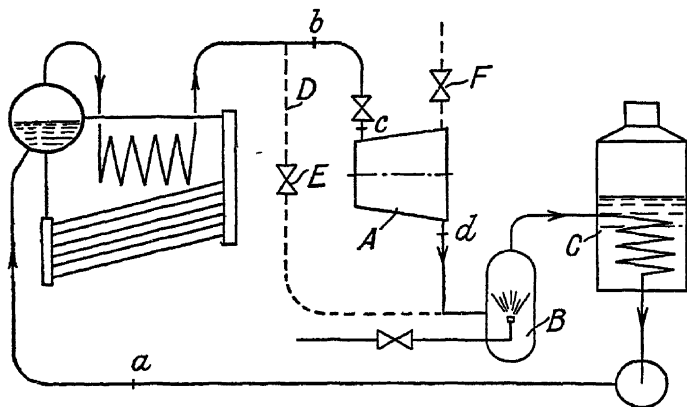


FIG. 148.—Diagrammatic Arrangement of Combined Power and Heating Plant with Back-pressure Turbine

the small space occupied, steady speed, simple control devices, and good balance.

There are two types of turbine employed in combined power and process plants, the back-pressure turbine and the steam extraction, or "pass-out," turbine.

126. Back-pressure Turbines. The back-pressure turbine may be employed in cases where the power which may be generated by expanding steam from an economical initial pressure down to the heating pressure is equal to, or greater than, the power requirements. The lay-out of such a plant is shown diagrammatically in Fig. 148. Steam is generated in the boiler at a suitable working pressure and admitted to the turbine A. The exhaust steam from the turbine will normally be superheated and in most cases is not suitable for process work, partly because it is impossible to control its temperature (since this will vary with the initial superheat), and partly because of the fact that the rate of heat transfer from superheated steam to the heating surfaces is lower than that of saturated steam. For these reasons, a desuperheater B is often used. A jet of water, thermostatically controlled, is sprayed into the entering steam; the steam is cooled and the spray water evaporated. The now saturated steam enters the heaters C and is entirely condensed. The condensed steam may or may not be returned to the boilers, depending on the conditions.

It is unlikely that the steam required for power generation will always be equal to that required for process work, and some means of controlling the exhaust steam pressure must be employed if variations in the pressure—and, therefore, of the steam saturation temperature—are to be avoided.

The method of control employed depends on the circumstances. If the back-pressure turbine is the only power unit, then it is fitted

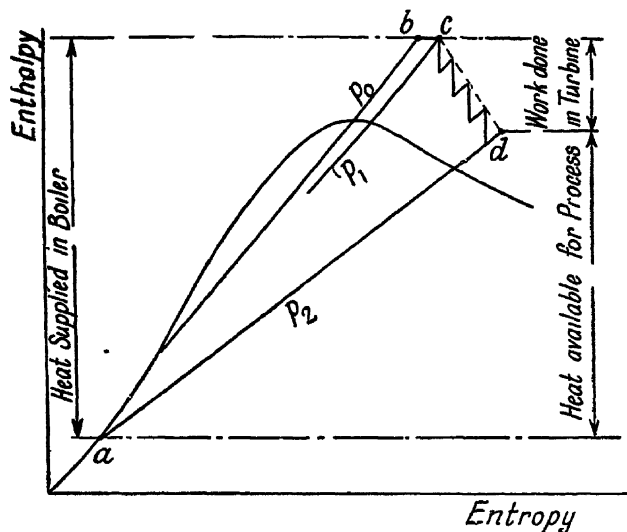


FIG 149

with an ordinary centrifugal governor and the quantity of available exhaust steam is controlled by the load on the turbine. If the available exhaust is too small, live steam may be passed through the reducing valve E into the desuperheater. If the quantity of exhaust steam is in excess of requirements, then the excess steam may be blown to atmosphere, or into a feed tank, etc., through the valve F. Valves E and F may be operated by hand to maintain the exhaust pressure constant, or, as is more usual, by automatic control gear. If, however, the back-pressure turbine is operating in parallel with other machines, then its output is controlled entirely by the heat load. In addition to the normal speed governor, there is a pressure regulator which controls the supply of steam to the turbine so as to maintain a sensibly constant exhaust pressure.

The thermodynamics of the back-pressure turbine is extremely simple and may be explained with reference to Fig. 149, which is lettered to correspond with Fig. 148, so far as the state of the working substance is concerned. The boiler pressure is p_0 , the pressure at inlet to the turbine is p_1 , and the heating steam pressure is p_2 . bc shows the throttling at the throttle valve, cd the expansion in

the turbine, and da the subsequent condensation in the heaters. This diagram shows clearly that, apart from minor heat losses, the whole of the heat supplied in the boiler is usefully employed.

Let K_i = internal power of turbine in kW.

H = heat required in heaters, B.Th.U. per hour.

W = steam flow through turbine, lb. per hour.

Then $W(I_d - I_a) = H$

or
$$W = \frac{H}{I_d - I_a}$$

Also $3413K_i = W(I_c - I_d)$

$$\therefore K_i = \frac{H}{3413} \left(\frac{I_c - I_d}{I_d - I_a} \right) \quad (154)$$

Conversely
$$H = 3413 \cdot K_i \left(\frac{I_c - I_d}{I_d - I_a} \right) \quad (154a)$$

127. Pass-out Turbine. In many cases the power available from a back-pressure turbine through which the whole of the heating steam flows is appreciably less than that required in the factory. This may be due to the small heating or process requirements, to a relatively high back-pressure, or a combination of both. In such a case it would be possible to install a back-pressure turbine to provide the heating steam and a condensing turbine to generate the extra power; but it is possible, and usual, to combine the functions of both machines in a single turbine with obvious advantages and economies. Such a turbine is shown diagrammatically in Fig. 150. Live steam enters the turbine and expands through the H.P. stages before the extraction branch. Here a certain quantity of steam is continuously being extracted for heating purposes, the remainder passing through the pressure control valve into the L.P. part of the turbine.

The control gear must be designed to meet the following requirements—

- (1) the speed of the turbine must be kept sensibly constant; and
- (2) the pressure of the heating steam must be kept sensibly constant; both of these regardless of the variations in power and heating loads.

These requirements are adequately met by the use of (a) a speed governor of the normal centrifugal type which controls the admission of H.P. steam to the turbine, and (b) a pressure regulator, responsive to changes of pressure of the extracted heating steam and controlling the admission of steam to the L.P. stages. A change of either power load or heating load will cause both governors to operate. For example, suppose the generator load is increased; this causes a certain momentary drop of speed. In response to this

the speed governor increases the lift of the H.P. throttle valve, thus allowing more steam to enter the turbine. If no change were made in the position of the pressure control valve, the pressure in the heating steam pipe would now rise. This pressure rise actuates the oil relay in such a way as to increase the lift of the pressure control

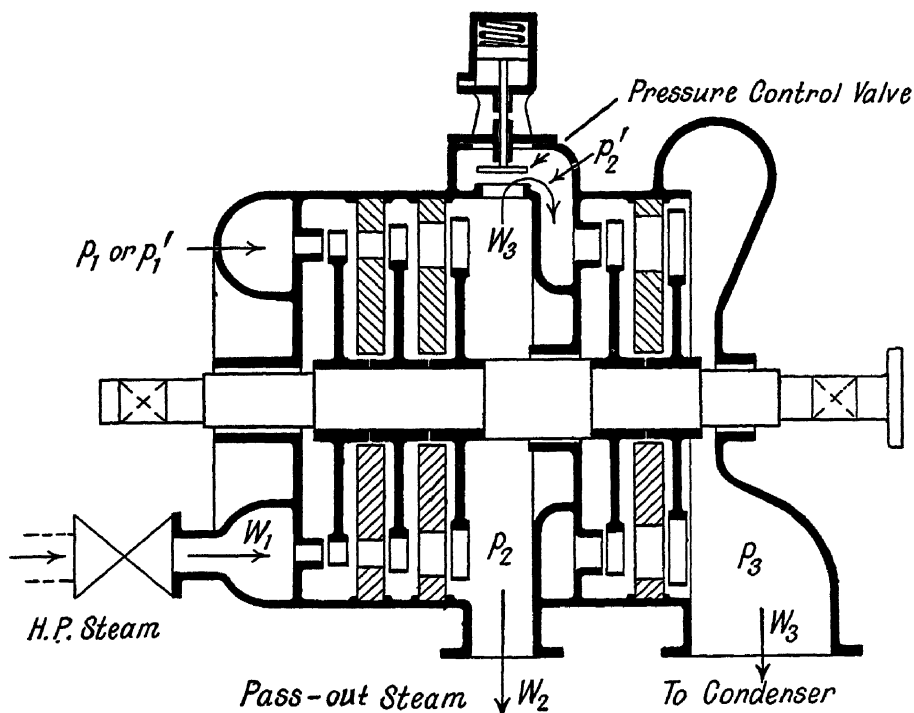


FIG. 150.—Diagrammatic Arrangement of Single-extraction Pass-out Turbine

valve and allow more steam to flow through the L.P. stages to the condenser. For a constant heat load, the weight of heating steam will remain nearly constant (it will vary slightly due to variations in its enthalpy), therefore the increase in steam flow through the L.P. stages will be practically equal to that through the H.P. stages. Thus, with a constant heat load, an increase in power load is followed by an increase in lift of the throttle and pressure control valves. Conversely, a reduction in power is followed by a reduced lift of both valves.

Suppose now that there is a sudden increase in heat load due, say, to coupling up another heater unit. The heating steam pressure will fall and so cause the lift of the pressure control valve to be reduced. This reduces the flow of steam through the L.P. stages with the

result that less power is developed in those stages, and, as the power output of the H.P. stages is as yet hardly affected, the speed of the turbine begins to fall. This causes the H.P. throttle to open, so admitting a greater quantity of steam to the turbine as a whole. It will be seen that a change in the heating load causes the lifts of the main throttle and pressure control valves to vary in opposite ways.

When the speed and pressure governors operate independently of each other it will be apparent that, provided the driven alternator is not connected electrically to any system, an increase in power

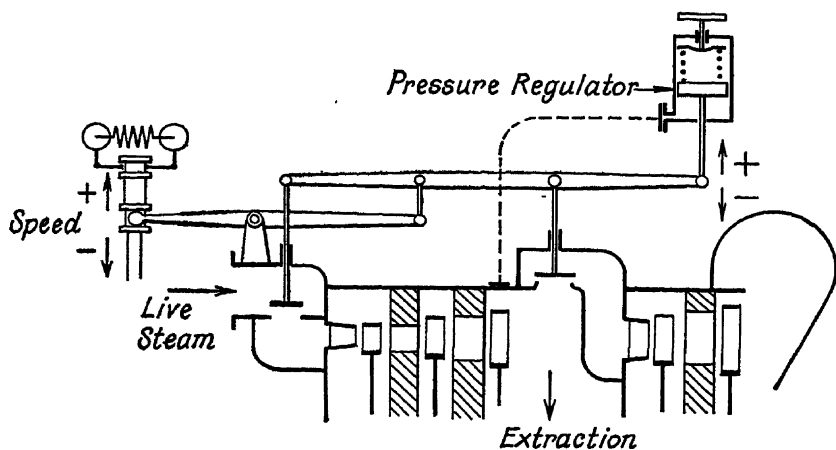


FIG. 151.—Diagrammatic Arrangement of Speed Governor and Pressure Regulator on Pass-out Turbine

load brings about an increase in pass-out steam pressure, whereas an increase in heat load causes a fall in speed. There is also a tendency to "hunting." This may be overcome by interconnecting the governor gear and the pressure regulating gear in such a way that both the throttle valve and the pressure control valve are operated simultaneously and in the correct direction. One form of such gear is shown diagrammatically in Fig. 151. In order to simplify the diagram, the oil relays have been omitted. It will be seen that a drop in speed causes both valves to lift, whereas an increase in heat load in producing a fall in heating steam pressure causes the pressure control valve partially to close and the throttle to open further.

Although both Fig. 150 and Fig. 151 show simple throttle valves controlling speed and heating steam pressure, regulation may be effected in either case, or in both cases, by some form of nozzle control gear. Moreover, steam may be extracted at two or more points.

128. Operation of Pass-out Turbine with Single Extraction. Consider the turbine shown in Fig. 150. Fig. 152 shows the Mollier

diagram when ideal nozzle-control governing is employed for both speed and pressure control, while Fig. 153 shows the Mollier diagram when throttle valves are used for both controls

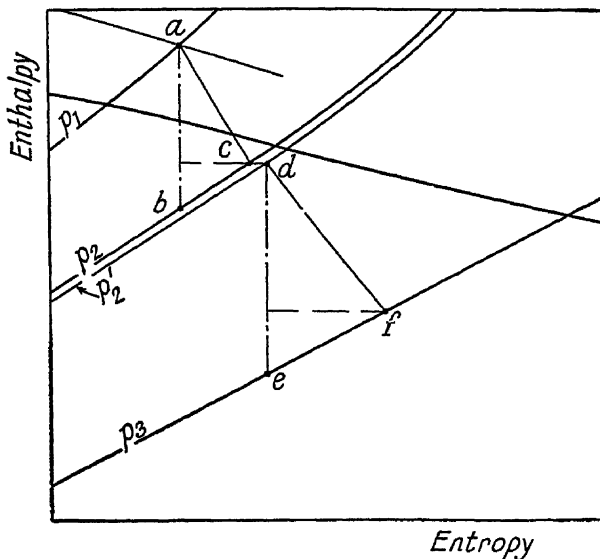


FIG 152 —Mollier Diagram for Pass-out Turbine with Ideal Nozzle-control Governing

Suppose that under any given conditions of operation—

W_1 = steam entering turbine, lb. per hour.

W_2 = steam extracted for heating, lb. per hour.

W_3 = steam entering L.P stages, lb. per hour.

$W_{1max.}$ = maximum quantity of steam entering turbine, lb. per hour.

$W_{3max.}$ = maximum quantity of steam entering L P stages, lb per hour

$W_{1max.}$ and $W_{3max.}$ will be fixed by the specified operating conditions, i.e. power and heating ranges

Let H = quantity of heat required, B.Th U. per hour.

K = total internal power, kW.

K_2 = power developed by heating steam in expanding through H.P. stages.

K_3 = power developed by the remainder of the steam (W_3) in expanding through the H.P. and L.P. stages.

Then with throttle valve control

$$p_1' = \frac{W_1}{W_{1 \max.}} \cdot p_1$$

and

$$p_2' = \frac{W_3}{W_{3 \max.}} \cdot p_2$$

With nozzle control governing there will be a small pressure drop $p_2 - p_2'$ at the control valves.

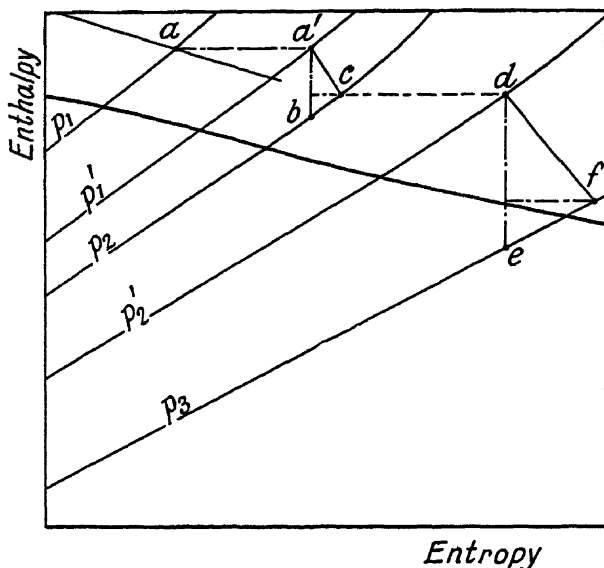


FIG. 153.—Mollier Diagram for Pass-out Turbine with Throttle Governing

The following are possible conditions of operation—

(1) *Full Extraction.* This is equivalent to back-pressure operation. Since the internal work done per lb. of steam = $I_a - I_c$,

$$W_1 = \frac{3413K}{I_a - I_c}$$

Let I_{w2} = enthalpy of water at p_2 .

Then heat rejected per lb. of steam = $I_c - I_{w2}$.

The total heat available is then

$$H = 3413K \left(\frac{I_c - I_{w2}}{I_a - I_c} \right) \text{ B.Th.U. per hour}$$

Alternatively, the power available with a heat load H is

$$K = \frac{H}{3413} \left(\frac{I_a - I_c}{I_c - I_{w2}} \right) \text{ kW}$$

This condition results in the largest value of the steam flow W_1 . Theoretically, the pressure-control valve is closed, but in practice a small amount of steam is allowed to flow through the L.P. stages in order that the heat generated in those stages by disc friction and blade windage may be continuously removed.

(2) *No Extraction* The turbine now operates as a normal condensing turbine, and $W_3 = W_1$. The pressure-control valve is full open and the condition curve on the Mollier diagram is practically a continuous curve. The steam consumption is

$$W_1 = \frac{3413}{I_a - I_f} \text{ K lb per hour}$$

This is the minimum steam consumption and is the lowest value of W_1 for any load. Consequently, with throttle governing for the H.P. section there is a considerable amount of wiredrawing at the H.P. throttle valve. Although this condition results in the minimum steam consumption for the turbine as a whole, the steam flow W_3 through the L.P. stages for any given power is greater than it is under any other conditions of operation. If the turbine is designed to develop its full output when running as a pure condensing machine, then the nozzles in the L.P. stages will be too large for most other operating conditions, and unless some form of nozzle-control governing is employed at the L.P. end there will be an appreciable amount of wiredrawing at the pressure-control valve with a corresponding loss of power. For this reason, the specified power output for no extraction should be kept as low as possible.

(3) *Partial Extraction.* (a) *Ideal Nozzle-control Governing* Consider a given heat load H B.Th U. per hour. Each lb of steam extracted gives up an amount of heat $= I_c - I_{w2}$ (Fig. 152).

$$\therefore W_2 = \frac{H}{I_c - I_{w2}}$$

Power developed by heating steam is

$$\begin{aligned} K_2 &= \frac{W_2}{3413} (I_a - I_c) \\ &= \frac{H}{3413} \left(\frac{I_a - I_c}{I_c - I_{w2}} \right) \end{aligned}$$

$$K_3 = K - K_2$$

Work done per lb. of steam expanding through H.P. and L.P. stages

$$\begin{aligned} &= (I_a - I_c) + (I_a - I_f) \\ &= I_a - I_f \end{aligned}$$

$$\therefore K - K_2 = \frac{W_3}{3413} (I_a - I_f)$$

The total steam consumption, lb. per hour, is

$$\begin{aligned} W_1 &= W_2 + W_3 \\ &= \frac{H}{I_c - I_{w2}} + \frac{3413 (K - K_2)}{I_a - I_f} \\ &= \frac{3413 \cdot K}{I_a - I_f} + \frac{H}{I_c - I_{w2}} \left(\frac{I_c - I_f}{I_a - I_f} \right) \quad . \quad . \quad (155) \end{aligned}$$

$$= \frac{3413 \cdot K}{I_a - I_f} + W_2 \left(\frac{I_c - I_f}{I_a - I_f} \right) \quad . \quad . \quad (155a)$$

(b) *Throttle Governing.* This presents some difficulty, since the pressures p_1' and p_2' are not known until the steam flows W_1 and W_3 are known. Two alternative methods are available. In the first, the pressures p_1' and p_2' are assumed, and the values of W_1 and W_3 calculated by means of the expressions given on page 281. The assumed pressures are now corrected and a recalculation made for W_1 and W_3 . These results will be approximately correct, but may be checked by a second approximation. Alternatively, a series of values of W_1 and W_3 may be taken and the corresponding heat and power output calculated. This is the more satisfactory method.

129. Steam Consumption Curves for Pass-out Turbine. These must show the relationship between three quantities, namely, the power output, quantity of pass-out steam, and the total steam consumption. The general form of the lines will be shown by means of an example.

EXAMPLE 20. Derive the steam consumption diagram for a single-extraction pass-out turbine operating under the following conditions—

Pressure of steam supplied	200 lb. per sq. in. abs.
Temperature of steam supplied	440° F.
Pressure of extracted steam	40
Pressure of exhaust steam	1
Maximum load at turbine coupling	1,000 kW.
Maximum load with no extraction	750 kW.
Internal efficiency of H.P. and L.P. stages	0.7
Mechanical losses of turbine	35 kW.
Maximum steam consumption under any conditions	25,000 lb. per hour.

(Steam consumption at no load) ÷ (Steam consumption at maximum output under given condition) = 0.12 for no extraction, and 0.14 for full extraction.

Assume ideal nozzle-control governing.

From the Mollier diagram we find—

Adiabatic heat drop from 200 to 40	130.5 B.Th.U./lb.
Useful heat drop from 200 to 40 . . .	91.4 „
Adiabatic heat drop from 40 to 1 . . .	230.0 „
Useful heat drop from 40 to 1 . . .	161.0 „

No Extraction

Total effective heat drop = $91.4 + 161.0 = 252.4$.

Total internal power at maximum load = $750 + 35 = 785$ kW.

$$\therefore \text{Maximum steam consumption} = \frac{785 \times 3413}{252.4}$$

$$= 10,600 \text{ lb. per hour}$$

$\therefore$ No-load steam consumption

$$= 0.12 \times 10,600 = 1,270 \text{ lb. per hour}$$

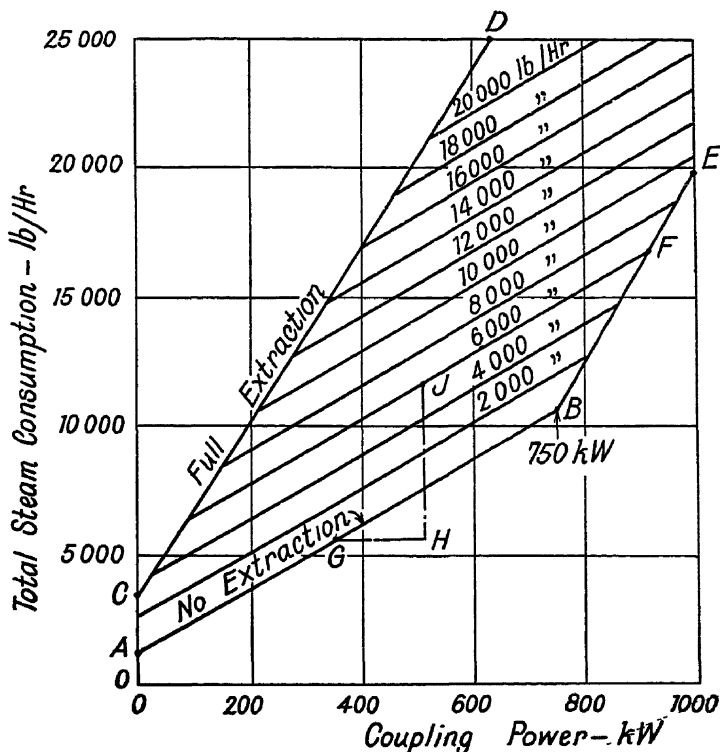


FIG. 154 —Steam Consumption Diagram for Pass-out Turbine

The steam consumption line is AB, Fig. 154. Since ideal nozzle-control governing has been assumed the steam consumption line would be slightly concave upwards. For the sake of simplicity a

straight line has been drawn. A similar assumption has been made with the other lines.

Full Extraction.

$$\text{Steam consumption} = \frac{3413}{91.4} = 37.3 \text{ lb. per kW-hr.}$$

$$\therefore \text{Internal power (maximum)} = \frac{25,000}{37.3} = 670 \text{ kW.}$$

$$\therefore \text{Coupling power} = 670 - 35 = 635 \text{ kW.}$$

No-load steam consumption

$$= 0.14 \times 25,000$$

$$= 3,500 \text{ lb. per hour}$$

The steam consumption line is CD.

Partial Extraction. Consider, for example, the extraction of 6,000 lb. of steam per hour. Suppose, in the first instance, that the L.P. nozzles are passing the maximum quantity of steam they are capable of passing, i.e. 10,600 lb. per hour. Then the L.P. nozzle-control valves are full open. The steam which passes right through the turbine will still develop the same internal power, i.e. 785 kW. The additional internal power developed by the extracted steam will be—

$$\frac{6000 \times 91.4}{3413} = 160 \text{ kW}$$

$$\therefore \text{Total internal power} = 785 + 160 = 945 \text{ kW.}$$

$$\text{Total coupling power} = 945 - 35 = 910 \text{ kW.}$$

The total steam consumption is $10,600 + 6,000 = 16,600$ lb. per hour. This condition is represented by the point F. By considering any other point G, say, on the line AB, the increment of power GH will be again 160 kW and the increment of steam consumption JH is 6,000 lb. per hour. Points such as F and J lie on a constant extraction line for 6,000 lb. of steam per hour. Such a line is obviously parallel to AB. Under the assumed conditions, the constant extraction lines are also equidistant when the increment of extraction is the same.

The straight line BE, Fig. 154, drawn through the limiting points on the constant extraction lines gives the maximum power output for a constant pass-out pressure. For all points on BE, the L.P. nozzle valves are full open, and only for steam consumptions greater than that at E are the nozzle valves partially closed.

EXAMPLES XV

1 Steam is supplied to a back-pressure turbine at a pressure of 400 lb per sq in abs and superheated 160° F. The exhaust steam from the turbine is utilized in heaters at a pressure of 40 lb per sq in abs. If the amount of heat required per hour is 30 million B Th U, and if the efficiency ratio of the turbine is 0.75, calculate the horse-power generated and the hourly heat consumption of the plant, assuming that the condensate from the heaters is returned to the boilers at 267.2° F, and that there are no heat losses. Assume a boiler efficiency of 80 per cent.

Calculate the hourly heat consumption of a plant consisting of a condensing turbine generating the same power, supplied with steam at the same pressure and superheat, exhausting at 1.0 lb per sq in abs, and having an efficiency ratio of 0.75, and separate low-pressure boilers generating dry and saturated steam at 40 lb abs from feed water at 267.2° F and supplying the same quantity of heat. Assume again an efficiency of 80 per cent for the high-pressure and low-pressure boilers.

2 In a factory where L.P. steam is required for heating purposes and electrical energy is required for power purposes, it is proposed to install a back-pressure turbine to operate under the following conditions—

Initial steam pressure	250 lb per sq in abs
Initial steam temperature	600° F
Exhaust pressure	20 lb per sq in abs
Efficiency ratio of turbine	70 per cent

If the amount of heat required per hour equals 50 million B Th U, calculate the total power available in kW if the generator efficiency is 94 per cent. It may be assumed that the condensate drains from the heaters return to the boiler at the condensing temperature.

3 Steam is supplied to a back-pressure turbine at 200 lb per sq in abs and 500° F. It is exhausted at 20 lb per sq in abs, the internal efficiency ratio of the turbine being 0.75. If the coupling power is 1,000 kW and the mechanical losses amount to 20 kW, calculate the total amount of heat available, assuming that the heater condensate temperature is 228° F.

Calculate the coupling power and the heat available when the quantity of steam is reduced to one-half of that at 1,000 kW coupling power. Assume that the turbine is governed by throttling, that the internal efficiency ratio, exhaust pressure, mechanical losses, and heater condensate temperature remain as before.

4 Calculate the maximum and minimum hourly steam consumptions of a pass-out turbine which develops 1,000 kW and which has a heat load varying from nil to 18 million B Th U per hour. The operating conditions are—

Initial steam pressure	300 lb per sq in abs
Initial steam superheat	240° F
Pass-out pressure	45 lb per sq in abs
Exhaust pressure	1.0 lb per sq in abs

The heating steam gives up heat by cooling and condensing only. The efficiency ratio of the H.P. and L.P. groups of stages may be assumed to be 0.70 under all conditions of operation, and it may be further assumed that regulation is effected by varying the number of nozzles in operation.

5 A pass-out turbine operates under the following conditions—

Initial steam pressure	300 lb per sq in abs
Initial steam temperature	700° F
Pass-out steam pressure	40 lb per sq in abs.
Exhaust steam pressure	1.0 lb per sq in abs.

When the turbine is developing its normal full load and giving out the

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normal quantity of heating steam, the total consumption is 40,000 lb. per hour, and the weight of extracted steam is 16,000 lb. per hour.

If the total quantity of steam supplied to the turbine is 26,000 lb. per hour and the quantity of steam extracted is 14 million B.Th.U. per hour, calculate the power available. The internal efficiency ratio of the H.P. and L.P. groups of stages is 0.72. The heating steam is condensed but the resulting condensate is not cooled below the saturation temperature. The admission of H.P. steam is effected by nozzle control, but the pass-out pressure is regulated by a simple throttle valve.

6. A factory has an average load of 1,000 kW and requires 15 million B.Th.U. per hour for process work. The heat is to be supplied by saturated steam which condenses in heaters at 30 lb. per sq. in. abs.

It is proposed to install a pass-out turbine to operate under the following conditions—

Initial pressure	.	.	.	.	200 lb. per sq. in. abs.
Initial superheat	.	.	.	.	120° F.
Exhaust pressure	.	.	.	.	0.7 lb. per sq. in. abs.

The turbine efficiency ratio for the pressure ranges 200 to 30 and 30 to 0.7 is 0.76, and the boiler efficiency may be taken as 0.80. If the calorific value of the coal used is 12,600 B.Th.U. per lb., calculate the daily coal consumption of the factory, allowing 10 per cent for auxiliaries.

7. A factory has an average load of 1,200 kW and requires 18 million B.Th.U. per hour to be supplied by steam which condenses in heaters at 30 lb. per sq. in. abs.

Steam for power purposes is to be generated at 250 lb. per sq. in. abs. and superheated 140° F. Compare the probable fuel consumptions in the following cases—

(1) If the steam expands in the turbine to a final pressure of 0.8 lb. per sq. in. abs. and the heating steam is separately generated at 30 lb. per sq. in. abs.

(2) If the supply of heating steam is extracted from the turbine at 30 lb. per sq. in. abs., the steam not required for heating purposes being expanded to 0.8 lb. abs.

It may be assumed that the boiler efficiency is 0.76 in each case, that the turbine efficiency ratio for any range of pressure is 0.78, and that the calorific value of the coal is 12,800 B.Th.U. per lb. Both speed and pressure are regulated by nozzle control.

CONSTRUCTION OF NOZZLES AND DIAPHRAGMS

130. Introductory. The nozzle is one of the most important elements in a steam turbine, for in it takes place the conversion of pressure energy into kinetic energy. The shape and finish of the nozzle should be such that this conversion takes place with the highest possible efficiency. The basic requirements may be enumerated as follow—

(1) Sudden changes in direction of the flowing steam, especially at high velocities, should be avoided.

(2) The inlet of the nozzle should be designed to utilize the carry-over energy from the previous stage to the greatest possible extent.

(3) The wall surfaces should be as smooth as possible in order to reduce friction, especially where the steam velocity is high.

(4) The design should be such as to permit of ease of manufacture and finishing, and to enable accurate channel sections to be obtained, especially at the high-pressure end of the turbine.

There are two types of nozzle, namely, (1) the simple convergent nozzle, which is suitable for expanding steam from any given pressure to any pressure higher than the corresponding critical pressure (i.e. to a pressure greater than $0.577 p_1$ for saturated steam, or $0.546 p_1$ for superheated steam), and (2) the convergent-divergent nozzle, which is suitable for expanding steam from any given pressure to any pressure lower than the corresponding critical pressure. From the theoretical analyses given in Arts. 38 and 44, it will be clear that a given convergent-divergent nozzle will only be suitable for a definite ratio of final steam pressure to initial pressure. Suppose, for example, a convergent-divergent nozzle is designed to operate with steam at a certain initial pressure p_1 and a certain outlet pressure p_2 . If the actual pressure in the space into which the nozzle discharges is higher than p_2 , then as the steam in the nozzle will expand down nearly to p_2 , the steam is said to be "over-expanded." If, on the other hand, the pressure on the discharge side is lower than p_2 , since the nozzle is restricted in its capacity for expanding steam the steam in the nozzle is said to be "under-expanded." In both cases there are fairly serious losses involved. Consequently, every convergent-divergent nozzle requires to be carefully designed for the conditions under which it has to operate.

The convergent nozzle is not subject to the same restrictions. Provided the pressure on the outlet side does not fall below the critical pressure, the nozzle will function in a satisfactory manner in spite of variations in outlet pressure. This statement refers to efficient operation and not to mass flow.

The great majority of turbine nozzles are of the convergent type; the convergent-divergent nozzle is hardly ever used except in small turbines and in conjunction with velocity-compounded wheels.

CONSTRUCTION OF CONVERGENT NOZZLES

131. **First-stage Convergent Nozzles.** Fig. 155 shows a segment of nozzles suitable for the first stage of an impulse turbine of

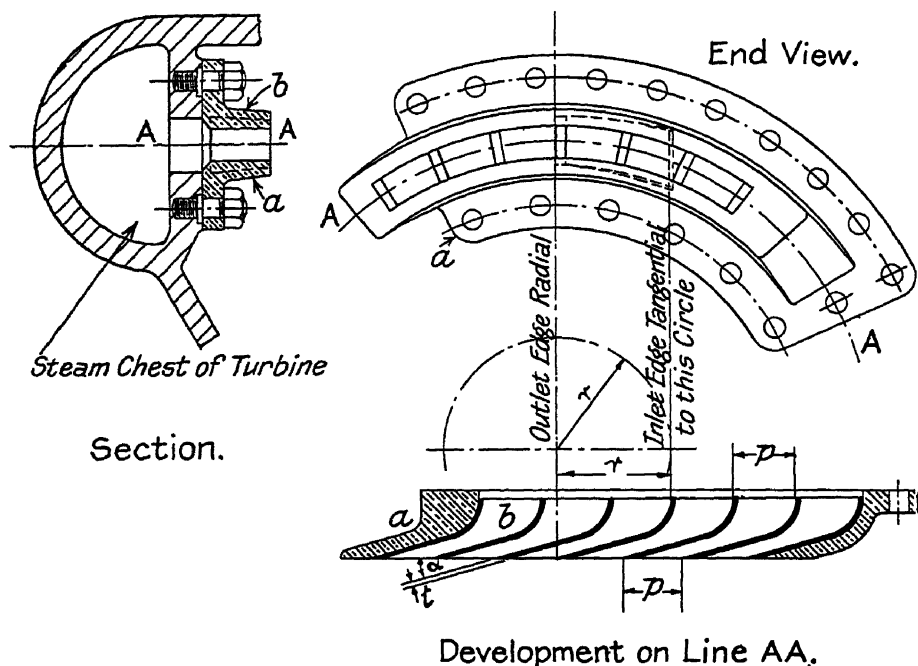


FIG. 155

medium power. The segment contains six nozzles and comprises a casting *a* into which the nozzle guide vanes *b* are embedded by "casting in." The procedure adopted in the manufacture of such cast-in guide blades varies somewhat, but is generally as follows. The guide vanes are made from sheet metal of uniform thickness which is first cut to shape and then curved in a press. After certain preparation (which will be described shortly) the guide vanes, now of the correct shape and having the correct curvature at inlet, are embedded in a sand core which is to form the steam passage in the nozzles. This may be effected either by means of a core box in which the guide vanes are set, or by forming the core on a circular table, the guide vanes being set up at the correct angle and the correct distance apart. Each end of the vanes projects outward from the core, as shown in Fig. 156, which shows a section through

the mould for the nozzle. When the molten metal is poured into the mould the projecting edges of the guide vanes are surrounded; on freezing of the metal the vanes become firmly held.

The circumferential pitch p of the nozzles is normally about 2 in. The outlet angle α varies from 12° to 20° . The thickness t

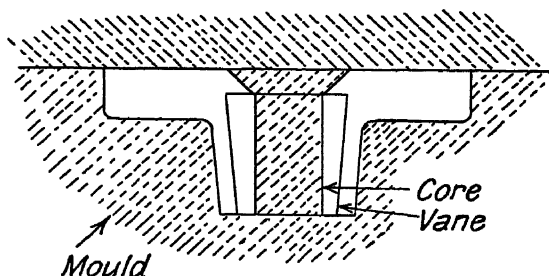


FIG 156

of the guide vanes should be made as small as possible for the reasons given in Art. 60. It varies from 15 L.S.G (0.072 in.) to

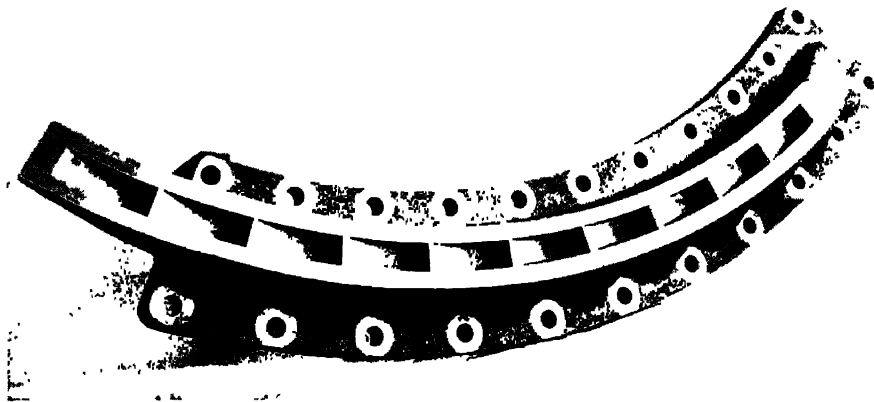


FIG 157 —Curtis Turbine Nozzles—Exit Side

8 L.S.G (0.160 in.). The larger thickness would only be used with long blades where strength to resist bending is required

Fig 157 shows a photograph of such a nozzle segment having ten nozzles.

132. Materials and Preparation of Nozzle Guide Blades. Rolled copper guide blades cast in gunmetal nozzle segments may be used for saturated steam. For superheated steam it is essential to use steel or alloy steel vanes. The metals employed include low-carbon sheet steel, 3 to 5 per cent nickel steel, stainless iron, and Hadfields' Hecla A T V. steel. The late Dr. O. Lasche, in his book,

Materials and Design in Turbo-Generator Plant, states that "low-carbon double-pickled steel stamping sheet, or soft tin plate made from Siemens-Martin steel of not more than 34 kg. per sq. mm. tensile (21.6 tons per sq. in.), with about 24 per cent elongation, has shown itself to be thoroughly reliable. Such material does not easily rust. Also when cast into the iron frame (diaphragm) it can take up carbon from the fluid cast iron without becoming hard or brittle."

Steel guide vanes are cleaned with acid and the ends tinned. The tinning prevents oxidation and assists in the partial alloying of the cast iron and the steel guide blade during the appreciable time that the iron is molten.

Special precautions must be taken when casting in guide blades of Hecla A.T.V. and other highly alloyed steels of this general type into brass or bronze nozzle segments or diaphragms, owing to the fact that vanes of these materials may develop cracks when quickly brought into contact with molten alloys containing zinc and tin. There is a "heat shock" effect of the hot alloy on the steel setting up strains which may cause cracking if, at the same time, an alloying action is allowed to take place between the steel and the molten metal. The measures suggested by Messrs. Hadfields, Ltd., for the lessening of the heat shock and for the minimizing or prevention of the alloying action are (1) to smoke the blades or the mould with the blades in position before closing; this may be done by means of the smoky flame of a paraffin lamp, or (2) if the above is found not to be convenient the following coatings may be used, either (a) graphite, or red lead, bonded with 15 per cent aqueous solution of molasses, painted evenly and dried at a temperature below 180° C., or (b) graphite, or red lead, mixed into a paste with palm oil rubbed evenly on the warmed blade, and then heated to 150°–180° C. The selection of the method depends upon the degree to which the brazing is desired to be carried. The use of red lead ensures a tight joint. Cracking may also be avoided by preheating the steel to a temperature of 550° C. The manufacturers also recommend a slow rate of pouring so as to lessen the heat shock. These precautions are not necessary when casting Hecla A.T.V. steel vanes in cast-iron nozzles or diaphragms owing to the absence of any alloying action.

Steel guide vanes which have been strained due to cold bending should be normalized.

133. Built-up Nozzle. An accurate nozzle segment may be obtained by the built-up construction shown in Fig. 158. This has been used in turbines constructed by Metropolitan-Vickers, Ltd., and comprises a number of vanes A, which are machined all over, placed between curved angles B, likewise machined all over. The ends of the segments are closed by pieces C of suitable shape. The guide vanes are attached to the angles by rivets D and small spigots E riveted over.

134. Diaphragm Nozzles. For high-pressure diaphragms, the built-up construction originated by the Erste Brunner Maschinenfabrik is used by many manufacturers. Details of one form of construction are shown in Fig. 159. Each diaphragm consists of a steel centre A, in halves, to which are riveted several nozzle elements B. Each of these is machined all over. The abutting edges are machined radial so that when the elements are set in the groove of the diaphragm centre they fit together and form a complete ring.

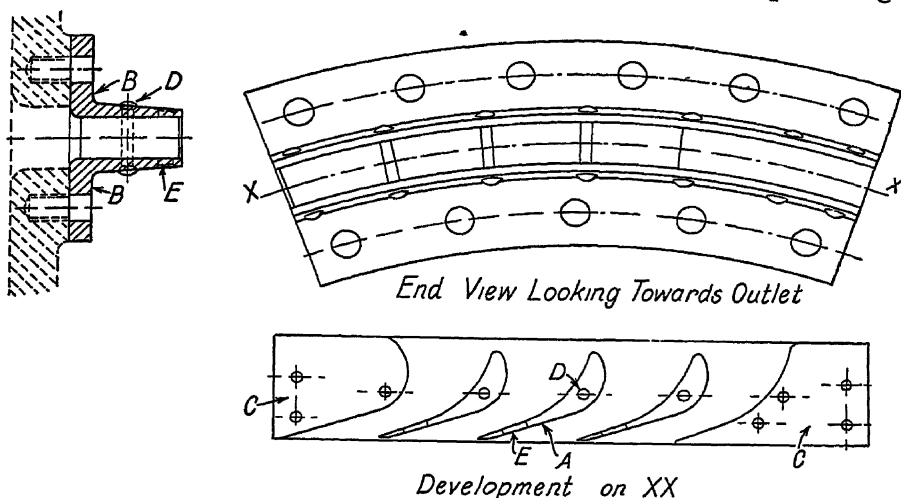
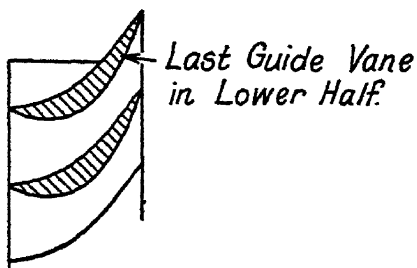
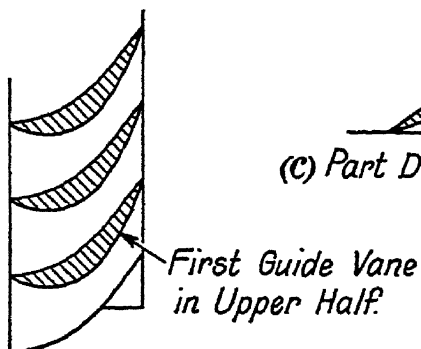
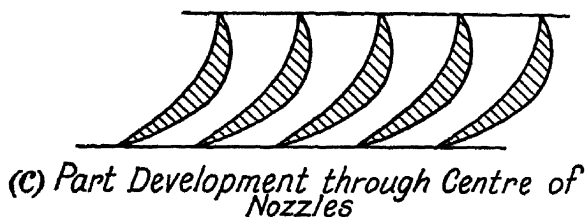
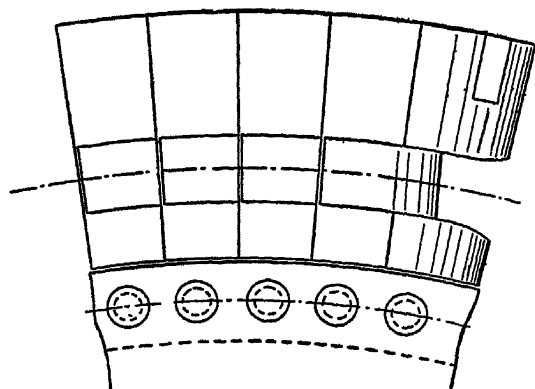
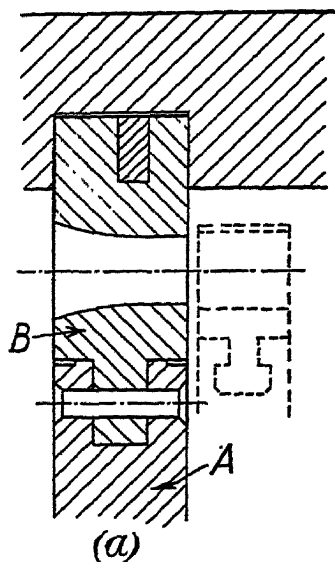


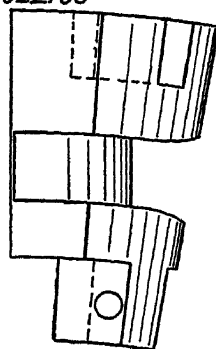
FIG 158.—Metropolitan-Vickers Type of Built-up Nozzle

Where the admission is partial, blanks are fitted in place of guide blades. Fig. 159 (d) shows how the guide blades may be arranged at the horizontal joint so that a complete and uninterrupted ring of nozzles may be obtained. On the side opposite to that shown, the guide vanes would protrude from the upper half. The advantages of the construction are (1) an accurate profile of good shape is secured for the guide blade, (2) the nozzle areas are accurate; (3) where required, a nozzle may be constructed with a relatively small radial height, thus enabling full admission to be obtained and losses due to partial admission eliminated, (4) a robust construction, yet occupying a small axial length, is assured; and (5) the nozzle surfaces may be made smooth.

In turbines of medium power and in the low-pressure stages of turbines with a high output the nozzle plates in the diaphragm are frequently of the "cast-in" type already described in Art. 131. A typical construction where the ratio of nozzle height to mean diameter is relatively small is shown in Fig. 160. A method of setting out the guide vane is illustrated in Fig. 160 (c). The straight outlet parts of the vanes are first drawn at the angle α to the face



(d) Arrangement of Guide Vanes at Horizontal Joint



(e) Detail of Nozzle Element

FIG. 159.—Erste Brünner Type of Built-up Nozzle

of the diaphragm, and the perpendicular bc is drawn from b to the adjacent guide blade. It seems desirable to provide the nozzle with a short parallel throat to control the jet, and for this purpose

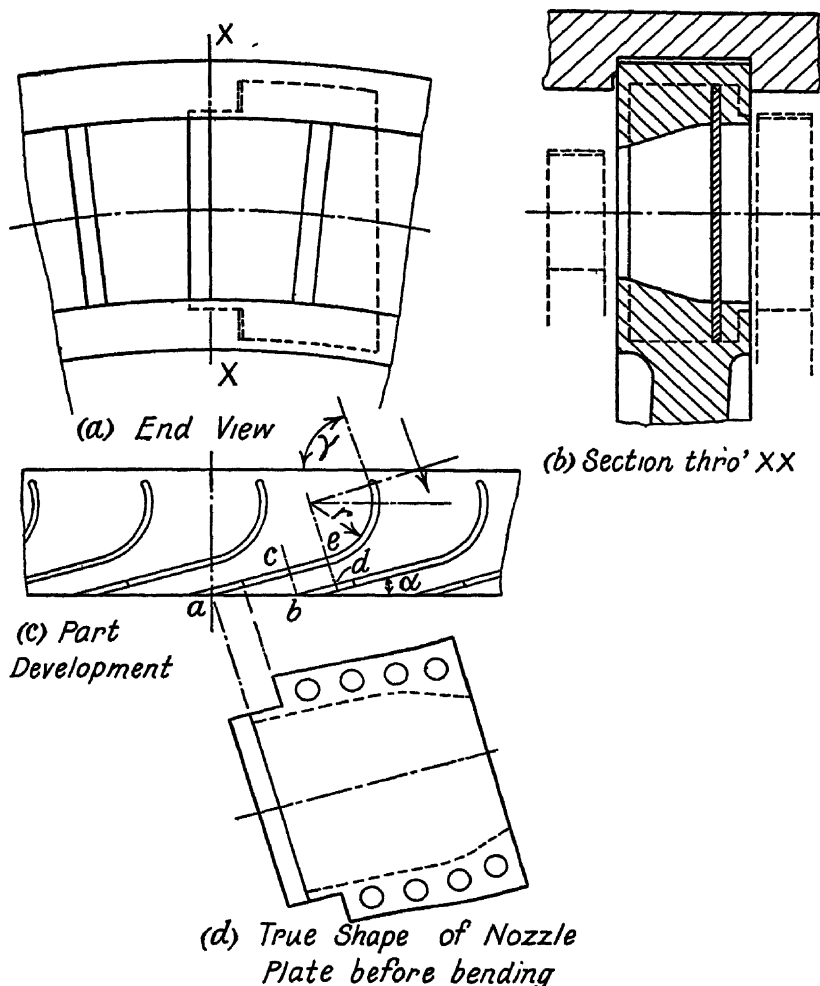


FIG. 160.—Details of Cast-in Type of Diaphragm Nozzle

de is drawn parallel to bc , the length of the parallel part being ec . For the effect of such a parallel throat on nozzle efficiency, the reader is referred to Art. 60. For manufacturing purposes it is desirable to shape the guide blades on a round former, and for this purpose the curved inlet portion is given a suitable radius r , the centre of curvature lying on de produced. The inlet angle γ of the

guide blade should be made to agree with the angle γ given by the velocity diagram, Fig. 65, page 123. The true shape of the nozzle plate before bending may be obtained by simple projection and development. In order to increase the union of the guide blade with the diaphragm casting the edges of the nozzle plates are either drilled, as shown, or scalloped.

CONSTRUCTION OF CONVERGENT-DIVERGENT NOZZLES

135. **The de Laval Nozzle.** The nozzle used in the single-stage de Laval turbine is invariably of the convergent-divergent type. The construction is shown in Fig. 161. The nozzle is of gunmetal and

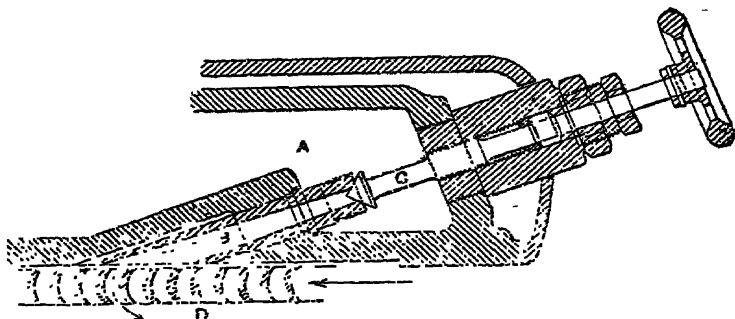


FIG. 161

is provided with a rounded inlet, which forms the short convergent part of the nozzle, and a gradually diverging part. The nozzle is an easy force fit in the cylinder, the steam chest end being screwed so that the nozzle may be withdrawn when required. In a steam turbine which may sometimes run condensing and at other times non-condensing, special nozzles are supplied for either condition of operation. Owing to the greater expansion ratio in the condensing nozzles and to the fact that the outlet area is the same in both types—being limited by the same blade height—the throat area of the condensing nozzles is smaller than that of the non-condensing type. Consequently, even allowing for the smaller specific steam consumption when operating condensing, the number of condensing nozzles required is about twice the number of non-condensing nozzles. The individual nozzles may be opened or closed by means of the needle-valve shown. Although the cylindrical or conical form of nozzle affords a simple construction and easy machining operations, the resulting elliptical outlet in the wheel plane cannot be ideal from an efficiency point of view, since the blade channels other than that directly in line with the nozzle axis are only partially filled with the issuing steam jet and their performance will be adversely affected by ejector action and eddy effects.

136. Machined Type of Nozzle. With the introduction, now several years ago, of velocity-compounded wheels at the high-pressure end of marine reaction turbines, the problem of constructing suitable nozzles for the impulse stage presented itself. The pressure drop was large, often from about 180 lb. per sq in. gauge down to about 40 lb. gauge, the number of jets was sometimes large, and the nozzles had to be so constructed as to be as efficient as possible. Moreover, there was little or no previous experience with such turbines in marine practice and the possibilities of corrosion and erosion had to be faced. One type of nozzle construction is shown in Fig. 162. The nozzle casting is of gunmetal, while the vanes are of a special metal. A number of slots are machined in the main casting or box to receive the vanes, which are machined all over, and the shape of which is indicated in Fig. 162 (c). That part of the vanes which is not embedded in the slots is machined to a taper, as shown in Fig. 162 (a), so as to give the required sectional areas at the throat and mouth of the nozzle. The chief advantages of this rather costly form of construction are—

(1) The vanes are machined all over and thus offer the minimum resistance to the flow of steam.

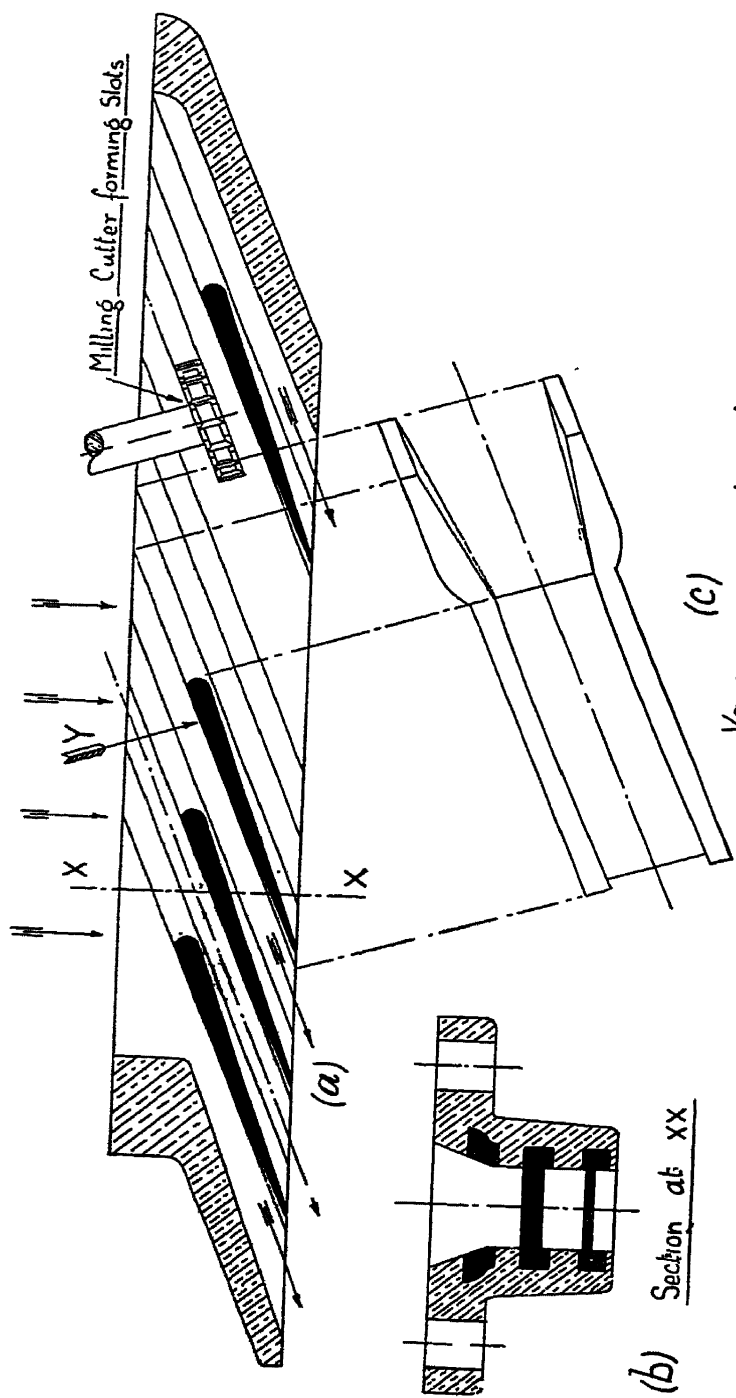
(2) The machining of the slots and vanes ensures correct nozzle angle and correct throat and mouth areas.

(3) The vanes may, if badly corroded or eroded, be replaced by new ones.

137. "Cast-in" Type of Convergent-divergent Nozzles. Another form of construction which has been largely adopted in marine impulse turbine stages is shown in Fig. 163. Guide blades are rolled to the section shown by the dotted lines in the upper part of the figure, are then cut to shape and cast into the nozzle segment in the usual manner. After marking-off and machining, the guide vanes are thinned down to the final section by filing.

138. Built-up Nozzle. An interesting form of nozzle construction has been used in many steam turbines constructed by the well-known Swiss firm of Brown, Boveri & Co. It consists essentially of three parts—a segmental strip *a* (Fig. 164), in which the nozzle passages are machined, a covering segment *b*, and a wedge piece *c*. These parts are bevelled as shown, and the downward pressure of the locking screws *d* on the ring *c* causes the strips *a* and *b* to be forced against the steam chest, so keeping the joints steam-tight. The set-screw *d* is covered by a cap nut *e* to prevent steam leakage. The construction has all the advantages of other built-up nozzles, namely, accurate angles, form, and areas, together with a good finish. It also facilitates examination and replacement, if necessary.

139. Loss Due to Steam Entering Blades Obliquely and Special Form of Nozzle to Minimize such Loss. Consider the form of nozzle most frequently employed in impulse turbines. This has already been described, and comprises a number of sheet metal guide vanes



Vane as seen when looking
in direction of arrow Y.

FIG. 162

cast into a diaphragm or nozzle block. A longitudinal section shows a passage which is symmetrical about a straight line XX parallel to the turbine axis, Fig. 165. The outlet edge YY of the vane is, in such cases, radial, while the inlet edge $Y'Y'$ is tangential to a circle having the radius s . The path of a particle of steam in passing

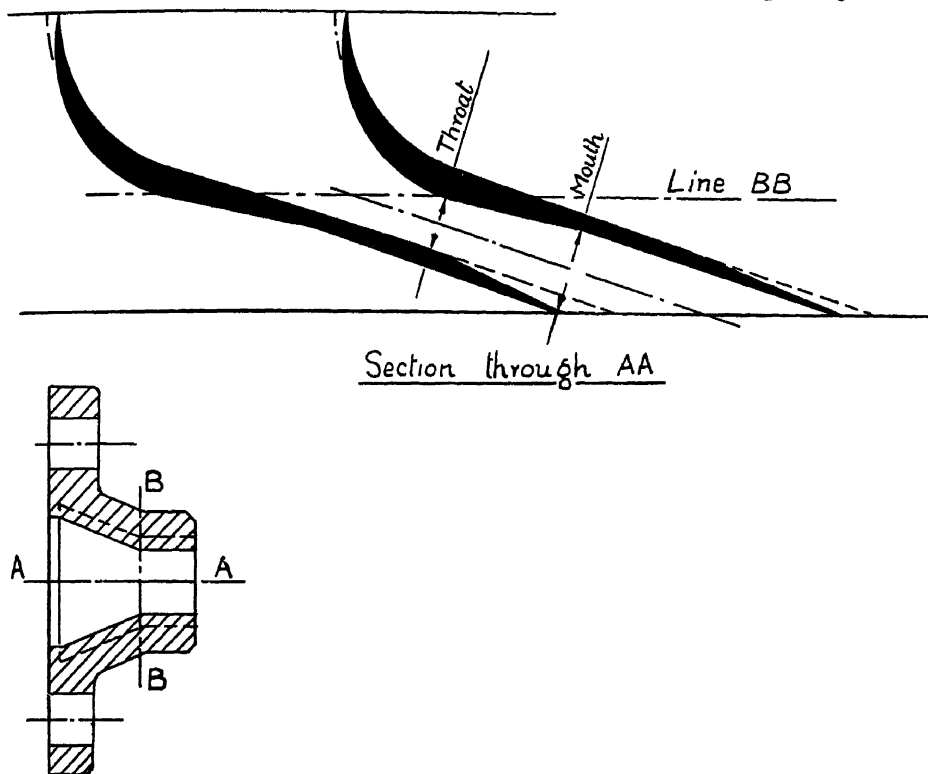


FIG. 163

through the nozzle is a helix, but at the outlet the particle is moving along the straight line indicated by the arrows in plan and end elevation. There is always a certain clearance c between the nozzle and the inlet edge of the blading, with the result that the blade reaches the position shown before the steam enters it. The distance

$x = \frac{c}{\sin \alpha}$ and is greater with small nozzle angles than with larger angles. Inspection of the figure shows—

(1) That the entry of the steam into the moving blade takes place at the angle ϕ to the tangent plane at the point of entry, such that

$$\tan \phi = \frac{x}{R} = \frac{c}{R \cdot \sin \alpha}$$

As a result there is a slight but somewhat indeterminate reduction in work done in the blades, together with secondary losses.

(2) If the radial height of the blade be the same as that of the nozzle, and if both are symmetrically placed about the line XX, a

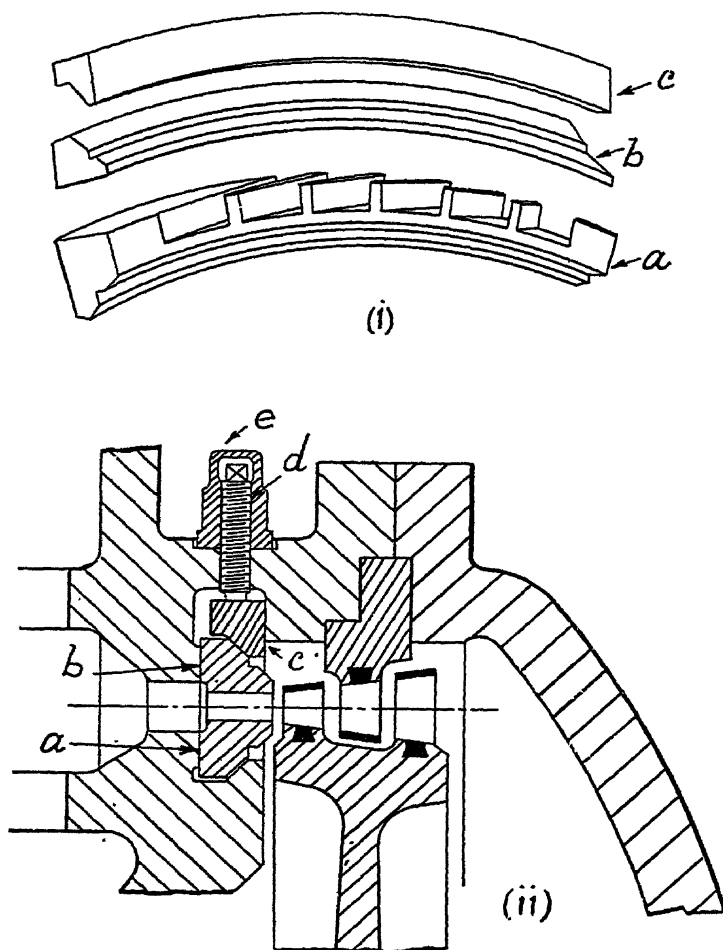


FIG. 164.—Built-up Nozzle by Messrs. Brown, Boveri & Co.

proportion $\frac{AB}{BC}$ of the steam flowing through the nozzles will never enter the blades at all, but will blow out into the space around the shrouding without doing any work in that row of blades. This loss may be easily avoided either by displacing the blade radially outward or, as is more usual, by making the radial height of the blades slightly greater than that of the nozzle (see also Art. 67).

The losses due to oblique entry may be minimized by laying down the nozzle section as shown in Fig. 166. For any given nozzle angle α , the blade will be in the position shown in plan when the particles of steam passing along the surface of the vane just reach the inlet edge of the blade. For normal entry, the trajectory of a particle of steam flowing along the axis of the nozzle must be tangential to the circle of (mean) radius R at the point reached by the

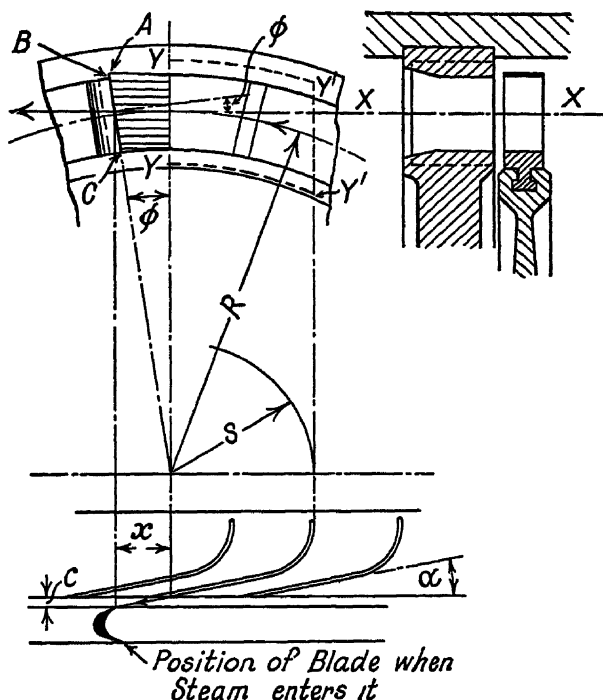


FIG 165

blade when the steam enters it, i.e. the axis of the nozzle must lie along the line DE. It is clear, then, that in a longitudinal section through the nozzle the axis will not now appear straight, but will be a curved line, curving away from the turbine axis when moving in the direction opposite to that of steam flow. The equation to the curve may be found as follows—

Consider a transverse section distant x from the inlet edge of the moving blades. The point A lies on the axis, and the distance $BA = x \cdot \cot \alpha$.

But

$$BA = ED \text{ (approximately)}$$

$\therefore$

$$ED = x \cdot \cot \alpha$$

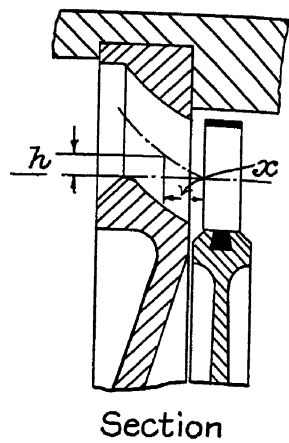
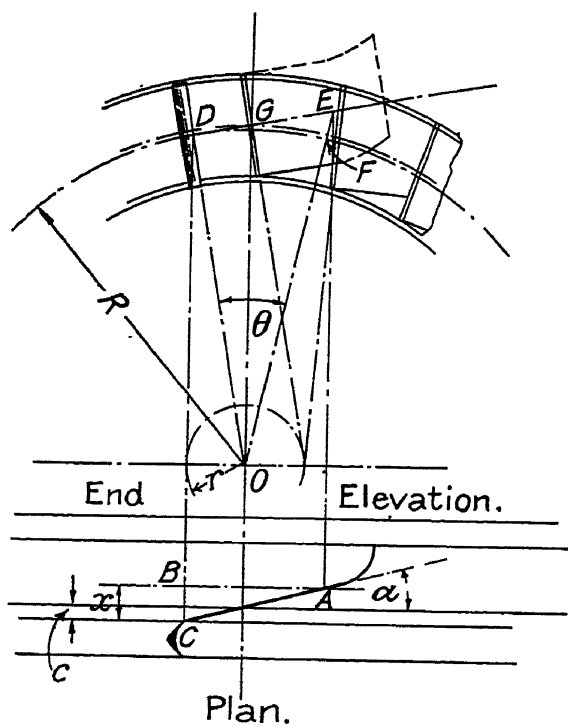


FIG. 166

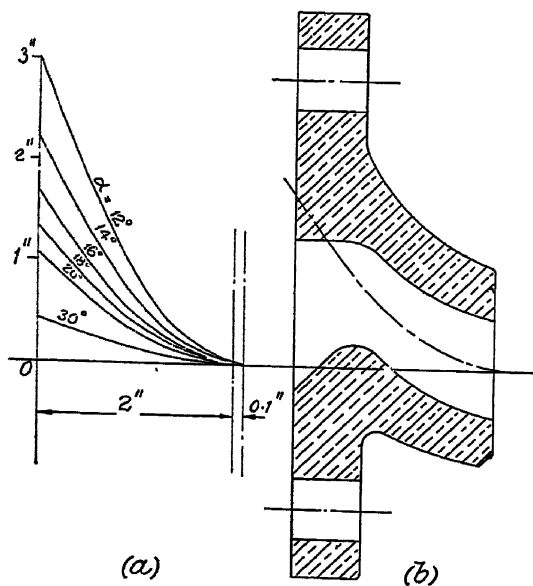


FIG. 167

Let h = height of centre line above the surface of a cylinder co-axial with the turbine and having a radius equal to that of the mean blade circle.

$$\begin{aligned}\text{Then} \quad h &= EF \\ &= OE - OF \\ &= \sqrt{(R^2 + x^2 \cdot \cot^2 \alpha)} - R\end{aligned}$$

Values of h have been calculated for a mean radius of 15 in., a clearance of 0.1 in., and various values of α . The results are shown in Fig. 167 (a). The axial length chosen, i.e. 2 in., is common in practice, and the modification for the usual angles is fairly considerable. In Fig. 167 (b) the actual longitudinal section of a 16° nozzle with cast-in guide vanes is shown.

CONSTRUCTION OF TURBINE BLADES AND BLADE ATTACHMENTS

140. Centrifugal Stress in Turbine Blades of Uniform Cross-section. Consider a turbine blade having a constant cross-sectional area A sq. in., a mean diameter D in., a length l in., and composed of material having a density w lb. per cub. in. Let the rotational speed

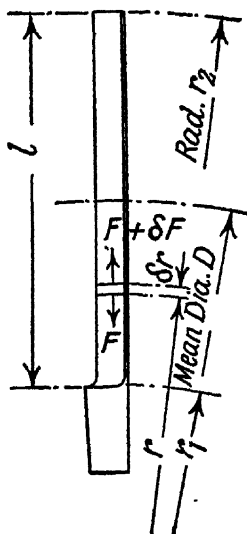


FIG. 168

be N r.p.m. and the corresponding angular velocity ω radians per sec.

Let F = internal force in blade at radius r (Fig. 168).

$F + \delta F$ = internal force in blade at radius $r + \delta r$.

Volume of blade element = $A \cdot \delta r$.

Centrifugal force on blade element = $\frac{wA \cdot \omega^2 \cdot r \cdot \delta r}{g}$

Then
$$F + \delta F + \frac{wA\omega^2 r \cdot \delta r}{g} = F$$

or
$$\delta F = - \frac{wA\omega^2 r \cdot \delta r}{g}$$

Then, if F_1 = internal force at root
and F_2 = internal force at tip, due to centrifugal force on shrouding—

$$\begin{aligned} F_1 - F_2 &= \frac{wA\omega^2}{g} \int_{r_1}^{r_2} r \, dr = \frac{wA\omega^2}{2g} (r_2^2 - r_1^2) \\ &= \frac{wA\omega^2 D l}{2g} \end{aligned}$$

If the stress $f_s = \frac{F_2}{A}$ due to the shrouding be calculated separately, then the stress due to the inertia of the blade alone is

$$f_c = \frac{F_1}{A} = \frac{w \cdot \omega^2 D l}{2g} \text{ lb per sq. in.}$$

Note that $g = 32.2 \times 12 \text{ in per sec.}^2$

For practical purposes, we have—

For steel blades, $w = 0.288 \text{ lb. per cub. in.}$

$$f_c = 4.09l \left(\frac{D}{100} \right) \left(\frac{N}{100} \right)^2 \text{ lb per sq. in.} \quad . \quad . \quad (156)$$

For brass blades, $w = 0.31 \text{ lb per cub. in.}$

$$f_c = 4.4l \left(\frac{D}{100} \right) \left(\frac{N}{100} \right)^2 \text{ lb per sq. in.} \quad . \quad . \quad (156a)$$

Writing $m = \frac{l}{D}$, and u for the mean blade velocity—

$$f_c = 0.215 m u^2 \text{ for steel blades} \quad . \quad . \quad (156b)$$

$$f_c = 0.231 m u^2 \text{ for brass blades} \quad . \quad . \quad (156c)$$

The resultant centrifugal stress at the root of the blade is equal to the sum of the stresses f_c and f_s , and is a uniform stress if the centroids of the blade sections lie on a radial line.

141. Bending Stresses in Symmetrical Impulse Blades of Uniform Cross-section. In addition to the centrifugal stress at the innermost blade section, there is a stress due to bending caused by the impulse. While this stress might not be large in itself, its effect on the maximum tensile stress must be considered in long blades.

It is first necessary to find the bending moment across the blade root. For this purpose, divide the nozzle and blade heights into any number, say four, equal parts, as shown in Fig. 169 (a) and (b), and calculate the steam flow through all the nozzles for each part. Let these be w_1, w_2 , etc., lb per sec. Now draw the velocity diagrams for each part of the blade, as shown in Fig. 169 (c), making the peripheral velocities u_1, u_2 , etc., equal to that at the mid-height of the part in question. In this way, the velocity changes V_1, V_2 ,

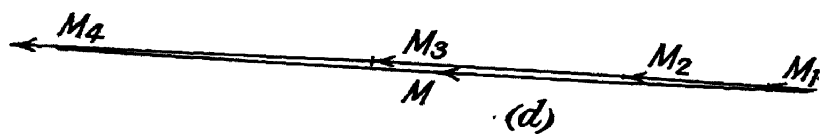
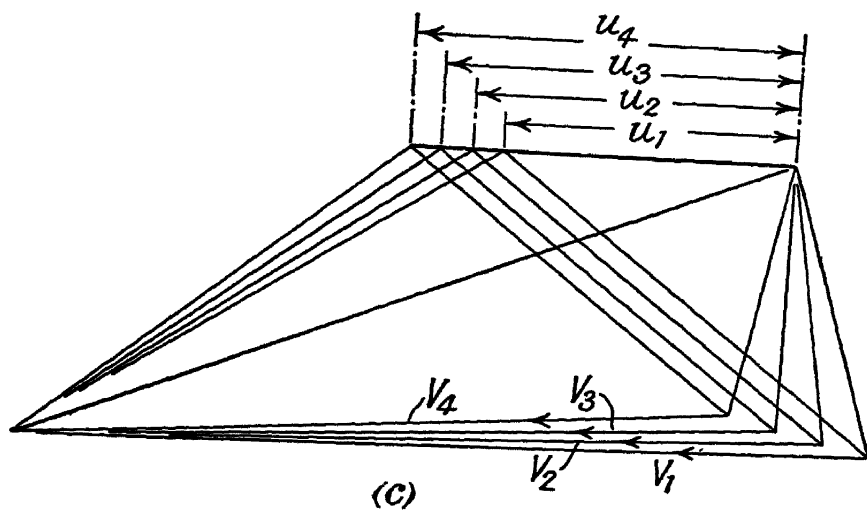
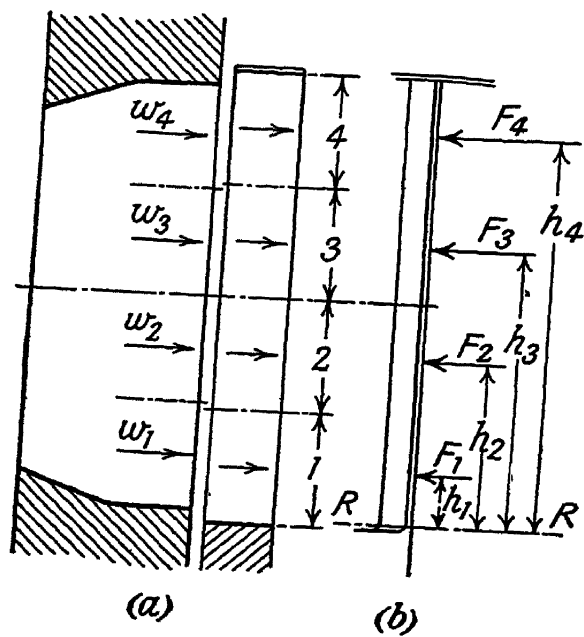


FIG. 169

etc., are obtained. Then if n is the total number of blades on the wheel, the impulses are equal to

$$F_1 = \frac{w_1 V_1}{gn}, F_2 = \frac{w_2 V_2}{gn}, \text{ etc}$$

The bending moments due to impulse are

$$M_1 = F_1 h_1, M_2 = F_2 h_2, \text{ etc.}$$

The resultant bending moment across R—R is then found vectorially as shown in Fig. 169 (*d*).

If the blades are profile blades constructed with unequal angles in accordance with Fig. 75, then the axis X—X, Fig. 170, is almost

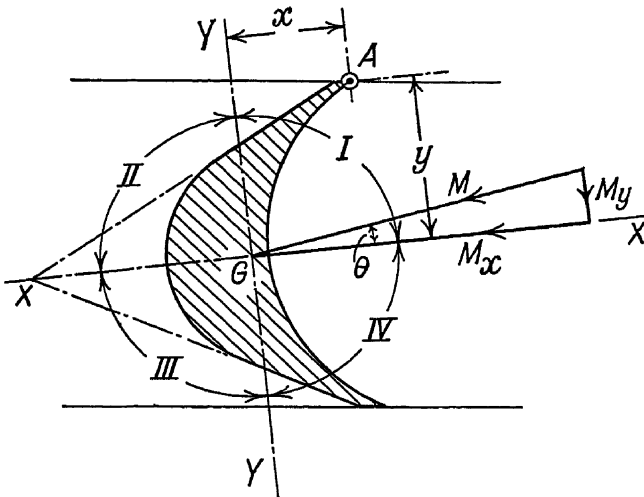


FIG 170

exactly an axis of symmetry. If G is the centroid of the blade section and YY is drawn perpendicular to XX, then XX and YY are the principal axes of the section. Suppose that the plane of M is inclined at the angle θ to XX, then M may be resolved into the two components M_x and M_y . It is obvious that M_x will produce tensile stresses in quadrants I and IV, and compressive stresses in II and III; while M_y will produce tensile stresses in I and II, and compressive stresses in III and IV. Hence, in general, the greatest tensile bending stress will occur at the point A and is equal to

$$f_b = \frac{M_x \cdot x}{I_y} + \frac{M_y \cdot y}{I_x}$$

Therefore the maximum tensile stress will also occur at A and equals $f_s + f_c + f_b$.

142. Graphics of Blade Sections. The position of the centroid is most easily obtained by the method shown in Fig. 171. The blade section is first drawn 10 or 20 times full size. The straight line XX is drawn through the end points of the horns as shown. Then, still considering the type of profile blade shown in Fig. 75, the line XX is perpendicular to the axis of symmetry. A straight line X_1X_1 is now drawn parallel to XX and at some convenient distance a from it. The position of X_1X_1 is quite arbitrary, but is preferably taken just outside the blade section. The drawing of the blade is now

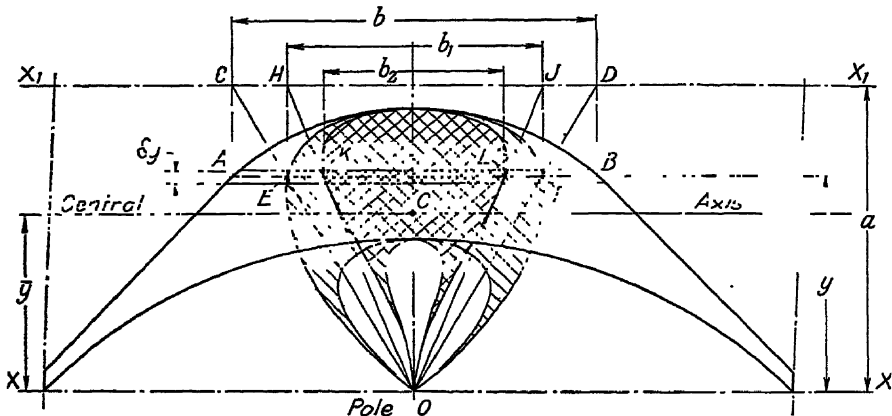


FIG. 171.—Graphical Determination of Position of Centroid and I_x

covered by a sheet of tracing paper and a number of lines such as AB are drawn parallel to XX'. Perpendiculars AC and BD are drawn and OC and OD joined, intersecting AB in E and F. The locus of points such as E and F is termed the "first derived figure."

Let A = area of blade section.

A_1 = area of first derived figure, shown by hatching.

Then $A_1 = \Sigma b_1 \cdot \delta y$, and since $b_1 = \frac{y}{a} \cdot b$

$$A_1 = \frac{1}{a} \sum b \cdot \delta y \cdot y = \frac{A \cdot \bar{y}}{a}$$

where $\bar{y}$ is the distance of the centroid G above XX.

Hence $\bar{y} = \frac{a \cdot A_1}{A}$

The second moment of area I_x about the axis XX may be found by an extension of the same method. Perpendiculars EH and FJ are drawn and OH and OJ are joined, intersecting AB in K and L .

The locus of points such as K and L is then drawn. This is termed the "second derived figure." Let A_2 be its area.

$$\text{Then} \quad A_2 = \Sigma b_2 \cdot \delta y$$

$$\text{But} \quad b_2 = \frac{y}{a} \cdot b_1 = \frac{y^2}{a^2} \cdot b$$

$$\therefore \quad A_2 = \frac{1}{a^2} \Sigma b \cdot \delta y \cdot y^2 = \frac{I_x}{a^2}$$

$$\therefore \quad I_x = a^2 \cdot A_2$$

The second moment of area about the central axis, viz. I_{CG} , may then be found by the well-known theorem—

$$I_{CG} = I_x - A \cdot \bar{y}^2$$

This result is, however, likely to be considerably in error, because the value of $A\bar{y}^2$ is commensurate with that of I_x , and a small error in the value of I_x leads to a large error in I_{CG} .

The value of I_{CG} may be found with sufficient accuracy by drawing the first and second derived figures with reference to the central

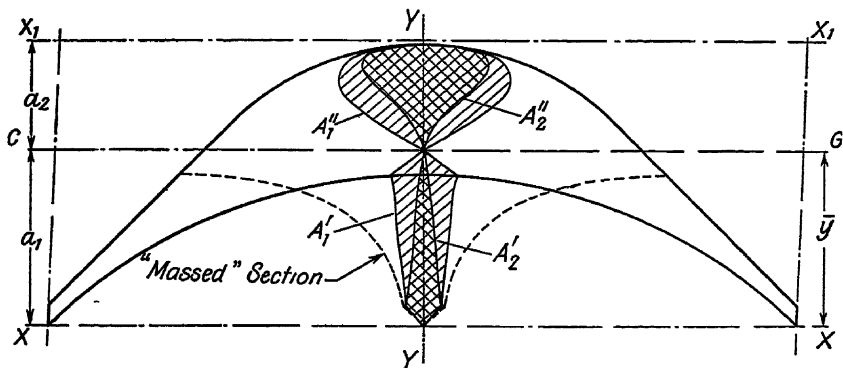


FIG. 172 — Graphical Determination of I_{CG}

axis after this has been determined. The method is illustrated by Fig. 172. Two lines XX and X_1X_1 are drawn parallel to CG at distances a_1 and a_2 from it. The original straight line XX may be conveniently used. In order to obtain a more manageable diagram, the lower part of the section is "massed" about the axis YY as shown by the dotted lines. The first and second derived figures are drawn by the method described above. Let A'_2 and A''_2 be the areas of these latter

$$\text{Then} \quad I_{CG} = A'_2 \cdot a_1^2 + A''_2 \cdot a_2^2$$

Where required, the second moment of area about the principal axis YY , Fig. 173, may be found by the same method. The section

is preferably massed about a line X_1X_1 . In general, the component M_v of the bending moment (see Fig. 170) is so small that the stress due to it may be neglected.

When the impulse blade has no axis of symmetry, the principal axes must be found by the method given in the next article.

143. Bending Stresses in Reaction Blades. Owing to the fact that there is a drop in steam pressure in the channels of the rotating blades of a reaction steam turbine, there is an axial thrust on the blades due to steam pressure, in addition to the dynamic thrust due

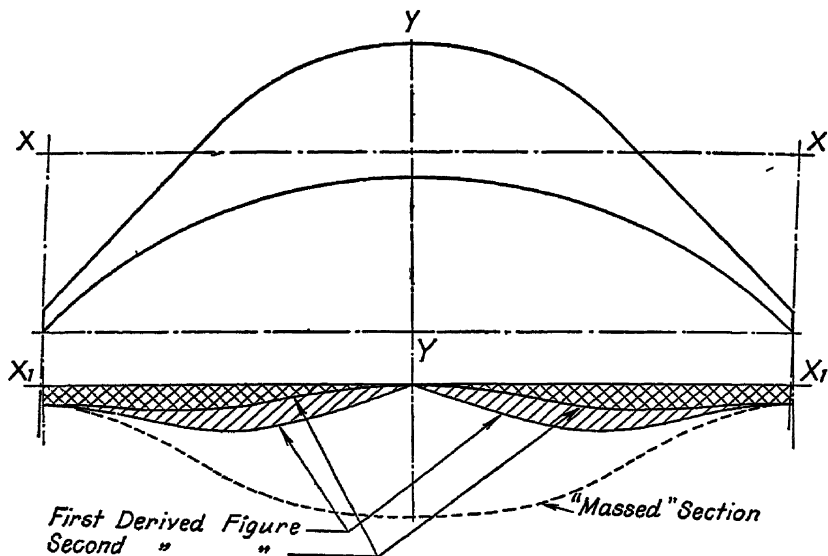


FIG. 173.—Graphical Determination of I_y

to change of momentum. With the usual half-degree reaction, the pressure drop across the moving blades is practically equal to that across the stationary blades in any given pair. Consequently, in a group or "expansion" comprising s pairs of blades or stages, if p_1 and p_2 are the steam pressures at inlet to and outlet from the group, the pressure drop across any ring of blades is given, approximately, by the relation

$$\Delta p = \frac{p_1 - p_2}{2s} \quad (157)$$

Let n = number of blades in one ring.

θ = angle subtended by one blade (Fig. 174).

$$\theta = \frac{2\pi}{n} \text{ radian.}$$

Then area of shaded element = $r \cdot \theta \cdot \delta r$.

Axial force on shaded element = $\Delta p \cdot r \cdot \theta \cdot \delta r$.

Moment of axial force about blade root

$$\begin{aligned} &= \Delta p \cdot r \cdot \theta \cdot \delta r (r - r_1) \\ &= \Delta p \cdot \theta (r^2 - r_1 r) \delta r \end{aligned}$$

$\therefore$ Total moment due to static pressure—

$$\begin{aligned} M_s &= \theta \cdot \Delta p \int_{r_1}^{r_2} (r^2 - r_1 r) \cdot dr \\ &= \theta \cdot \Delta p \left\{ \left(\frac{r_2^3 - r_1^3}{3} \right) - \frac{r_1}{2} (r_2^2 - r_1^2) \right\} \\ &= \frac{\theta \cdot \Delta p \cdot l^2}{6} (2r_2 + r_1) \end{aligned}$$

or

$$M_s = \frac{\pi \cdot \Delta p \cdot l^2}{3n} (2r_2 + r_1) \quad . \quad . \quad . \quad (158)$$

Note. If the dimensions are in inches and Δp is in lb. per sq. in., M_s will be a moment in lb.-in. units.

The bending moment M_d due to dynamic forces is found by the

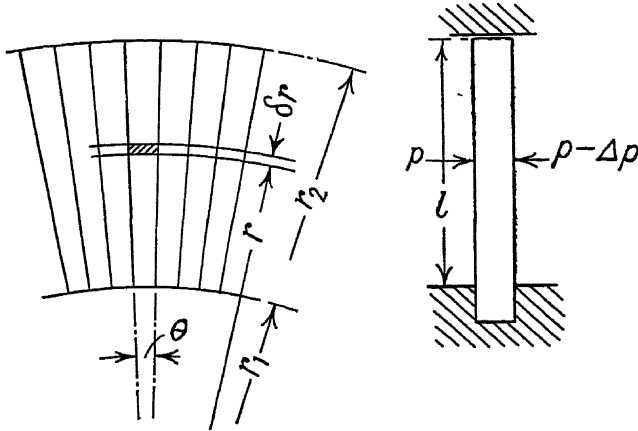


FIG 174

method given in Art. 141 for impulse blades. The plane of this moment is nearly transverse to the turbine axis.

The moments M_s and M_d are now combined by the vector diagram, Fig. 175, to give the resultant moment M , the plane of which is perpendicular to the moment vector. It is possible that the plane of M might not pass through the centroid, in which case there will be, in addition to the bending action, a torque on the blade. It will, however, be assumed that such torque, if any, is negligibly small. Let UU and VV be the principal axes of the section. Then

M may be resolved into the components $M_u = M \cdot \cos \phi$ and $M_v = M \cdot \sin \phi$. The tensile stresses at A and B, due to bending only, are then—

$$f_b \text{ at A} = \frac{M_u \cdot u_A}{I_v} + \frac{M_v \cdot v_A}{I_u}$$

$$f_b \text{ at B} = \frac{M_u \cdot u_B}{I_v} - \frac{M_v \cdot v_B}{I_u}$$

The resultant stress is obtained by adding the centrifugal stress f_c to the bending stress f_b .

The reaction blade section has no axis of symmetry; consequently, the principal axes of the section must be found by one or

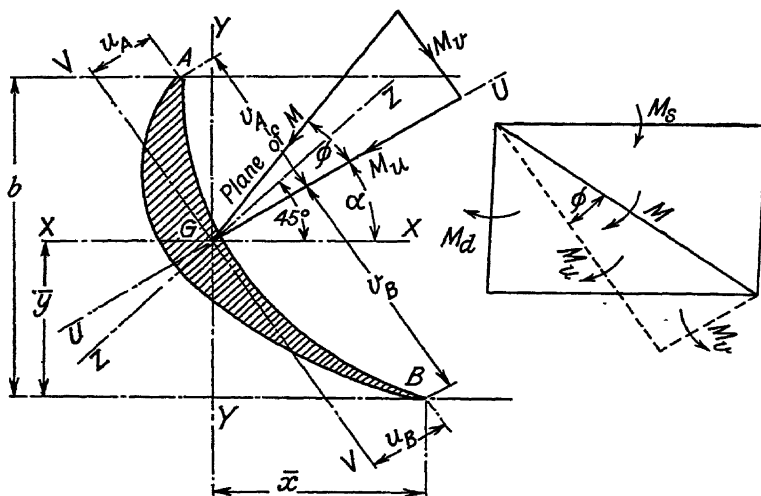


FIG. 175.—Bending Moments on a Reaction Blade

other of the methods available. As graphical integration is most easily effected the following is probably the simplest method. The centroid G is first found by the method given in Art. 142. The axes XX, YY, and ZZ are then drawn through G, XX and YY being at right angles to each other and ZZ making an angle of 45° with XX. Then if UU and VV are the principal axes of the section, UU makes an angle α with XX, such that—

$$\tan 2\alpha = \frac{2I_{zy} - (I_x + I_y)}{I_x - I_y}$$

Also $I_u = \frac{1}{2}\{(I_x + I_y) + (I_x - I_y) \sec 2\alpha\}$
 and $I_v = \frac{1}{2}\{(I_x + I_y) - (I_x - I_y) \sec 2\alpha\}$

In order to avoid shock losses due to incorrect entry, the blade inlet angle is frequently varied along the length of the blade, the actual blade inlet angle being made to agree, as nearly as possible, with the diagram angle at any radius.

Furthermore, the stress at the innermost section of such long blades would often be excessive were the area of the blade section made uniform throughout, and in order to reduce the root stress the blade is tapered, either in width or thickness, or both. Such a blade is shown in Fig. 176, which is diagrammatic. The centroids of the various cross-sections should, if possible, lie on a straight line which is both radial and perpendicular to the axis of the turbine. In this way any bending moment due to inertia forces is eliminated. By setting the line of centroids a little forward of the radial line the bending moment due to the inertia forces may be utilized partially to counteract the bending moment due to impulse. Blades of the type shown in Fig. 176 are sometimes drop-forged, a small allowance being left on for finishing. In other cases they are machined.

The cross-sectional area of tapered blades may be varied in such a way that the centrifugal stress is uniform over the greater part of the blade length. Using the same symbols as in Art. 140, let A be the cross-sectional area of the blade at radius r , and $A + \delta A$ the cross-sectional area at radius $r + \delta r$. Suppose that f is the uniform centrifugal stress.

The inertia force on the element is

$$\delta F = - \frac{w \cdot A \cdot \omega^2 \cdot r \cdot \delta r}{g}$$

Then since $F = fA$, and $F + \delta F = f(A + \delta A)$

$$\delta F = f \cdot \delta A$$

$$\therefore \delta A = - \frac{w \cdot A \cdot \omega^2 \cdot r}{gf} \cdot \delta r$$

$$\text{or} \quad \frac{\delta A}{A} = - \frac{w \cdot \omega^2 r}{gf} \cdot \delta r$$

$$\therefore \int_{A_2}^{A_1} \frac{1}{A} \cdot dA = \frac{w \cdot \omega^2}{gf} \int_{r_1}^{r_2} r \cdot dr$$

$$\log_e \frac{A_1}{A_2} = \frac{w\omega^2}{2gf} (r_2^2 - r_1^2)$$

$$\text{i.e.} \quad \log_{10} \frac{A_1}{A_2} = \frac{0.4343 \cdot w \cdot \omega^2}{2gf} (r_2^2 - r_1^2). \quad (159)$$

In using this formula, one must adopt consistent units; e.g. if r_1 and r_2 are expressed in inches, then $g = 32.2 \times 12$, w for steel = 0.288 lb. per cub. in., and the stress f would be in lb. per sq. in.

145. General Notes on the Attachment of Turbine Blades. There is probably no detail in the construction of a steam turbine which affects the reliability of the machine more than the attachment of the blades to the rotor. The forces brought to bear on the attachment are relatively large, especially at the low-pressure end of turbines of large output. In turbines running at 3,000 r.p.m., the ratio of the centrifugal force to weight is $255r$, where r is the radius

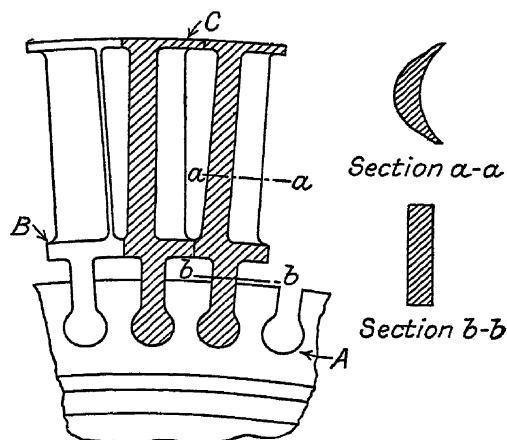


FIG 177

in inches. Thus if the peripheral velocity is, say, 700 ft per second, the radius is 26.75 in, and the centrifugal force on the blade is no less than 6,820 times its weight

It is often difficult and occasionally impossible to estimate the stresses in the attachment. Approximate calculations are possible in many cases, but there is always the possibility of stress concentrations at sharp corners. In this connection, the use of a blade material having a definite yield point would seem to offer some safeguard. For these reasons the calculated stresses should be kept reasonably low.

The form of the attachment has sometimes an influence on the shape of the wheel rim and a particular design of root may involve a fairly heavy rim, thus adding to the stresses in the disc (or, alternatively, requiring a heavier disc) and possibly increasing the length of the turbine. As a general principle, the form of the attachment should be such that the centrifugal force on the blade is transmitted to the disc in the simplest and most direct manner. Furthermore, the design of the attachment should preferably be such that the security of the attachment is, so far as possible, independent of the skill of the mechanic.

146. De Laval Blade Attachment. This is illustrated in Fig. 177. A number of holes A are drilled in the rim of the wheel parallel to the axis. The slot for the blade tang is then finished by milling. The

transition from the working section of the blade (section $a-a$) to the rectangular tang (section $b-b$) is obtained by the rectangular part-B, which forms a continuous ring when all the blades have been inserted in the wheel. The tips of the blades are provided with integral shroudings C, which are carefully fitted so that these also form a continuous ring. The centroid of the blade section is arranged in the same radial line as that of the blade tang, so that the latter is practically free from bending stresses. The bending moment due to impulse is almost negligible. Light caulking of the wheel rim is sufficient to hold the blades in place.

147. Inverted-T Attachment. Various forms of this are shown in Fig. 178. (a) shows a blade with a separate spacer. This type of construction enables the blades to be cut from bars or strip of blade section formed either by rolling, drawing through dies, or by machining. The amount of machining necessary to finish the blade is small and the construction is relatively inexpensive. The spacers are shown in Fig. 178 (b). They are likewise cut from section bar and finished by milling. The leading edge of the spacer is given a taper, as shown, so that the blades will be set radial. The spacer stands proud of the rotor by a distance s . This facilitates the diaphragm construction and removes the section XX where the bending stresses are a maximum from the section YY where there is apt to be a concentration of stress at the points A.

The centroid of the section ZZ does not lie in the same radial line as the centroid of the blade section. Consequently the section is subjected to bending as well as direct tension. When the stress across section ZZ becomes excessive the blade and spacer are made from one piece, as shown in Fig. 178 (c). Such blades must be machined from bar of rolled "rhubarb" section, and are more costly to produce than the blades with separate spacers. In order to avoid the concentration of stress which is bound to occur at the points A, Fig. 178 (a) and (c), the blade tang is frequently tapered, as shown in Fig. 178 (d). The construction adds to the depth of the root, increases the cost of machining both blade and rotor, and also necessitates a somewhat heavier rim.

In long blades at moderate peripheral speeds, or blades of moderate length at high peripheral speeds, the resultant stress in the tang due to bending and direct tension is sometimes excessive when the root is constructed as shown in Fig. 178 (c). The bending stress may be practically eliminated by adopting the construction shown in Fig. 179. The root section is displaced circumferentially so that the resultant moment about the centroid G of the tang section due to the centrifugal force on the blade and on the part of the root above the tang is zero. Both the concave and convex parts of the root may be tapered with respect to the centre-line of the blade drawn radially through the centroid.

148. Stresses in Wheel Rims. The type of wheel rim commonly

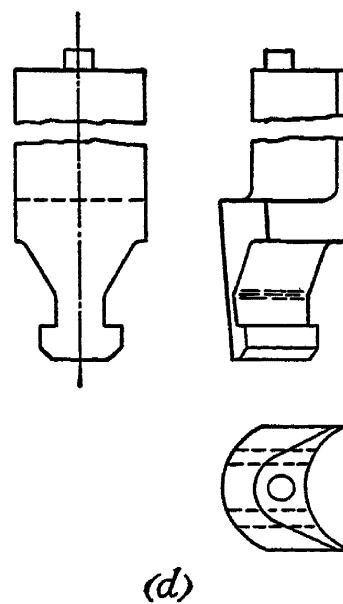
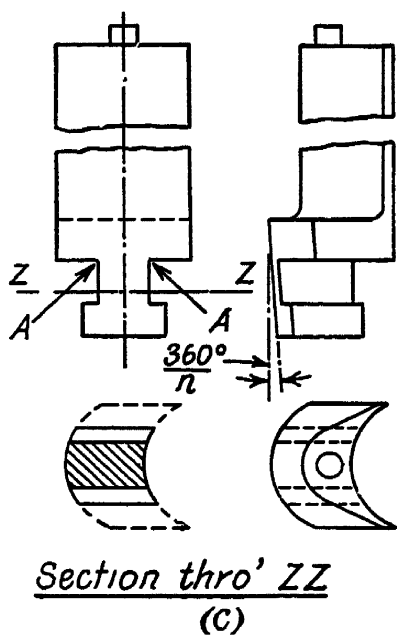
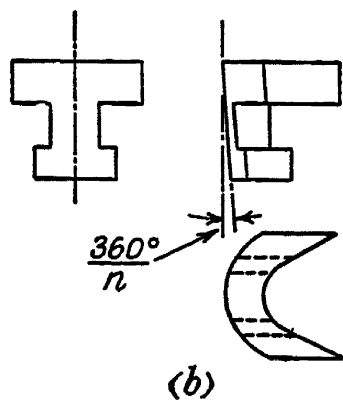
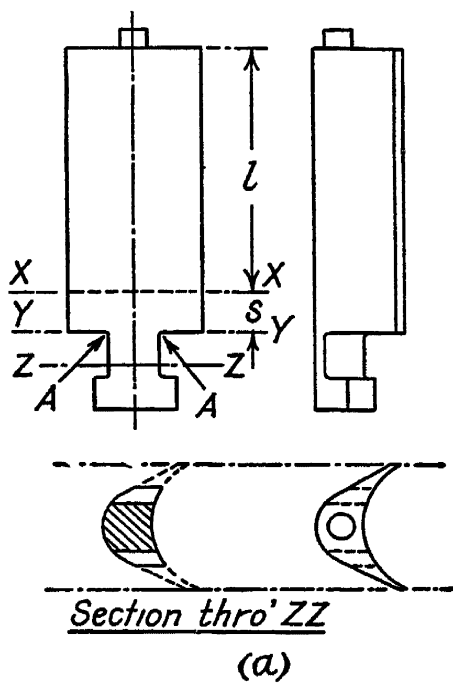


FIG 178

used with the inverted-T form of blade fastening is shown in Fig. 180 (a). The width of the wheel rim is an important dimension as it affects the stresses in the disc proper and, in certain cases, the

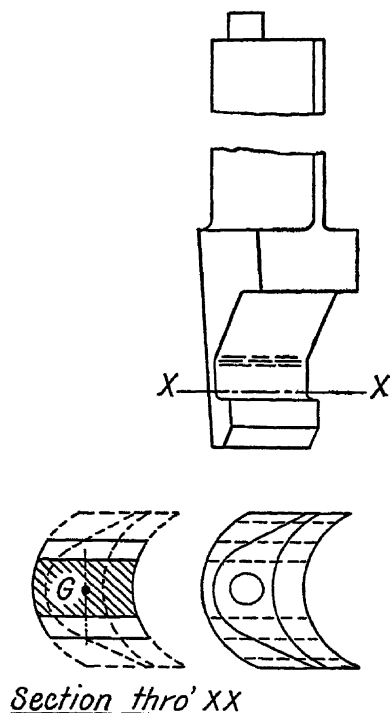


FIG. 179

overall length of the turbine. It depends on the dimension d , which may be found approximately in the following manner.

Consider the section AB on one side of the wheel rim and corresponding to one blade. This section has a circumferential width b and an axial depth d . Let G be the centroid of the section and F the total radial force on the shrouding, blade, and blade root. This force is transmitted to the rim as two equal forces $\frac{F}{2}$. By applying equal and opposite forces $\frac{F}{2}$ at the centroid G it is seen that across the section AB there are—

- (1) A force $\frac{F}{2}$ producing a uniform tensile stress equal to $\frac{F}{2bd}$.
- (2) A bending moment $\frac{Fa}{2}$ producing at B a tensile stress equal to $\frac{3Fa}{bd^2}$.

The resultant tensile stress at B is therefore

$$f = \frac{F}{2bd} + \frac{3Fa}{bd^2} \quad (160)$$

from which the value of d to limit the stress f to any given value may be found.

A little consideration will show that eq. 160 for the stress must err on the high side, because the strain which would accompany the

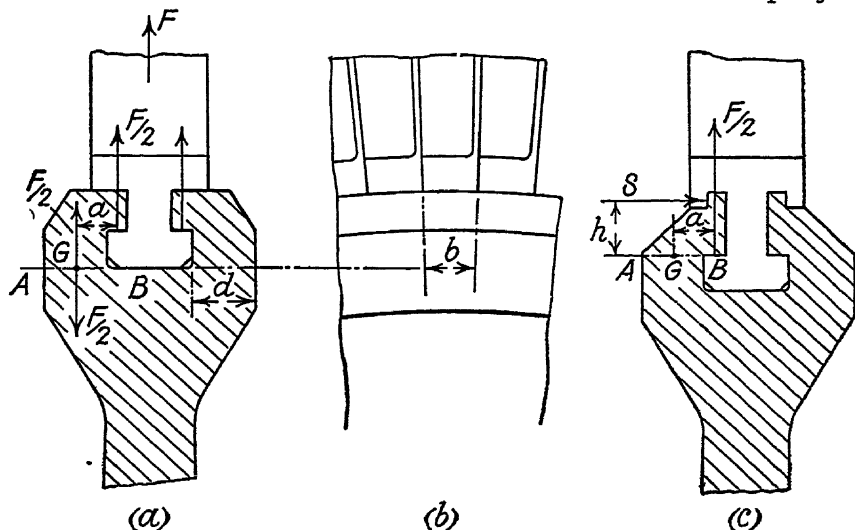


FIG 180

bending would cause the jaws to open out slightly. But if the jaws were to open out, the circumference of the outer parts of the rim would increase and this would induce hoop stresses in the rim (over and above the usual hoop stress) which would definitely restrict the amount of bending possible.

When the width of the rim becomes excessive, the form of attachment known as the "side-grip" and shown in Fig. 180 (c) may be used. It will be clear that the tendency for the rim to open under the action of the forces $\frac{F}{2}$ will be opposed by a shearing force S across each of the snugs on the blade spacer. As a result, the bending moment across AB is reduced to some value

$$M = \frac{Fa}{2} - Sh$$

149. Serrated Blade Root. Fig. 181 shows a form of attachment suitable for fairly large centrifugal forces. The blade and spacer must be made in one piece. Serrations are machined on both sides of the root and also in the blade groove. The depth of the serrations is about $\frac{2}{3}$ times the width of the groove and the angle 45° to groove.

150. **Fitting of Blades in Wheel.** In all the foregoing types of fastening—with the exception of the de Laval—it is necessary to

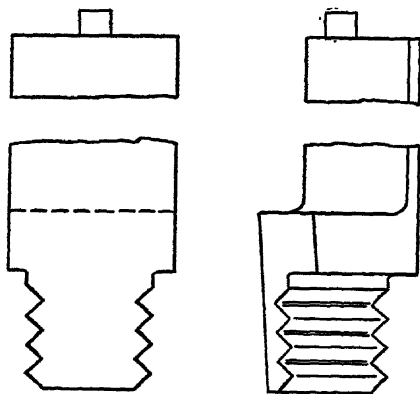


FIG. 181

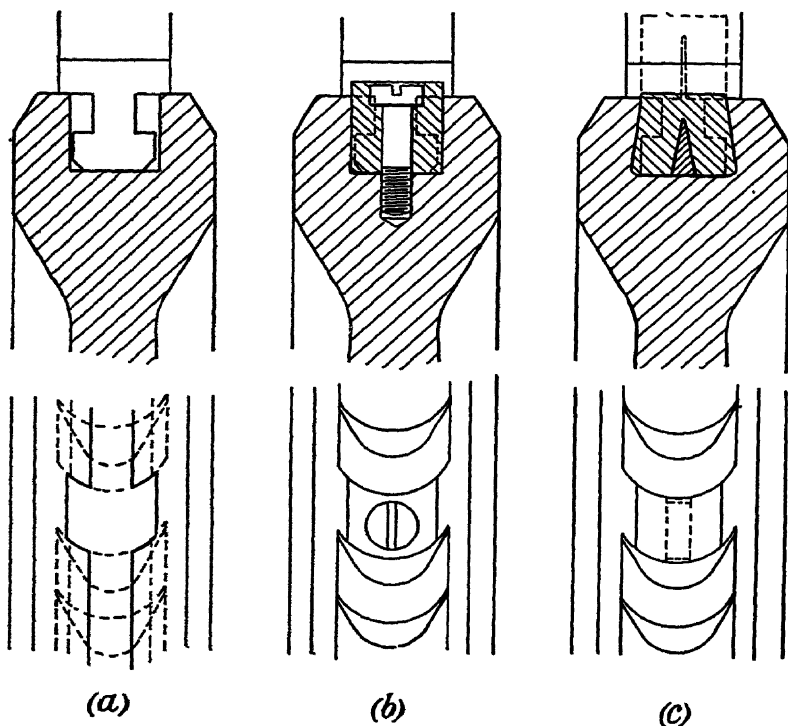


FIG. 182

cut an opening in the wheel rim for the insertion of the blades, as shown in Figs. 182 (a) and 182A. The blades are then inserted and driven round into place. The blades are driven hard up against a

stop inserted and wedged in the blade groove. When the blade ring is almost completed the stop is removed and a blade or two fitted in its place. The last one or two blades are carefully fitted to occupy the whole of the space except the opening. This is then filled.

Two of the several methods of filling the gap are shown in the figure. Fig 182 (b) shows a filling-up piece secured by a counter-sunk cheese-headed screw which is prevented from slacking back

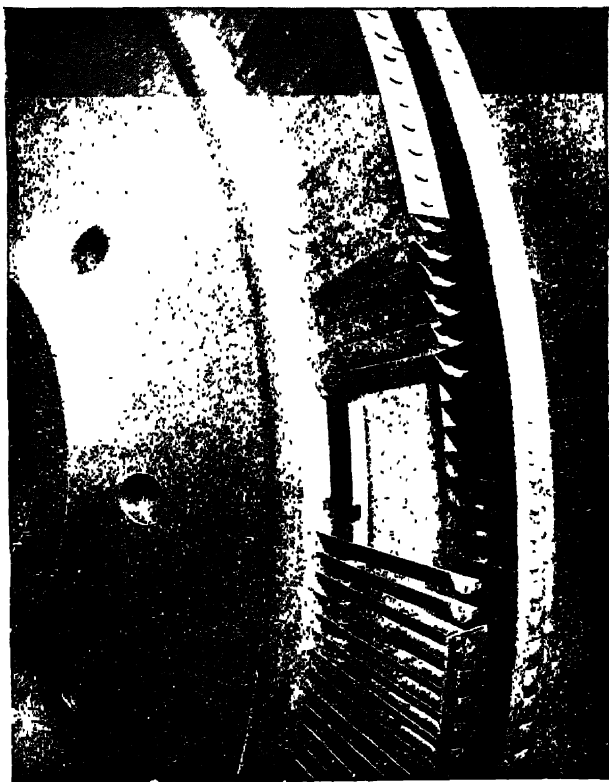


FIG 182A

by bruising the head of the screw or caulking the filling-up piece. The hole for the screw weakens the rim as it breaks the continuity in the circumferential direction. In Fig. 182 (c) the gap is cut with a taper. A filling-up piece of soft brass, copper, or mild steel is cut to the shape shown by the dotted lines and nicked on the under side. This is then driven on to the steel wedge, which forces the filling-up piece to fill the gap completely. When either of these or similar methods is used there is, of course, a blade missing at the gap. By rather more elaborate devices it is possible to insert a special blade at the gap and so preserve a complete ring of blades.

151. Blade Attachment for High-pressure Curtis Wheel. An interesting form of construction employed on a 36,000 kW turbine built by Messrs. Brown Boveri & Co., Ltd., is described in a report of a test by Professor E. Josse (1), and illustrated in Fig. 183. The turbine operates with steam at 1,700 lb. per sq. in. and 900° F. The rotor blades on the Curtis wheel are milled out of one piece

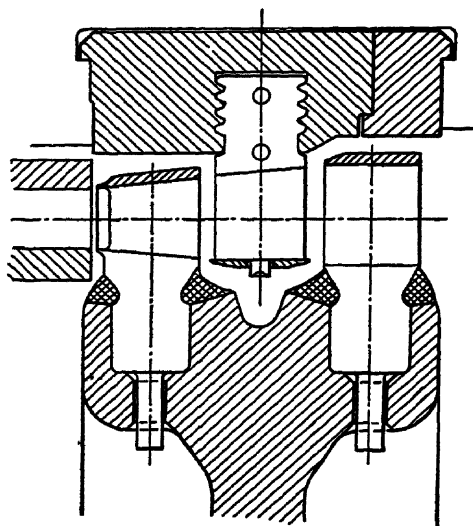


FIG. 183

along with the spacing and shrouding element, and have a very heavy profile. They are riveted into the disc and are welded together in pairs at the shrouding and are also welded to the rim of the wheel as shown. The advantages of the construction are: (1) The whole construction is very rigid; (2) it is not necessary to have a gap in the rim for the insertion of the blades; (3) the blade and wheel reach the same temperature quickly; and (4) there is no risk of the blades becoming loose in the wheel.

152. Straddle Attachment. Two forms of straddle attachment are shown in Fig. 184. For moderate speeds and short blades the type shown in Fig. 184 (a) is employed. The blade and spacer must be machined from one piece of metal. The blade is forked and fits tightly on to a projecting tongue on the wheel. The rivets are staggered, approximately one-half of each rivet being embedded in the blade, the effective attachment being one rivet in double shear. The rivets are slightly countersunk. When the shear stress in the rivets is excessive, as with higher speeds or longer blades, the attachment shown in Fig. 184 (b) is used, the rivets then being in quadruple shear. The straddle attachment enables a complete ring of blades to be employed as there is no necessity to cut any gap for

the insertion of the blades. Moreover, if the necessity should arise, one or more blades may be removed from the wheel without disturbing the remaining blades.

153. Modified Straddle Attachment. An attachment which is used in the last two or three stages of turbines of large output is shown in Fig. 185. It is really an inversion of that shown in Fig.

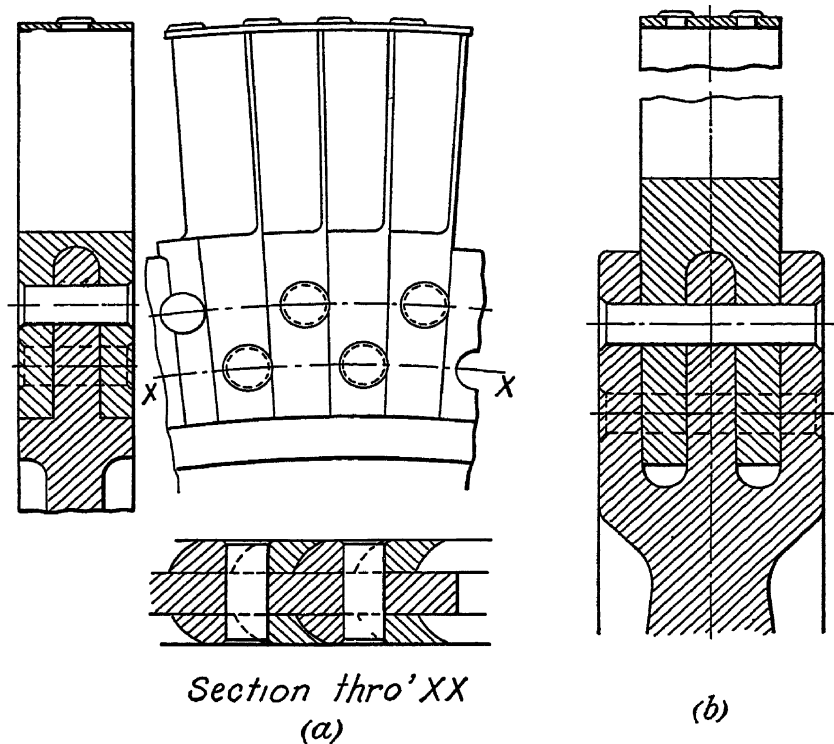


FIG 184 —Straddle Blade Attachments

180 (c) The periphery of the wheel is provided with a T-shaped section. The blade root is widened, as shown, and provided with two small snugs which prevent the fork from opening out. A shearing force S will be induced in each prong and the resultant moment across the section XX will be

$$M = \frac{Fa}{2} - S \cdot h$$

It is necessary to cut slots, shown by the dotted lines, in the T-head for the insertion of the blades, a blade being omitted at the slot. Suitable spacers, fitting tightly between the blades on either side of the slot, are fastened to the rim. By cutting two such slots at

diametrically opposite points of the wheel the balance of the wheel is not affected.

154. Side Entry Blades. These have been employed for very high peripheral speeds. The general features of the attachment are shown in Fig. 186. The rotor carries a comparatively heavy rim in which closely-spaced slots are cut, one to each blade. The slots

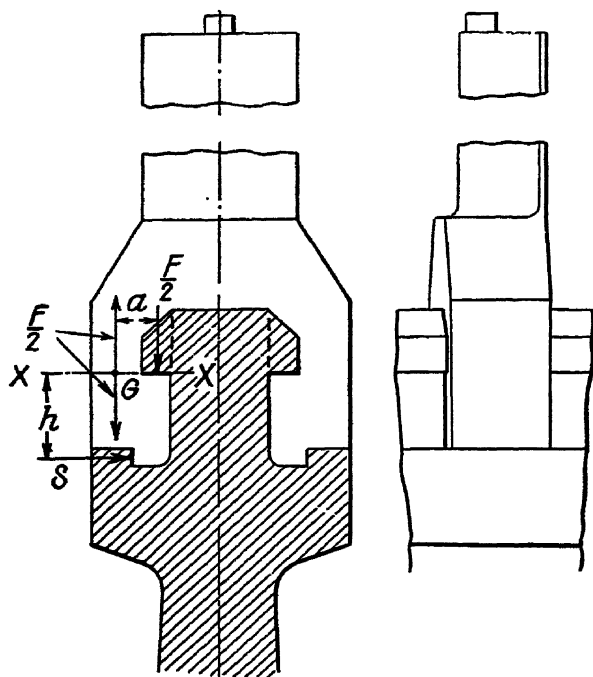


FIG. 185.—Modified Straddle Attachment

may be either parallel or skew to the axis, depending on the shape of the blade root profile. The design sometimes incorporates a principle which may be termed the principle of diminishing cross-sections. Considering the blade only, the area of the material under tension is made a maximum at section 1, where the tension in the root is the maximum. Part of the pull is taken by the outermost teeth, so that a smaller area may be employed at section 2. The teeth in line 2-2 take a further share of the pull so that the area across 3-3 may be still further reduced. On the same principle, the areas of the rim sections diminish from section 3-3 outwards. The blade root is lightly caulked to prevent it moving axially. The advantages of this form of attachment are: (1) The width of the wheel rim may be made greater than the width of the blade, so reducing the maximum stress in the fastening; (2) the bending across section 1-1 may be more or less eliminated by a suitable

positioning of the blade section with respect to section 1-1, (3) a complete ring of blades is obtained, and (4) the renewal of any blade is a relatively simple matter. The construction calls for excellent workmanship and is obviously expensive.

155. Attachment of Shrouding Strip. As the shrouding strip is attached to the blades at the extreme radius, the radial acceleration

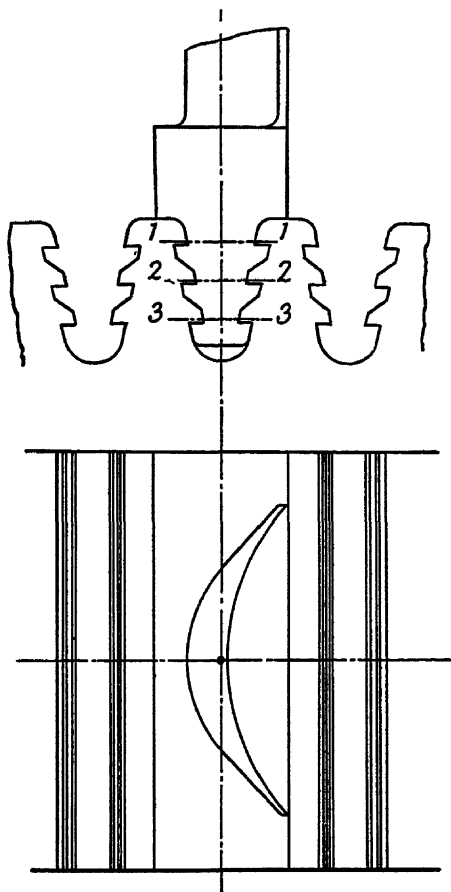


FIG 186 —Side Entry Attachment

is the maximum. Consequently, the centrifugal force is often very large and the thickness of the shrouding strip must be chosen with reference to the bending stress, the shrouding being in the condition of a continuous beam on several supports and carrying a uniformly distributed load. Typical attachments are shown in Fig. 187; (a) shows a round pin machined on the end of the blade, the centre being as near to the centroid as possible. In Fig. 187 (b) the "pip"

for the shrouding is usually formed by milling. Two types are shown. The holes in the shrouding strip are carefully punched at the correct distance apart. In the long tapered and twisted blades described in Art. 144, the outermost section of the blade is often so thin that the attachment of the shrouding is a matter of some difficulty. It

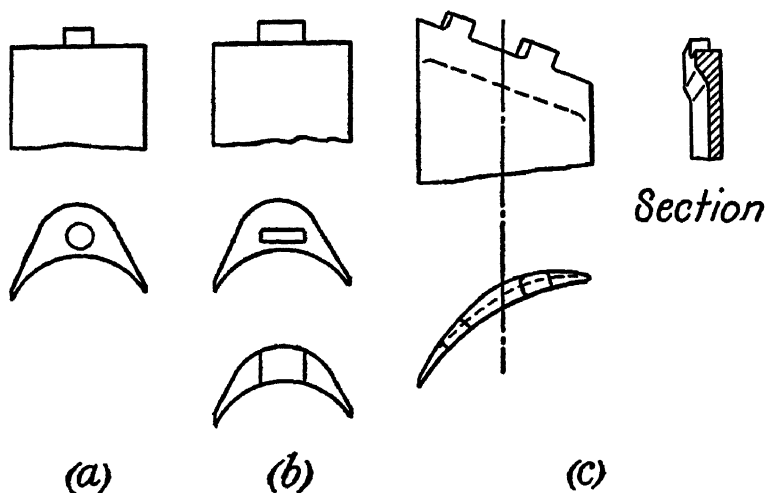


FIG. 187

is necessary to use two projections and to thicken up the blade end, as shown in Fig. 187 (c). The section is then gradually varied so as to avoid stress concentration.

156. Attachment of Parsons Reaction Turbine Blades. The Parsons turbine has passed through an orderly evolution since its invention in 1884, and many forms of blade fastening have been used. During the period of moderate steam pressures and temperatures, the blades were usually made of drawn brass, the spacers being made of soft brass. The blades were drawn through dies of standard section and fitted into grooves of standard width.

Fig. 188 (a) shows a cross-section through two pairs of reaction blades. Prior to 1912, the blades of Parsons turbines were open at the tips, no form of shrouding being used. To allow for the difference in expansion of the rotor and its blades and that of the turbine cylinder, and also to provide a certain "running clearance," a tip clearance c is arranged as shown in Fig. 188 (a). In consequence of the drop in steam pressure across each ring of blades, there is a certain leakage of steam through the clearance spaces between the fixed blades and the rotor, and between the moving blades and the cylinder. This leakage adversely affected the efficiency of the Parsons turbines of that time.

In order to minimize the rubbing pressure and the heat generated

in the event of accidental contact of the blades with the cylinder or rotor, the blade tips are thinned down, as shown in Fig. 188 (b). For the purpose of bracing the blades together, preserving the correct circumferential pitch, and preventing vibration, binding strip A is inserted into grooves machined in the inlet edge of the blade

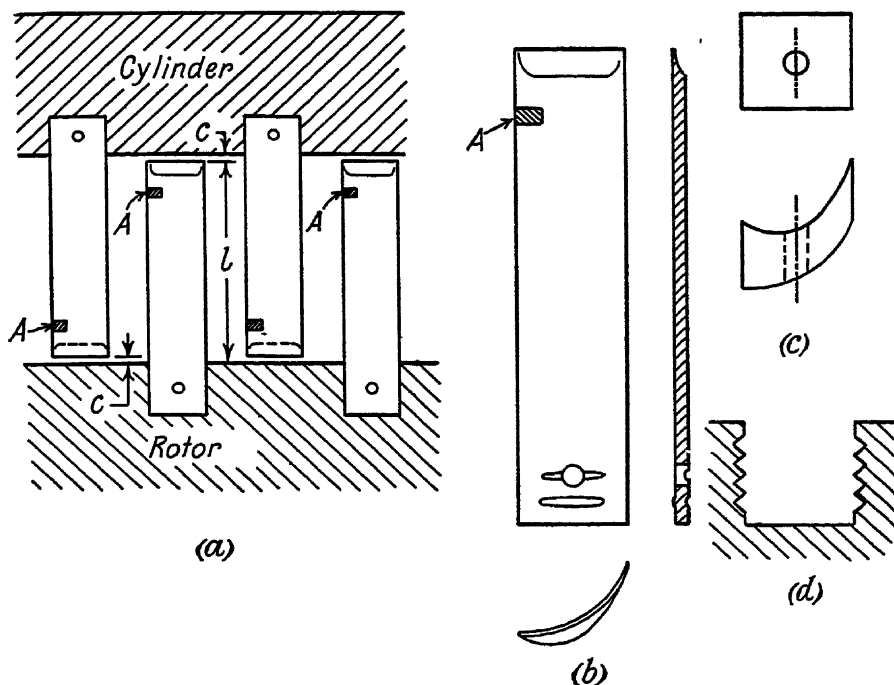


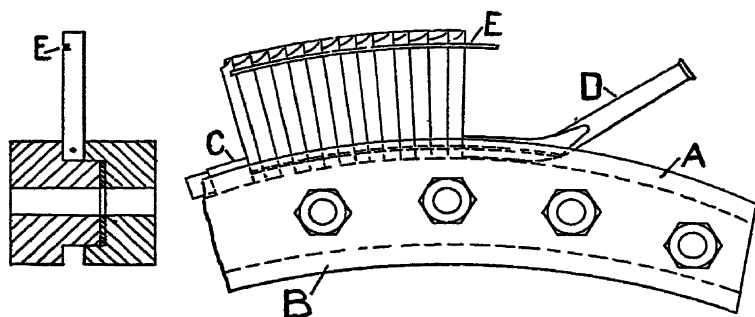
FIG 188

and silver-soldered to the blade. In long blades two, or three, rings of binding strip are used.

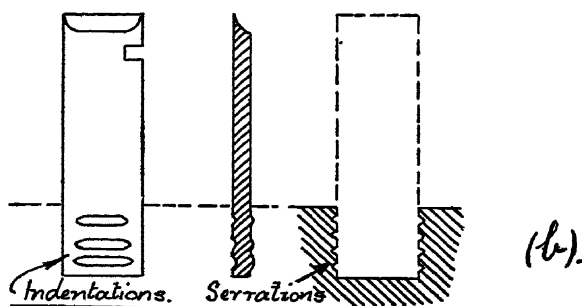
The method of assembling such blades is illustrated in Fig. 189 (a). The blades are first assembled in segments on a jig or "former," consisting of a cast-iron segment in which grooves are turned having the same depth and radius as those in the rotor and cylinder, A being a rotor groove and B a cylinder groove. The width of the grooves in the "former" is about 0.005 in. less than that of the rotor or cylinder grooves.

In commencing to build up a segment, a single packing piece is first threaded on to a wire which passes through the blade roots and spacers, and the end of the root wire at the back of the spacer is then riveted over. The spacer with the root wire attached to it is then inserted in the groove and driven hard up against a stop C,

which consists of a piece of steel, shaped at one end to fit the spacer and nicked at the other end and opened out by a fox-wedge. Blades and spacers are threaded on to the root wire alternately until a segment of the required length is obtained. After each four blades and their spacers have been threaded on to the wire, the whole is



(a).



(b).

FIG. 189

“end caulked” by means of the curved set D. When the segment is of the desired length, the end of the wire at the finishing end is either brazed or silver-soldered to the last blade.

After this operation, the blades are set radial by means of a square and twisted by a special tool to give the correct outlet angle. The binding strip E, which binds all the blades in the segment together, is now silver-soldered to the blade. The segment of blades is now removed from the former by slacking back the nuts and is ready for fitting in the rotor or cylinder, as the case may be. It is finally secured by radially caulking each spacer, so causing the soft metal to flow and fill the serrations (shown in Fig. 188 (d)) in the

grooves, and the notches in the blades. This system has long been known as the "rosary" system.

When higher peripheral speeds were adopted in marine turbines due to the introduction of mechanical reduction gearing, the so-called "individual" blading, shown in Fig. 189 (*b*), was introduced. As the term implies, each blade is fitted in the rotor separately. The blade is prepared in the ordinary way, except that instead of drilling a hole near the bottom for the root wire the blade is indented on the concave side of the root with two or three indentations. These are produced in a press, the metal being forced out on the convex side. During the caulking process, the blade indentations are filled by the soft spacers, while the projections produced on the convex side of the blade are forced into the metal of the spacers. Thus the blade is securely fastened to the spacers, while the latter, thanks to the serrations in the sides of the rotor grooves, are securely fastened to the rotor. With ordinarily good workmanship, this method of blading produces a strong and reliable attachment.

The "individual" method of blading is somewhat slow and has been superseded by the "segmental" method. This is employed for the stationary blades and for rotor blades when the stresses are not high. The method is very similar to the "rosary" method previously described, in that the blades are built up in segments on a former. The roots are brazed into a solid mass by a mechanically timed immersion into molten brazing alloy, the soundness of the brazing thus being made independent of the skill of the operator. The segments are fixed in position by means of short serrated side-locking pieces (somewhat similar to those shown in Fig. 190) which are driven up circumferentially.

157. Parsons End-tightened Blading. In the year 1912, Messrs C. A. Parsons & Co., Ltd., introduced the so-called "end-tightened" blading, in which the clearance which governs the leakage past the blades is axial instead of radial. This form of blade is now used in all the high-pressure and intermediate-pressure stages, the thin-tipped radial-clearance blades being employed only in the low-pressure stages. The earlier form of end-tightened blading is shown in Fig. 190. Short sections of blading *A* are first silver-soldered at the root to the deep spacers *B*, after which a strip of shrouding is silver-soldered to the blade tips, both operations being carried out with the blade segment assembled in a former. Serrations are then milled on one side of the roots and shallow grooves on the other, the blade segment being clamped to a radius bar during the milling operation. The rotor groove is serrated at *D* for the blade roots and at *E* for the packing pieces *F*. The packing pieces are inserted at the end of a blade segment and driven into position in the circumferential direction.

The shrouding strips are thinned down to a fine edge so as to

reduce the rubbing surface as much as possible. It is impossible to machine the shrouding so that the axial clearance will be the same in all rows of blades. This is achieved, however, by bringing the turbine as near to its working temperature as possible and allowing the rotor to turn at a low speed. The rotor is then very gradually moved towards the steam end. In this way the proud shrouding strips are gradually worn down. When all the strips are

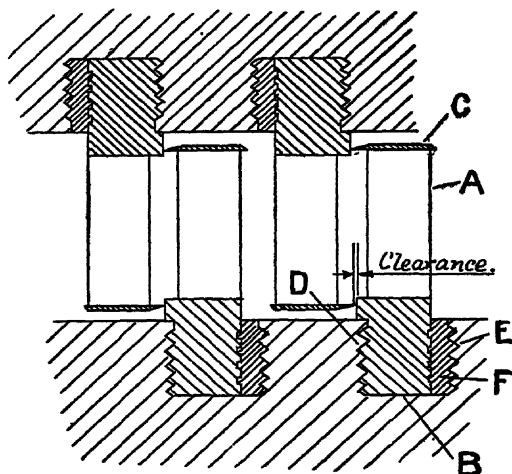


FIG. 190.—Details of Parsons "End-tightened" Blading

in simultaneous contact, the rotor is moved back until there is a suitable clearance, say from 0.008 to 0.015 in., between the shrouding and the blade roots. A liner of the necessary thickness is then fitted on the steam-end side of the Michell adjusting block so that the clearance can never be less than the prescribed amount. Another liner is fitted on the exhaust-end side of the adjusting block, a clearance of about $\frac{1}{32}$ in. being left between the block and the liner. By means of a hand-operated gear the adjusting block may be moved towards the exhaust end of the turbine a distance equal to this clearance. In this way the running clearance between the shrouding strip and blade roots may be increased during the time that the turbine is being warmed up. When the turbine has been running for some time and all parts have reached a steady temperature, the rotor is moved forward and the clearances thus reduced to the proper amount.

From a glance at Fig. 190 it will be apparent that the flow of steam through the clearance between the fixed shrouding strip and the rotor blades will disturb the main steam flow into the moving blades. This is avoided by the device shown in Fig. 191 and first introduced by Messrs. Parsons in 1930. In addition to the ordinary shrouding strip A, attached to the stationary blades, there is a

second strip B which has a tapered edge serving to deflect the leakage steam and so minimize the disturbance produced. The blades are built up in short segments consisting of about twelve blades, blades and spacers being threaded alternately on a root wire and assembled on a former. The end-tightening shrouding strip is riveted on while the blades are in the former. The whole is then slowly immersed by a cam into a brazing bath, after which

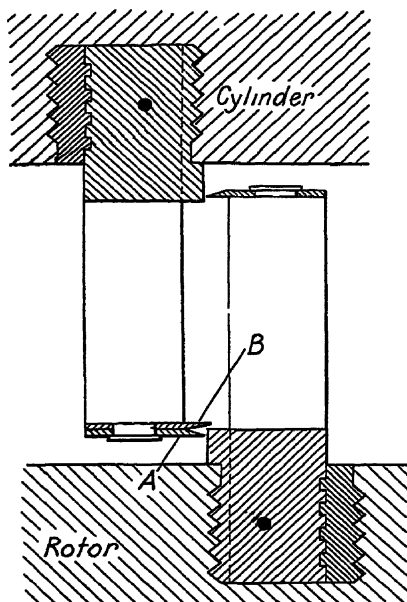


FIG 191 —Improved "End-tightened" Blading

the blade segment is dressed up and machined. In still more recent designs the root wire has been dispensed with, the blades and spacers being electrically welded together while still in the former, after which they are brazed.

158. Parsons Integral Blades. In all the various attachments described above the inertia force on the blade must be transmitted through the spacers to the rotor, the bond between the blades and spacers varying from the shearing strength of the root wire together with that of the spacer material embedded in shallow indentations to that provided by brazing and partial welding.

For long blades running at high speeds, the blade and spacer are made from one piece, as shown in Fig. 192, the root being lozenge-shaped. Such blades are rolled by a discontinuous process. Pieces of rectangular bar of the correct size for the root portion are either pressed in a die or milled on the sides so as to form a T-shaped

billet. The profile rolls have each a gap cut in the rim and the two rolls are so geared together that the gaps coincide and form a space which just takes the root portion of the billet. The billet is passed through the rolls a number of times until the stem is reduced to the

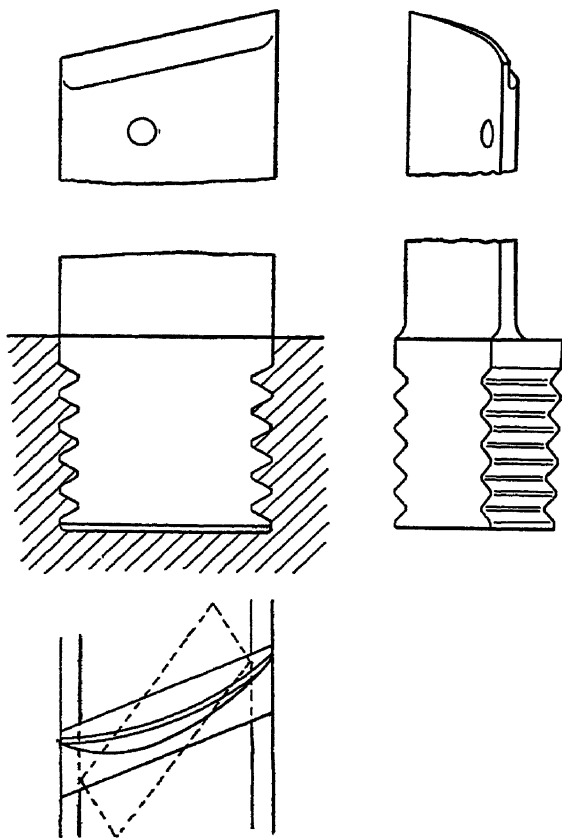


FIG. 192.—Parsons Integral Blading

finished blade size, the form of the root remaining unchanged. Blades manufactured in this way require no machining on the stem except cutting to length and tipping. The blade is pickled, polished, and the serrations machined on the root.

159. Blade Materials. The choice of materials for turbine blades is a somewhat difficult one and is guided by many considerations, among which are the following—

(1) *Conditions of Operation.* In the H.P. stages of certain turbines and in the I.P. stages of reheat turbines, where the temperature is high, it is essential to use a material which will be stable at high temperatures. In many cases the stresses are moderate, but

if the stresses are at all appreciable then attention must be paid to the strength of the selected material at the operating temperature and, in particular, to its resistance to creep.

In the L.P. stages, where the steam is wet, and especially in the neighbourhood of the dew point, it is necessary to employ a material which resists corrosion well. In the L.P. stages of turbines of high output the stresses in the blades are very high and a material having a high tensile strength is essential. The blades in the L.P. stages should also be capable of resisting the erosive action of the water drops. Experiments have shown that this resistance is fairly well measured by the Brinell hardness number of the material. When the peripheral velocity is very high it is not generally possible to obtain a material which, besides being hard enough, has other desirable mechanical properties such as toughness and ductility. In that case the necessary resistance to erosion is obtained by brazing special erosion shields to the blades. For further information on this side of the subject the reader is referred to the author's book, *Steam Turbine Operation*.

(2) *Methods of Manufacture* The principal methods of manufacturing turbine blades are cold drawing, machining, drop forging, rolling, and combinations of these methods. Certain blades, such as plate and profile blades with separate spacers, and reaction blades, may be produced in strip form by cold drawing provided the material is suitable. Impulse blades with integral spacers are usually machined from bar. This is a comparatively expensive process. Drop forging is generally restricted to long blades of complicated shape, such as tapered and twisted blades. The required dies are expensive to make and costly to maintain. Rolling may be applied to impulse and reaction blades with separate spacers. The blades are passed many times through profile rolls which are gradually brought together. Smaller sections are rolled cold and frequently annealed to remove strain. The heavier blade sections are rolled hot, the strip being annealed, and pickled to remove scale. The final rolling is carried out cold between highly polished rolls which give the blade a bright and smooth surface. When brazing or hard soldering has to be done, as in the attachment of bracing wire a material which becomes hot short at the brazing temperature should not be used.

(3) *Cost.* Certain materials have highly desirable properties, but are costly. Consequently their use is restricted to particular conditions where their properties are essential.

The following are among the better-known blading materials. The figures given for the composition are percentages.

Brass. The brass used for blading usually contains 70–72 copper and 28–30 zinc. The soft spacers are made of 63–37 alloy. Brass may be cold drawn, and it resists corrosion well. The strength falls away rapidly as the temperature is increased, and brass is unsuitable

for temperatures higher than 400–450° F. It was formerly much used in reaction turbines, but is now no longer used in large turbines.

Phosphor Bronze is a copper-tin alloy to which a small amount of phosphorus is added during the melting, usually in the form of phosphor tin or phosphor copper. The phosphorus acts as a deoxidizer and its addition in small amounts improves the properties of the bronze. For a hard bronze the approximate composition is 86 copper and about 14 tin, with up to 1 phosphorus. For a softer and more ductile bronze, the composition may be 92 copper, about 8 tin, and traces of phosphorus. Phosphor bronze may be forged, rolled, or cold drawn. It resists corrosion well and was formerly regarded as a satisfactory blading material. It is not suitable for high steam temperatures.

Manganese Copper. This is an alloy containing about 95–96 copper, 4–5 manganese, and small fractions of iron, carbon, and lead. It may be cold drawn and cold rolled. The strength falls off with increasing temperature, but not to the same extent as the copper-zinc or copper-tin alloys. It is suitable for blades under low stress up to steam temperatures of 600° F.

Nickel Brass is an alloy containing about 50 copper, 40 zinc, and 10 nickel. It may be cold drawn, giving a hard and smooth surface. The temperature limit is about 400–450° F.

Copper Nickel is an alloy containing about 80 copper, 19 nickel, and fractions of iron, manganese, etc. It may be cold rolled or drawn.

Monel Metal is produced from a Canadian ore without the separation of its chief constituent metals, nickel and copper. It contains about 67 nickel and 28 copper, together with iron, carbon, manganese, etc. Monel metal possesses very good resistance to the corrosive agents which may be present in impure steam and, in addition, it retains a large proportion of its strength at high temperatures.

It is supplied by the manufacturers, Messrs. Henry Wiggin & Co., Ltd., for turbine blading in three forms: (1) As cold-rolled rectangular bar from which certain impulse sections are produced by machining; (2) in the form of cold-rolled profile strip which is now made in all the normal Parsons impulse and reaction sections, and also in the Brown Curtis impulse sections; and (3) as Monel integral root blades, somewhat similar to the Parsons integral root blades described in Art. 158. In the integral root form, the blade part is cold drawn to an accurate profile, while the root is left rough so that the turbine builder can machine it to give any wing angle desired. Monel blades have been widely used in marine work, especially in H.P. turbines.

Mild Steel is not now much used for turbines, although it has been used for the blading of certain notable turbines in the past. It is an inexpensive metal, but corrodes too easily in a damp atmosphere.

Nickel Steel Steels containing from 3 to 5 per cent nickel have been much used in steam turbines. The addition of nickel to steel increases the ultimate tensile strength and also increases the ratio of the yield point to the ultimate strength. Nickel steel is more resistant to corrosion than is mild steel. It may be forged and machined, but not welded. Steels containing 25 to 30 per cent nickel have also been tried in steam turbines, but with unsatisfactory results.

Stainless Steel is an alloy of iron, chromium, and carbon, containing 12-14 chromium and the normal percentages of carbon. Besides having valuable anti-corrosive properties, it is a very hard material and offers a relatively great resistance to erosion. Stainless steel is air-hardening and may not be used for blades which have to be brazed during assembly. The most useful alloy of this type in turbine work is that known as *stainless iron*. This contains about 0.1 per cent of carbon. It is not an air-hardening alloy and does not, therefore, become brittle when heated. Stainless iron can be forged, rolled, easily machined, and also welded and brazed. It can be stamped and pressed and is often used for cast-in nozzle guide blades.

A.T.V. Steel This is an alloy steel made by Messrs. Hadfield's, Ltd., of Sheffield, the brand Hecla/A.T.V. having been specially developed for steam turbine blades and nozzle guide vanes. The alloy was first developed in France about the year 1916, the letters A.T.V. being based on the words *Acier Turbines à Vapeur*. The composition of A.T.V.1 has been published (2) as: Nickel 36, chromium 12, carbon 0.25, and the rest iron. It is claimed for Hecla/A.T.V. steel that it is highly resistant to both corrosion and erosion, and that it maintains its strength well at high temperatures. It can be supplied in either the hard or soft condition according to requirements. The advantage of the harder steel is the higher ratio of yield point to ultimate stress. The mechanical properties given in Table XI are the minimum, test figures on samples always being higher. Hecla/A.T.V. work hardens fairly rapidly, and the shrouding pin at the blade tip should not be made larger than necessary to secure a reliable attachment.

The mechanical properties of blading materials are given in Table XI. The figures are not claimed to be more than a guide, as the strength of the selected materials depends so much on the actual composition and on the thermal and mechanical treatment to which the material has been subjected. The approximate working stresses at the normal speed of rotation are based on factors of safety of 3 on the yield point, 5 on the ultimate stress, and a mean value of 4 for the factors based on the yield point and the ultimate strength. The last value of the working stress is based on the suggestion of Mr. K. Baumann (3).

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EXAMPLES XVII

- 1 The following particulars relate to a stage of an impulse turbine—

Mean diameter	40 in
Nozzle height	7.4 in
Blade height	7.9 in
Nozzle angle	24 deg
Number of nozzles (complete admission)	62
Thickness of nozzle guide vanes	0.104 in
Steam velocity at nozzle outlet	1,310 ft per sec
Specific volume of steam at nozzle outlet	297.2 cub ft per lb

Dividing the nozzle height into four equal parts, calculate the mass flow per second through each part

2 Using the data given in Example 1 and assuming that the rotational speed is 3,000 r p m, divide the blade height into four equal parts and from the velocity diagrams calculate the change of velocity of the steam at the mid-height of each part. Assume $\beta_2 = 40^\circ$, $k = 0.85$, and that the number of blades on the wheel is 250. Hence calculate for each part the force on each blade and the bending moment about the root. Combine the bending moments graphically and find the resultant bending moment about the blade root section

3 The impulse blade for the turbine in Examples 1 and 2 has the following proportions—

Inlet angle	44 deg.
Outlet angle	40 deg.
Blade width	1 in
Thickness of inlet and outlet edges	0.02 in
Radius of concave part of blade	0.673 in
Radius of convex part of blade	0.345 in

Draw the section of the blade enlarged to a suitable scale, and derive the following—

- (a) Area of blade section
- (b) Distance of centroid from axis XX (Fig 171)
- (c) Value of I_y (Fig 170).
- (d) Value of I_x (Fig 170).

4 Using the notation of Fig 170 for the turbine and blade of Examples 1, 2, and 3, find the bending moments M_x and M_y , and calculate the bending stresses at the point A. Calculate also the centrifugal stress at the innermost section of the blades, neglecting the shrouding, and hence find the maximum tensile stress in the blades. Assume that the blading material is steel

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5. The following are the particulars relating to a stage of a reaction turbine comprising one row of fixed blades and one row of moving blades—

Mean diameter of blades	72.875 in.
Height of blades	12.875 in.
Axial width of blades	1.5 in.
R.p.m.	1,800
Steam pressure before moving blades	6.0 lb. per sq. in. abs.
Steam pressure after moving blades	5.0 lb. per sq. in. abs.
Effective outlet angle of blades	20 deg.
Steam velocity at exit from guide blades	740 ft. per sec.
Steam velocity at exit from moving blades	860 ft. per sec.
Number of moving blades per row	254
Steam flow	76.1 lb. per sec.
Material of blades	Steel

Assuming that the blades have the section given in Fig. 91, and referring to Fig. 175, calculate or derive the following—

- (a) Dynamic moment about blade root section.
- (b) Static moment about blade root section.
- (c) Moment component M_v .
- (d) Moment component M_u .
- (e) Centrifugal stress at blade root.
- (f) Bending stress at A.
- (g) Bending stress at B.
- (h) Resultant stress at A.
- (i) Resultant stress at B.

CONSTRUCTION OF TURBINE ROTORS

160. De Laval Rotors. Two types of rotor are employed in the de Laval turbine, the type used depending on the peripheral speed which, in turn, depends chiefly on the power of the turbine.

In turbines up to about 100 h p the type of wheel shown in Fig 193 is used. The hub of the wheel is bored out and a tapered steel bush is secured by a nut screwed in at one end. The enlarged end of the bush is provided with two flats which fit into a recess in the face of the hub. The spindle is provided with a short enlargement which fits tightly in the bush. A pin passing through the bush and the spindle serves to transmit the torque.

The profile of the disc is hyperbolic, and a light rim is added to carry the blades. In experiments—intentional and otherwise—in which wheels of the form used in the de Laval turbine have been run at high speeds to destruction, it has been found that the wheels break up into two or three segments, and that the centrifugal force on such parts is sufficient to do considerable damage, even after having smashed the turbine casing. To prevent any likelihood of such a disaster, a shallow groove, termed the “safety groove,” is turned out at each side of the disc just under the rim. The wheel is now weakest at this section, the stresses there being some 50 per cent greater than at the most highly stressed point in the disc. Then if the turbine should be severely overspeeded, the rim parts company from the disc and usually breaks up into a number of relatively harmless fragments. In such an event the wheel will inevitably be thrown out of balance, dangerous bending of the spindle is then prevented by the casing, which is arranged to surround the hub with only a small clearance (Fig. 193).

It will be shown in Art. 176 of the next chapter that a rotating disc is very considerably weakened by drilling even a small hole at its centre. In de Laval turbines above about 100 h.p., with a mean blade velocity of about 1,130 ft. per sec, the type of disc shown in Fig. 194 is used. The disc is continuous through the centre and the thickness is so varied that the disc profile is that of the “disc of constant strength” described in Art. 179. In order to attach the lens-shaped disc to the shaft, a departure is made from the theoretical profile by providing bosses at the sides of the disc. The spindle is made in two parts, each provided with an enlarged end, recessed into the boss and secured by stud bolts.

161. Construction of Disc Rotors. A familiar form of construction employed in impulse turbines and frequently in the L.P. stages of reaction turbines comprises a number of separate discs

on a shaft. The latter may either be stepped or of uniform diameter, according to the method employed for attaching the discs.

The wheel consists of three parts: the rim which is designed to hold the blades, the hub which is attached to the shaft, and the disc proper which connects the rim and the hub.

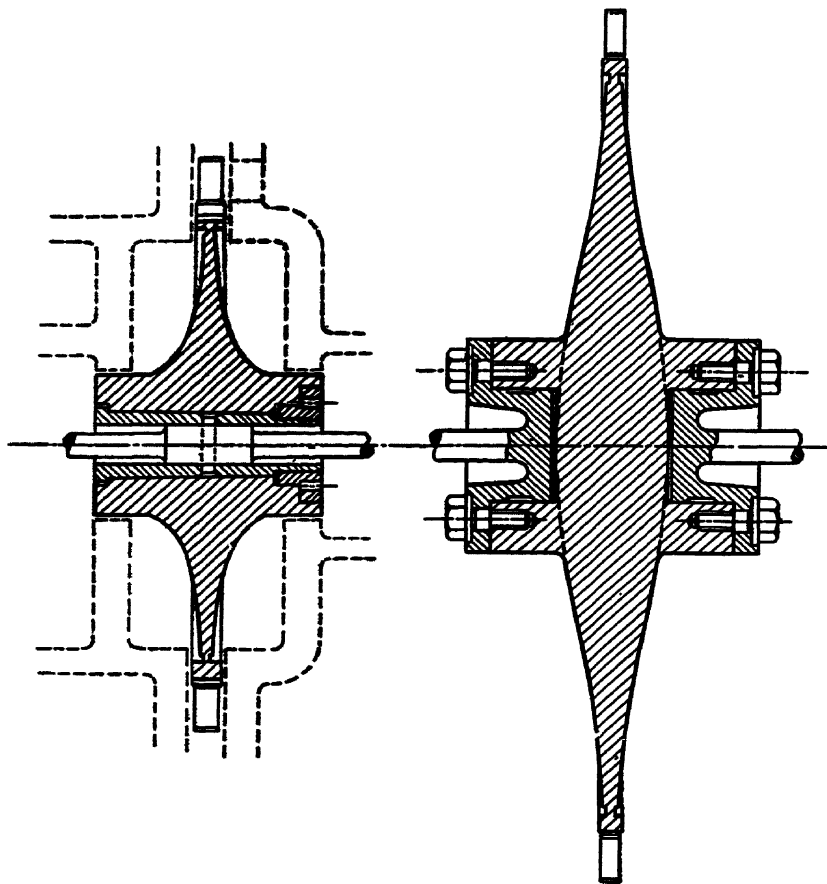


FIG. 193

FIG. 194

is prepared from a cheese of steel which is forged or pressed into disc form. The wheel is then rough machined and annealed to relieve it of internal stresses. It is afterwards carefully machined to give it the correct profile. The sides of the disc should be given a very smooth finish so as to avoid stress concentration. In the construction shown in Fig. 195, the wheels are pressed on to the shaft by means of a hydraulic press, the bore of the wheels being slightly smaller than the diameter of the shaft on which they are to be pressed.

This difference in diameter must be very carefully calculated and adhered to in the machining operations, so that even when the wheel is running at the overspeed (normally 10 per cent above normal speed) there will still be some mutual pressure between the wheel and the shaft. In starting a cold turbine, the discs will reach the

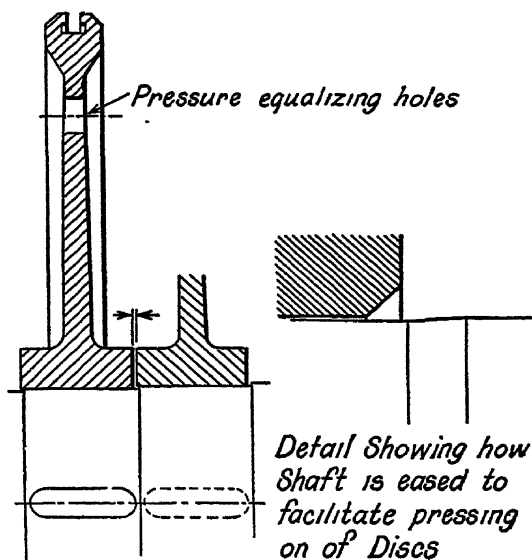


FIG 195

working temperature sooner than the shaft. As a result, the discs expand slightly more than the shaft, and unless there is a sufficient force fit allowance they may become loose. The forcing on and removal of the discs are facilitated by heating the discs.

The torque is transmitted by sunk keys. When the torque is moderate, one key only is used for each wheel, the keys for adjacent wheels being arranged at 180° to each other. For greater torques, two keys are used. In very large units, six, and even eight, keys have been used for each wheel.

A small axial clearance is left between adjacent discs to allow for expansion due to temperature differences. The shaft is stepped so that each wheel may be easily passed over the shaft until it reaches its own seat. In order to prevent any appreciable difference of steam pressure being set up on the two sides of the disc, either at starting or during normal working (due to ejector action of the steam passing between the nozzles and blades), pressure-balancing holes are drilled in the discs. About six such holes are drilled in a part of the disc where the stresses are moderate. The edges of such holes should be well rounded off to prevent stress concentration as far as possible.

162. Mounting of Wheels on Shaft. The objections to forcing the wheels on the shaft are that the stresses in the wheel are increased due to the heavy force fit, and also that, in the event of necessity, the removal of the wheel is a matter of some difficulty. Various alternative methods have been devised and employed.

The method illustrated in Fig. 196 has been used by a number of turbine builders, chiefly on the Continent. The shaft is not stepped, and the wheel hubs are bored out slightly larger than the diameter of the shaft. Recesses are turned in each end of the wheel hubs for "supporting rings" A and B. Each ring is carefully machined to fit the shaft and the hub and is recessed to pass over

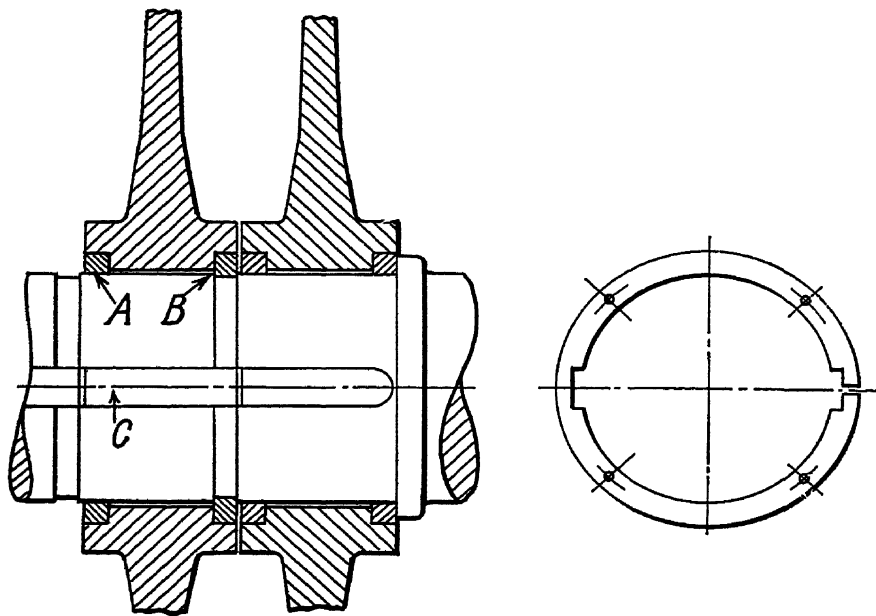


FIG. 196

the keys C. The rings are also split. A groove is turned in the spindle and the ring B is sprung into its groove just as a piston ring is. The rings B serve to locate the wheel axially. It is stated that when the wheels are rotating at their normal speed, the strains in the hub are such that the ends of the hub nip the supporting rings so that the discs are held quite tightly. Whether this is the case when the thickness of the disc at the hub approaches that of the hub itself would appear to be very doubtful. The torque is transmitted from the central part of the hub only. As there is a clearance between the shaft and the bore of the wheel and also an axial clearance between the wheels, shaft flexure does not tend to cause slackness.

A modification of the same principle is shown in Fig. 197. This method has been successfully used by Messrs. Brown Boveri & Co., Ltd. The wheels are bored out to an easy press fit on the shaft and are provided with a deep recess at each end. U-shaped chrome steel rings are forced into these recesses and are thus heavily compressed. This, while relieving the rotor of a considerable amount of stress which would be induced by the usual force fit, enables the wheel to expand slightly under stress and temperature without becoming loose or running out of centre.

The attachment shown in Fig. 198 consists of a split conical bush which is pressed into the conically-bored hub. In this way,

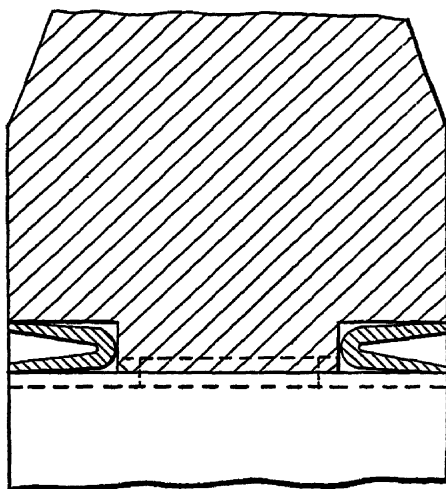


FIG 197

the pressure between the hub and the shaft is under some control. A liner is fitted between the bush at the larger end and the adjacent disc. The bush is screwed internally at its larger end for the attachment of withdrawing gear. This mounting is used by the well-known firm of A.E.G.

Under high temperatures, steel and other alloys, when subjected to stress, undergo continually a very small but definite strain. This phenomenon is known as creep. Thus a turbine wheel rotating in high-temperature steam and thus subjected to radial and tangential stresses would gradually change in size, the diameter increasing at an extremely slow, but steady, rate. Such change of dimensions, due to creep, will obviously reduce the mutual pressure due to force fit, and the wheel will tend to become loose.

The construction shown in Fig. 199 (British Thomson-Houston Co., Ltd.) is designed to avoid this. A steel bush is pressed into the hub and a number of radial holes are drilled through the two.

Radial pins are inserted in the holes and the bush pressed on to the shaft, one or more keys serving to transmit the torque. It will be seen that even if the bore of the wheel should increase to such an extent that contact between the wheel and the lightly stressed bush should cease, the wheel will still remain concentric with the shaft.

163. Rotors Machined from Solid Forgings. The difficulties associated with the use of high-temperature steam and which are

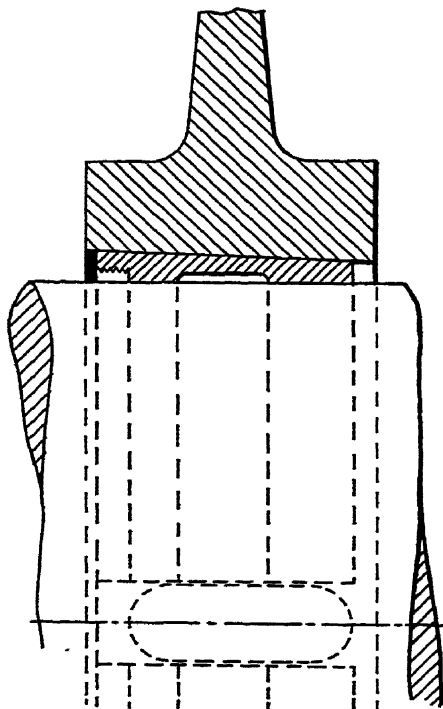


FIG. 198

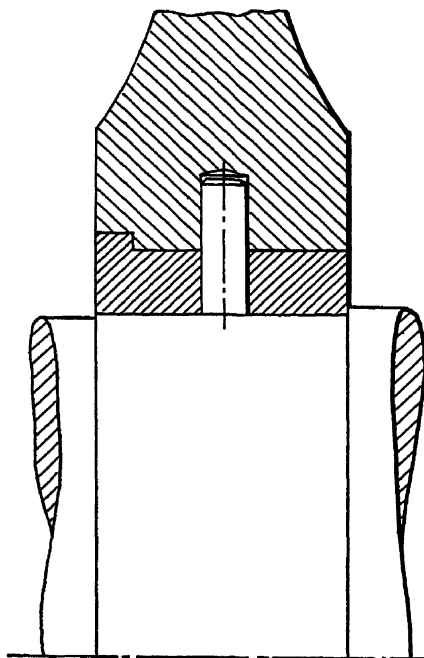


FIG. 199

briefly outlined in the preceding article led the well-known Czecho-Slovakian firm, Erste Brünnner Maschinenfabrik, some years ago, to initiate the practice of machining impulse turbine rotors from solid forgings. Such a rotor is shown in Fig. 200. The high-pressure discs are frequently made of uniform thickness. Where the stresses demand it, the thickness may be slightly increased near the shaft. The blades have the same axial width as the discs, thus saving length. The advantages of the construction are obvious. Trouble due to a loose disc is impossible. Wheel hubs are avoided and thus the diameter of the diaphragm glands is the least possible and the leakage of steam is reduced. The saving in axial length is appreciable

and generally leads to a reduction in spindle diameter when compared with the normal construction.

On the other hand, it is more difficult to obtain a sound forging for such a rotor than for a single wheel of the same diameter. This limits the size of rotor for which the construction may be employed.

164. Rotors for Reaction Turbines. In the majority of reaction turbines the rotor, whatever form of construction may be employed,

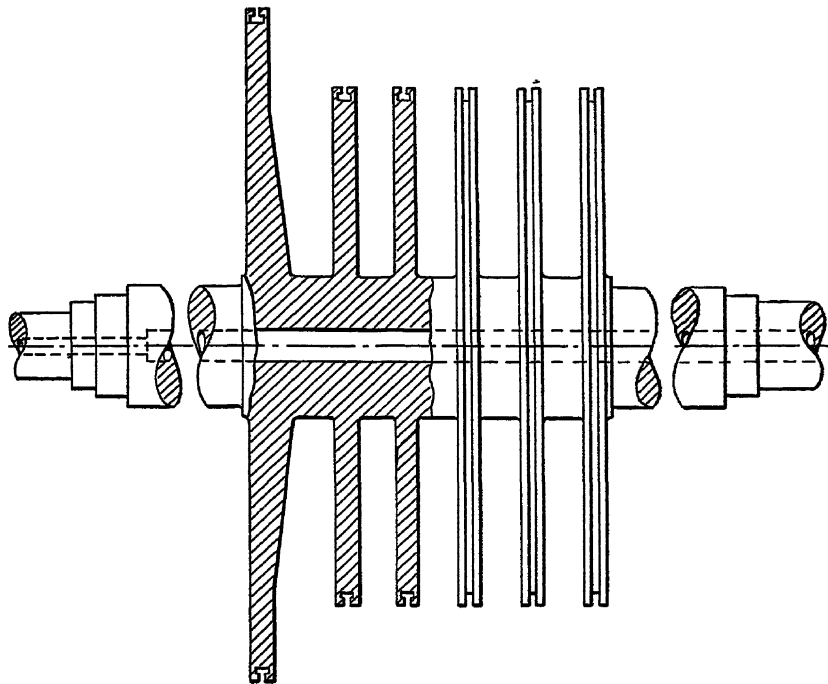


FIG 200 —Turbine Rotor Machined from Solid Forging

is arranged to have a continuous cylindrical or conical surface. The principal exceptions to this rule are the L.P. reaction turbines constructed by Messrs. Brown Boveri & Co., Ltd.

Various forms of Parsons turbine rotor are shown in Figs. 201 to 204. Fig 201 shows half-sections through the H.P. and L.P. rotors of a two-cylinder tandem turbine built in 1912. The H.P. rotor consists of a hollow forged steel element comprising the rotor drum and part spindle, and a spindle having an enlarged end. The former is shrunk on to the latter and further secured by screwed pins. The H.P. and I.P. dummy pistons are formed out of the rotor element, while a separate disc is used for the L.P. dummy. The mean diameter at the L.P. end is 71.5 in., the speed of rotation 750 r.p.m., and the mean peripheral speed 234 ft. per sec. The L.P. rotor

comprises a forged steel drum 86 in. external diameter and $2\frac{1}{8}$ in. thick. This is shrunk on to a cast-steel centre spider wheel which, in turn, is shrunk on to the rotor shaft. The ends of the drum are supported by mild steel discs. The mean diameter of the last blade ring is 105 in., the speed 750 r.p.m., and the mean blade velocity 344 ft. per sec.

The drum type of rotor is somewhat limited in its application owing to the stresses becoming excessive at high peripheral speeds.

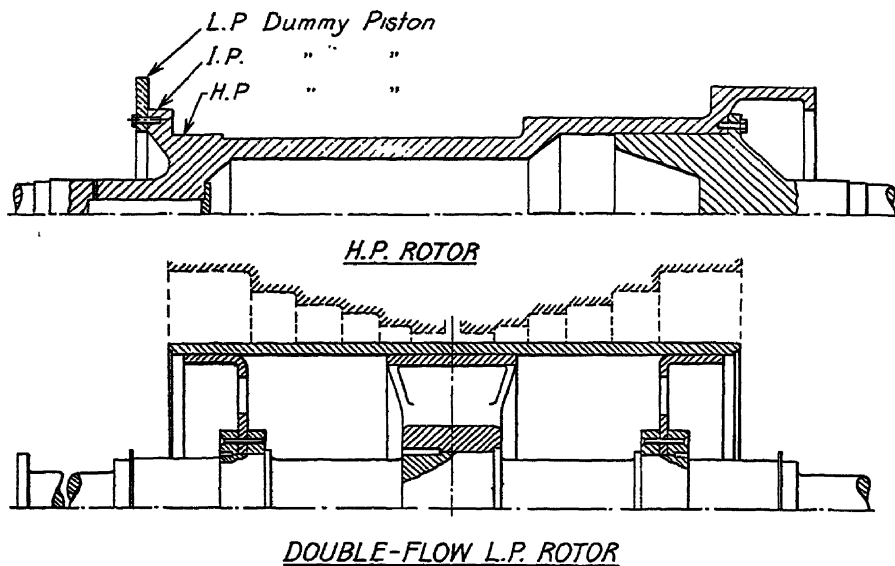


FIG. 201

The construction used in a turbine built by Messrs. C. A. Parsons & Co., Ltd., in 1916 is shown in Fig. 202. The H.P. turbine rotor is a solid forging, maximum mean blade velocity 239 ft. per sec. There are two dummy pistons, the H.P. dummy being turned from the solid forging, and the I.P. dummy being formed by two separate discs. The L.P. rotor is of the double-flow type and comprises eight heavy discs, with wide rims and hubs, shrunk on to the shaft. The mean peripheral velocity for the last row is 392 ft. per sec.

A modified form of double-flow L.P. rotor is shown in Fig. 203. The rotor is machined from a single forging. The centre part, which carries the shorter blades, is drum shaped, the outer parts, which carry the longer blades, are discs machined out of the solid. Such a construction offers great strength, especially when there is no central hole through the rotor and, of course, where the material is sound. Mean blade velocities in such a rotor appear to exceed 600 ft. per sec. In the H.P. turbine of recent two-cylinder tandem

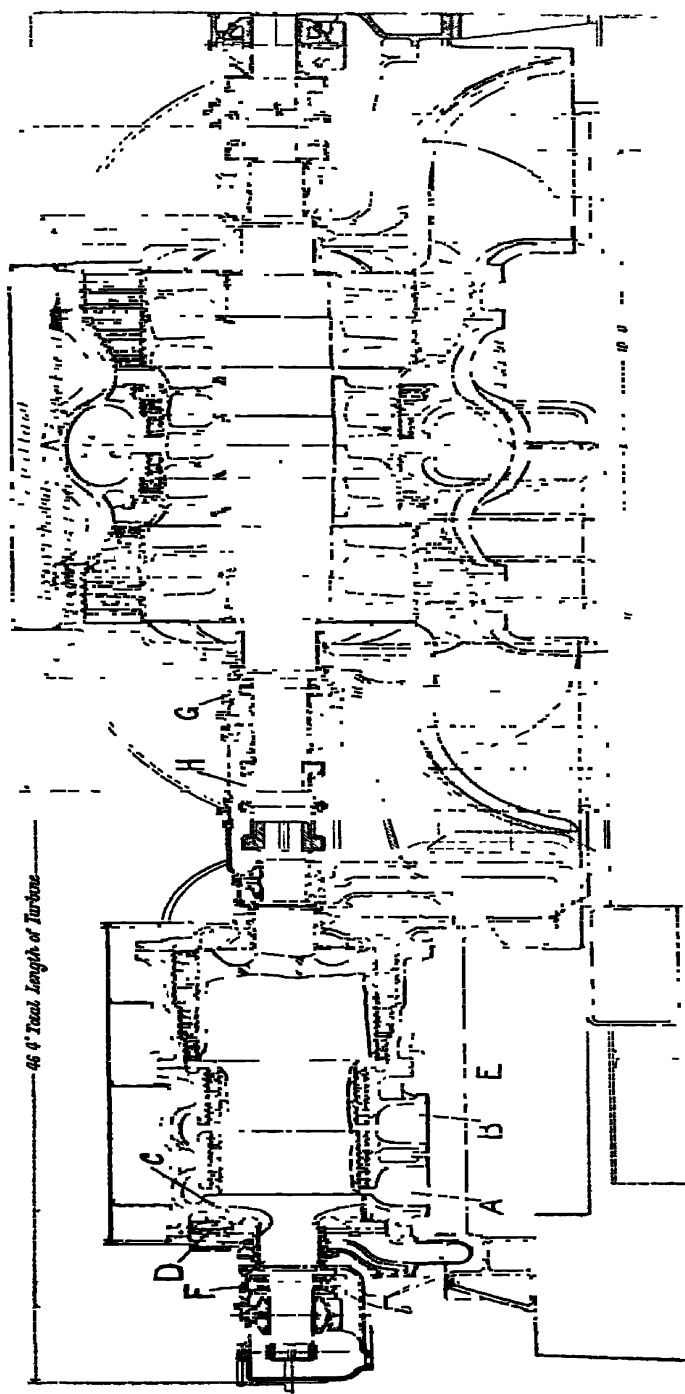


Fig 202 —15,000 kW 1,000 r p m Paicons Steam Turbine
 Installed in the Lots Road Power Station of the Metropolitan, District and London Electric Railway Companies

turbines, Messrs. Parsons have employed the construction shown in Fig. 204. Part of the rotor, including the dummy pistons, is turned from a solid forging. At the outlet end of the rotor, where the blades

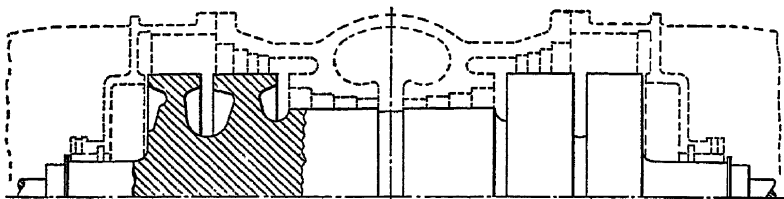


FIG. 203.—Double-flow L.P. Rotor for Parsons Turbine

are longer and the stresses induced in the solid rotor would become excessive, a number of separate discs are shrunk on to the stepped shaft. The discs are of a relatively heavy and robust design and are



FIG. 204.—Parsons Turbine Rotor

butted together at the rim to eliminate any possibility of transverse vibration. A hole, not less than 2 in. diameter, is bored through the shaft from end to end. The inner surface of this hole is very carefully examined by means of an instrument comprising a mirror,

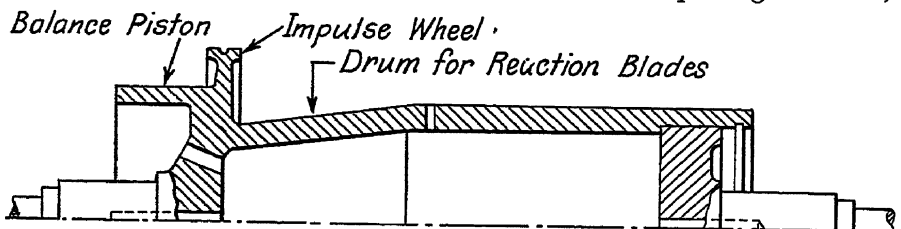


FIG. 205

lamp, and telescope, and known as a "borescope," for the purpose of detecting any minute flaws in the centre of the forging.

In recent single-cylinder turbines a similar construction has been adopted. At the L.P. end, however, discs having a diameter somewhat larger than that of the solid rotor are shrunk on.

165. Brown-Boveri Drum Rotor. A drum rotor for combined impulse and reaction turbines is shown in Fig. 205. This is used for powers up to about 3,000 kW and for moderate peripheral speeds. The steam end of the shaft, balance piston, Curtis wheel, and drum are made from a single forging, the exhaust end of the

shaft and its disc being likewise made from a forging. After the internal machining is completed, the drum is shrunk on to the L. P. end disc. Holes for equalizing the pressure behind the balance piston and that at an intermediate stage are bored in the drum end and through the drum itself.

166. Brown-Boveri Welded Rotor. About the year 1930, Messrs. Brown Boveri & Co introduced a welded rotor construction, one form of which is shown in Fig 206. The rotor comprises a number of discs and two shaft ends which are welded together at the periphery as shown in the detail view. The wheels are continuous discs,

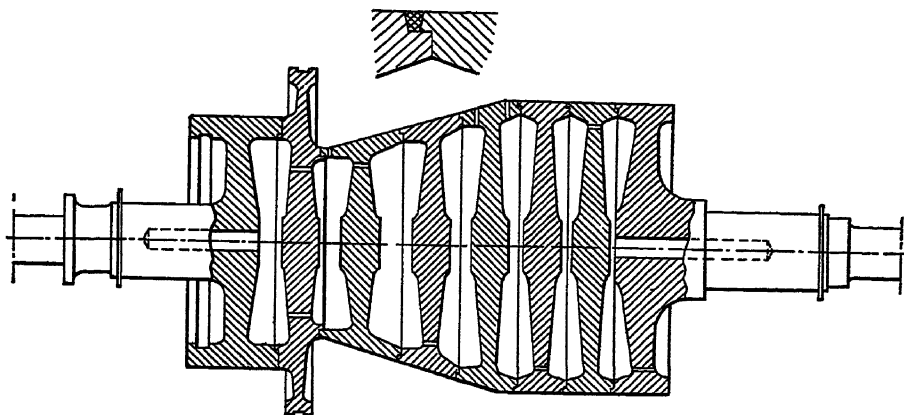


FIG 206 —Brown-Boveri Welded Turbine Rotor

ie they have no central hole and are therefore much stronger than discs which have to be bored and pressed on a shaft. For an explanation of this statement, the reader is referred to Art. 176. Moreover, there are no keys, and the possibility of loose discs is entirely eliminated. The transverse rigidity of the rotor is very great and the critical speed is well above the running speed. Small holes are drilled through the discs just under the rims and also through the rims. The spaces between the discs are, therefore, filled with steam at the working temperature; this tends to prevent strains and other troubles due to differential expansion at starting and during change of load.

As shown in the inset detail of Fig. 206, the rotor discs are spigotted into one another and the remaining V filled by weld metal. The stresses in the welded seams are said to be only 700–1,500 lb per sq. in. Owing to the large mass of the rotor, there is a considerable conduction of heat from the steel in the neighbourhood of the weld during welding, and the rate of cooling is such that the steel is hardened. In order to remove such effects the rotor is annealed at 600 to 650° C. for a prolonged period.

Before the construction was adopted as standard practice the

welded rotors were subjected to severe tests. The welds were tested in tension by applying a hydraulic pressure between the discs, in shear by heating one disc and cooling the next, while the whole rotor was subjected to impact by falling weights.

167. Balancing of the Steam Thrust on the Moving Blades of Reaction Turbines. In a pure impulse turbine the axial thrust on the blades is entirely due to the change of momentum of the steam

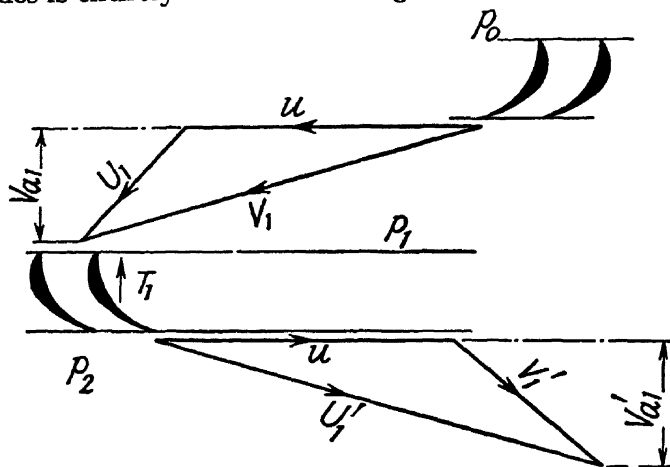


FIG. 207

in the axial direction and is always a very small force. In the reaction turbine, however, the steam is falling in pressure during its passage through each ring of blades, fixed and moving alike, so that in addition to any force due to change of momentum there is a force due to the pressure drop across the blades. The latter force may be very large and it is customary to provide some means of balancing it by steam pressure.

Consider a single row of fixed or moving blades, as shown in Fig. 207. The steam pressures at inlet and outlet are p_1 and p_2 respectively.

Let A = annular area of blade ring, sq. in.

We shall indicate the quantities referring to successive rows of blades, including fixed and moving, by the suffixes 1, 2, 3, etc. Then—

V_{a1} = axial component of velocity entering row No. 1.

V_{a1}' = " " " leaving " " 1.

V_{a2} = " " " entering " " 2.

V_{a2}' = " " " leaving " " 2.

Let T_1 = axial thrust exerted by the steam on the blades in row No. 1. This is also, of course, the thrust exerted by the blades on the steam.

w = mass flow of steam through blades, lb. per sec.

Then effective force on steam equals change of momentum per second.

$$\text{Or} \quad (\dot{p}_1 - \dot{p}_2)A - T_1 = \frac{w}{g} (V_{a'1} - V_{a1})$$

$$\text{and} \quad T_1 = (\dot{p}_1 - \dot{p}_2)A - \frac{w}{g} (V_{a'1} - V_{a1}) \quad . \quad . \quad (161)$$

This expression would give the resultant steam thrust on the first row of blades (say, fixed). Similarly, the thrust on the second row (which would be the first moving row in the group) would be—

$$T_2 = (\dot{p}_2 - \dot{p}_3)A - \frac{w}{g} (V_{a'2} - V_{a2})$$

In the same way we may write down the expression for the thrust on all the fixed and moving rows in the group. Signifying the last row by the suffix n and the exhaust pressure by $\dot{p}_e$, we have—

$$T_1 = (\dot{p}_1 - \dot{p}_2)A - \frac{w}{g} (V_{a'1} - V_{a1})$$

$$T_2 = (\dot{p}_2 - \dot{p}_3)A - \frac{w}{g} (V_{a'2} - V_{a2})$$

$$T_3 = (\dot{p}_3 - \dot{p}_4)A - \frac{w}{g} (V_{a'3} - V_{a3})$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$T_n = (\dot{p}_n - \dot{p}_e)A - \frac{w}{g} (V_{a'n} - V_{an})$$

Reference to the velocity diagram shows that since the axial component of the steam velocity at exit from one ring of blades is the axial component at inlet to the following row, $V_{a2} = V_{a'1}$, $V_{a3} = V_{a'2}$, etc. Hence, by addition, we have—

Total thrust on all fixed and moving rows of blades

$$= \Sigma T = (\dot{p}_1 - \dot{p}_e)A - \frac{w}{g} (V_{a'n} - V_{a1})$$

The thrust on the rotor is that due to the moving blades and is one-half of the total thrust given above

$$\therefore \text{Rotor thrust } T = \frac{1}{2}(\dot{p}_1 - \dot{p}_e)A - \frac{w}{2g} (V_{a'n} - V_{a1}) \quad . \quad . \quad (162)$$

The second term on the right-hand side of (162) amounts to about 1 per cent only of the total thrust and may, therefore, be neglected.

$$\text{Hence} \quad T = \frac{1}{2}(\dot{p}_1 - \dot{p}_e)A \quad . \quad . \quad . \quad (162a)$$

168. Proportions of Dummy Balance Pistons. Fig. 208 shows a diagrammatic section of a combination turbine consisting of an

impulse wheel followed by six expansions of reaction blades. Suppose that the axial forces due to steam pressures are balanced by a single balance piston A which rotates in the "dummy cylinder" B. The leakage of steam through the narrow annulus between the piston and cylinder is reduced to a small value by means of labyrinth packing. Suppose that the space behind the balance piston is connected to the outlet from the third expansion by the pressure-equalizing pipe C. Such connection may be made to the exhaust

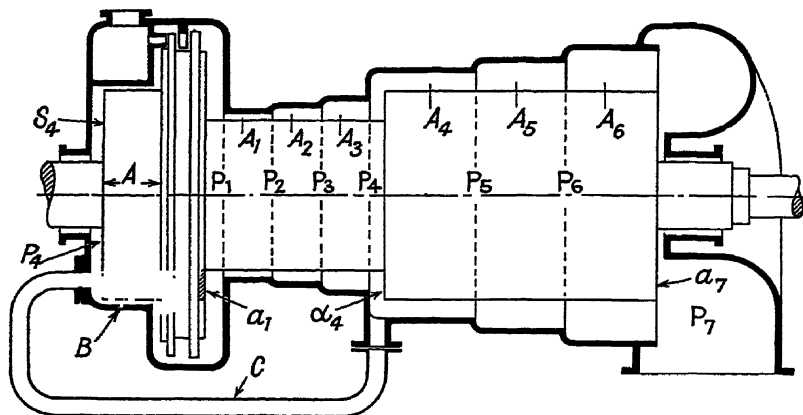


FIG. 208.—Diagrammatic Arrangement of Impulse-reaction Turbine with Single Balance Piston

branch or to any L.P. stage, but the loss due to leakage will vary according to the point of connection.

Let A_1, A_2 , etc., represent the annular areas of the blade rings in expansions 1, 2, etc.

Let a_1 = net annular area of the impulse wheel shoulder (in reality, of the dummy piston).

α_4 = area of rotor enlargement at outlet from third expansion.

a_7 = net area of L.P. end of rotor.

S_4 = net area of L.P. side of balance piston.

Suppose that P_1, P_2, P_3 , etc., are the steam pressures at inlet to expansions 1, 2, 3, etc., at full load. The pressure on the L.P. side of the balance piston will be P_4 . Then for equilibrium—

$$P_1 a_1 - P_4 \cdot S_4 = \frac{1}{2} \{ (P_1 - P_2) A_1 + (P_2 - P_3) A_2 + (P_3 - P_4) A_3 \\ + (P_4 - P_5) A_4 + (P_5 - P_6) A_5 + (P_6 - P_7) A_6 \} \\ + P_4 \alpha_4 - P_7 \cdot a_7$$

Since S_4 = (area of drum) + a_1 , the above equation enables the area a_1 to be found.

Provided the governing is effected by nozzle control and no steam is by-passed beyond the first expansion, then all the steam pressures, except P_7 , will vary directly as the steam flow through the turbine. Consequently, approximate balance will be obtained at all loads. The diameter of the dummy piston should be made rather less than the calculated value, so that there will be a definite thrust towards the exhaust end of the turbine. This will eliminate any tendency for the rotor to oscillate axially.

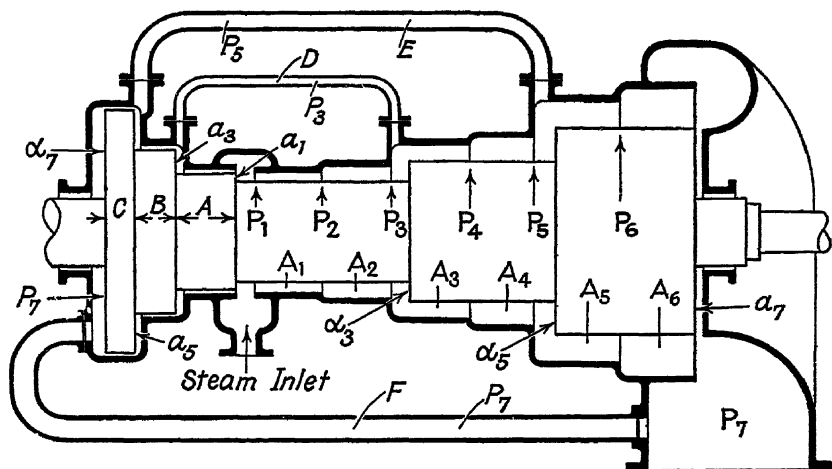


FIG. 209 —Diagrammatic Arrangement of Reaction Turbine with Three Balance Pistons

A turbine in which the rotor has three sections of different diameter is shown in Fig. 209. It will be assumed that the axial thrust is balanced by three balance pistons A, B, and C, the pressures on these being P_1 , P_3 , and P_5 respectively, while the pressure on the back of the L. P. dummy is the exhaust pressure P_7 . D, E, and F are the pressure-equalizing pipes. Using the notation given in Fig. 209 for effective areas under pressure, then—

$$P_1 a_1 = \frac{1}{2} \{ (P_1 - P_2) A_1 + (P_2 - P_3) A_2 \}$$

$$P_3 (a_3 - \alpha_3) = \frac{1}{2} \{ (P_3 - P_4) A_3 + (P_4 - P_5) A_4 \}$$

$$P_5 (a_5 - \alpha_5) + P_7 (a_7 - \alpha_7) = \frac{1}{2} \{ (P_5 - P_6) A_5 + (P_6 - P_7) A_6 \}$$

The solution of these equations in the order given enables the areas a_1 , a_3 , and a_5 to be found, and hence the mean diameter of the dummy pistons.

The quantity of steam leaking past the dummy piston A is usually greater than that leaking past B. Thus the excess steam will flow through D and do work in the last four expansions of the turbine. Similarly, part of the steam which leaks through B will do useful work in the last two expansions.

The same general methods may be applied to any other arrangement of rotor and balance pistons. Packing devices for dummy pistons are described in Chapter XXII.

EXAMPLE 21. The following particulars refer to a 2,500 h.p. reaction turbine—

Drum diameter, H.P. section	.	.	.	.	23 $\frac{5}{8}$ in.
" " L.P. "	.	.	.	.	33 $\frac{5}{8}$ "
" " L.P. "	.	.	.	.	47 $\frac{1}{4}$ "
H.P. Section.	1st expansion.	Blade height	=	$\frac{7}{16}$ in.	
"	2nd "	" "	=	$\frac{9}{16}$ "	
"	3rd "	" "	=	$\frac{3}{4}$ "	
I.P. Section.	1st "	" "	=	$\frac{1}{2}$ "	
"	2nd "	" "	=	$\frac{5}{8}$ "	
"	3rd "	" "	=	1 "	
L.P. Section.	1st "	" "	=	1 "	
"	2nd "	" "	=	2 $\frac{1}{4}$ "	
"	3rd "	" "	=	3 $\frac{3}{4}$ "	

The absolute steam pressures at entry to and exit from each expansion are set down in the diagram (Fig. 210). Calculate the diameters of the dummy pistons required in order that there may be no (appreciable) unbalanced steam thrust.

The annular areas of the blade rings are first calculated and set down as shown in Fig. 210.

$$\text{Then thrust on H.P. blades} = \frac{1}{2}\{(50 \times 33.13) + (38 \times 42.8) + (30 \times 57.49)\} = 2,504 \text{ lb.}$$

$$\therefore P_1 a_1 = 2,504$$

$$\therefore a_1 = \frac{2,504}{190} = 13.2$$

$$\text{Area of circle } 23\frac{5}{8} \text{ in. diameter} = 438.3 \text{ sq. in.}$$

$$\therefore \text{Area of H.P. dummy} = 438.3 + 13.2 = 451.5.$$

$$\therefore \text{Diameter of H.P. dummy (to nearest sixteenth of an inch)} = 24 \text{ in.}$$

$$\text{Thrust on I.P. blades} = \frac{1}{2}\{(22 \times 53.6) + (16 \times 67.25) + (10.5 \times 94.84)\} = 1,625 \text{ lb.}$$

$$\begin{aligned} \text{Thrust on shoulder between H.P. and I.P. sections of rotor} \\ = 72 \times 449.7 = 32,380 \text{ lb.} \end{aligned}$$

$$\therefore \text{Total steam thrust in I.P. section} = 34,005 \text{ lb.}$$

$$\therefore a_2 = \frac{34,005}{72} = 472.3 \text{ sq. in.}$$

$$\frac{\text{Area of end} = (a_1 + a_2 + a_3) + 438.3}{}$$

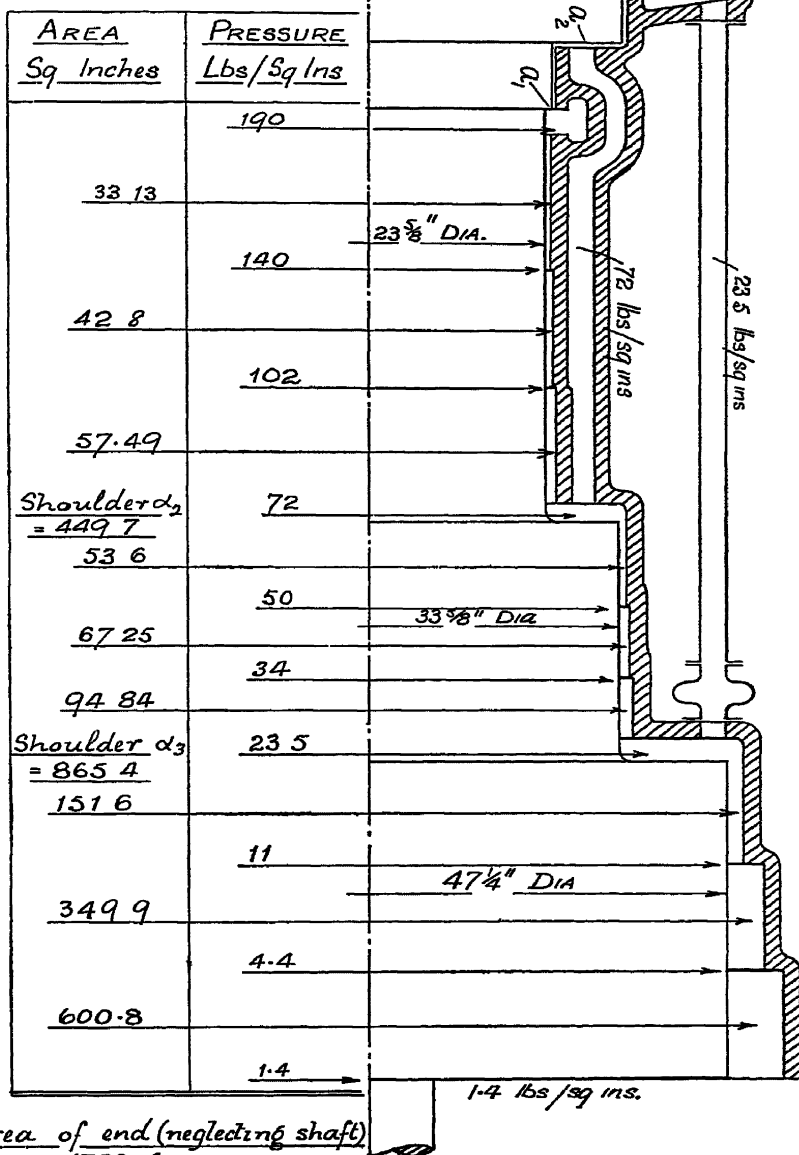


FIG. 210 —Illustrating Example on the Design of Dummy Balance Pistons

Area of circle 24 in. diameter = 452.39 sq. in.

$$\therefore \text{Area of I.P. dummy} = 472.3 + 452.39 \\ = 924.69 \text{ sq. in.}$$

$$\therefore \text{Diameter of I.P. dummy} = 34\frac{5}{16} \text{ in.}$$

$$\text{Thrust on L.P. blades} = \frac{1}{2}\{(12.5 \times 151.6) + (6.6 \times 349.9) \\ + (3 \times 600.8)\} = 3,003 \text{ lb.}$$

$$\text{Thrust on shoulder between I.P. and L.P. sections of rotor} \\ = 23.5 \times 865.4 = 20,330$$

$$\therefore \text{Total steam thrust on L.P. section} = 23,333 \text{ lb.}$$

$$\text{Back thrust on L.P. exhaust end of rotor} \\ = 1.4 \times 1753.4 = 2,455 \text{ lb.}$$

$$\therefore \text{Thrust to be balanced by L.P. dummy} \\ = 23,333 - 2,455 = 20,878 \text{ lb.}$$

$$\text{Full area of L.P. dummy} = 924.7 + a_3.$$

$$\therefore 23.5a_3 = 20,878 + 1.4(924.7 + a_3)$$

$$\text{or } 22.1a_3 = 22,173$$

$$\therefore a_3 = 1,004 \text{ sq. in.}$$

$$\text{Area of circle } 34\frac{5}{16} \text{ in. diameter} = 924.7 \text{ sq. in.}$$

$$\therefore \text{Full area of L.P. dummy piston} = 1928.7 \text{ sq. in.}$$

$$\therefore \text{Diameter of L.P. dummy piston} = 49\frac{9}{16} \text{ in.}$$

The diameters calculated above would be the mean diameters of the respective rings of labyrinth packing.

169. Couplings. Owing to the relative axial movement between a turbine spindle and the shaft of the driven machine, or between the shafts of the elements of a tandem turbine due to temperature changes, it is usually, but not always, necessary to join the shafts by means of a coupling which permits sliding in the axial direction. In some cases, there is also a change in alignment brought about either by differential expansion, slight settling of foundations, or flexibility of supports, and if the design of the coupling is such that it will permit of a small change of alignment as well, then so much the better.

A coupling suitable for turbines of small and medium power is shown in Fig. 211. The flanges are keyed to the shaft and sometimes secured by nuts as shown. The coupling bolts are screwed into one flange. Rubber bushes with metal cores are fitted over the plain ends of the bolts and have a small clearance in the holes of the other flange. This type of coupling allows for axial movement, slight lack of alignment, and also a slight torsional flexibility. Couplings of this kind have been used for the transmission of 3,000 h.p.

The well-known Parsons type of claw coupling, which has been used by practically every firm of turbine builders, is shown in Fig 212. This comprises two claw pieces A having a number of spokes

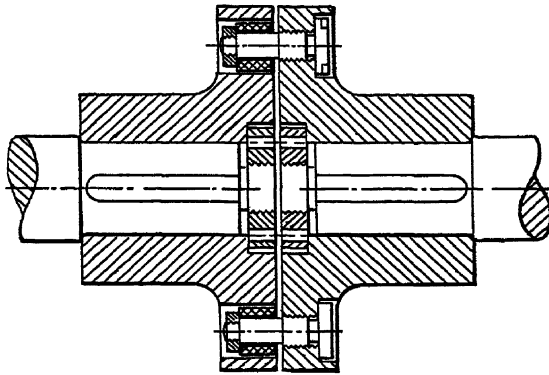


FIG 211 —Coupling for Turbines of Small and Medium Output

with parallel sides. The sleeves B are provided with an equal number of V-shaped projections which fit between the spokes. Rings C serve to limit the axial play of the coupling. The sleeves

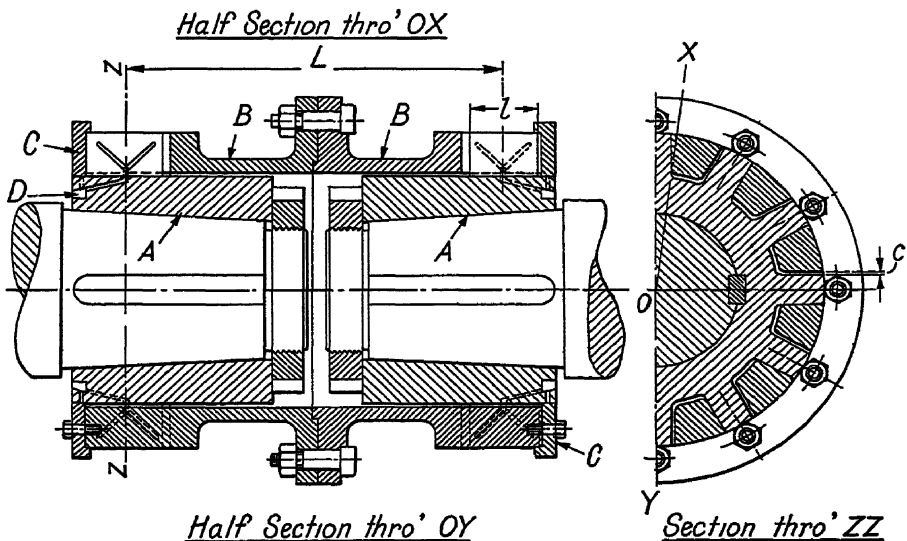


FIG 212 —Parsons Type Flexible Coupling

are spigotted together and bolted. The couplings must be very accurately made so that all the teeth are in contact simultaneously.

A small clearance c is provided between the claws, as shown.

If the shaft centres remain at the same height, this clearance permits the angular misalignment between them to reach the value $\theta = \frac{2c}{l}$ radians, or $\frac{114.6c}{l}$ deg. θ is normally 0.25 to 0.35 deg. If the shafts remain parallel, then the clearance permits a difference in height of $\frac{cL}{l}$ of the shaft centres. $\frac{L}{l}$ varies from 4 to 5.

For the purpose of lubricating the teeth on the driving side, oil is supplied—either from the adjoining bearing or by a special supply

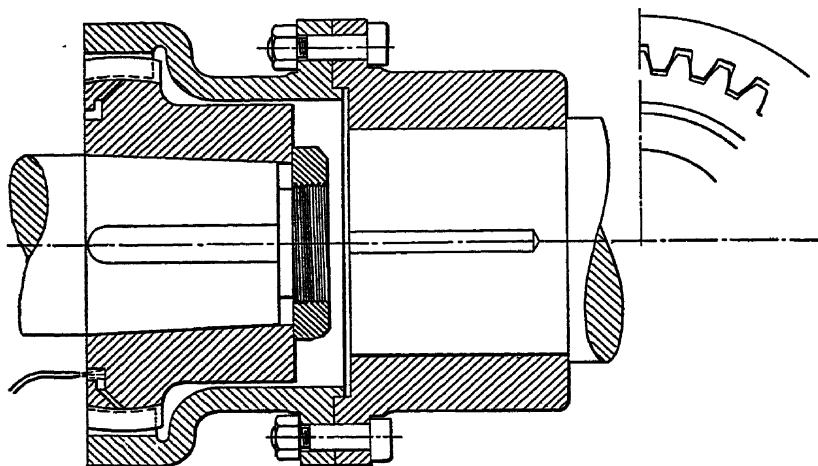


FIG. 213.—English Electric Type Flexible Coupling

pipe—to a groove D in the claw pieces. The oil passes through drilled holes to the distributing grooves on the faces of the teeth and is finally flung off by centrifugal action.

The tooth type of flexible coupling, in the form employed by the English Electric Co., Ltd., is shown in Fig. 213. A steel hub is keyed to the turbine shaft and is provided with several gear teeth at its periphery. A forged steel muff which is rigidly bolted to the driven shaft has internal teeth which gear with those on the hub. Means for lubrication are provided, as shown.

A type of coupling which has been employed successfully on some of the largest turbines built in this country is shown in Fig. 214. This is known as the Bibby coupling and is manufactured under licence from the Wellman Bibby Co., Ltd. It consists of two hubs keyed to the turbine shaft and driven shaft respectively, and provided with flanges. A number of accurately pitched grooves parallel to the axis are machined in the flanges. A tempered steel spring is bent to form a complete cylindrical grid. The segments of the spring are fitted in the grooves and the torque is transmitted from one

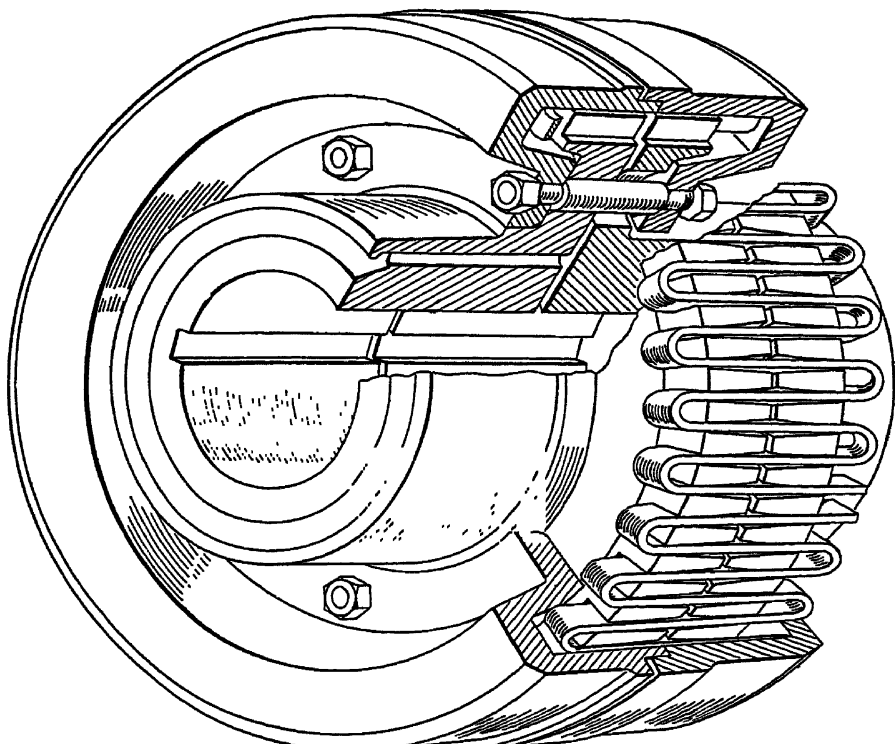


FIG 214 —Bibby Coupling

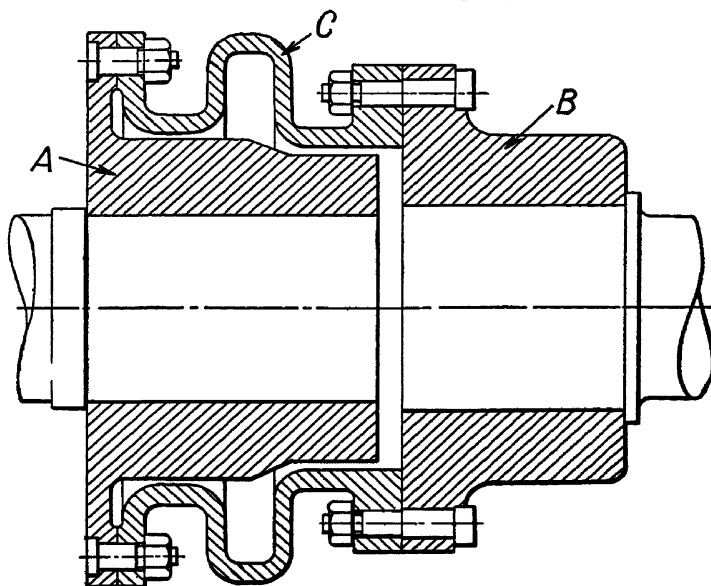


FIG 215 —Metropolitan-Vickers Semi-flexible Coupling

flange to the other through these flexible grids. The grooves widen somewhat towards the centre so that the spring fits closely only at its outer ends. Under a normal torque the drive takes place through the relatively long unsupported stretches of the spring members, which are very flexible and resilient in this condition. An increase in torque causes the springs to deflect, thus shortening their span and increasing their rigidity. This feature, while permitting flexibility in the torsional sense, gives freedom from resonance.

A semi-flexible coupling, constructed by the Metropolitan-Vickers Electrical Co., Ltd., to couple the L.P. turbine shaft and generator in two-cylinder tandem sets, is shown in Fig. 215. Hubs A and B, keyed to the turbine shaft and alternator shaft respectively, are connected together by a forged steel muff C, which gives rigidity in torsion but a limited degree of flexibility to allow for slight misalignment.

CHAPTER XIX

STRESSES IN TURBINE ROTORS

170. **Introductory.** The stresses in turbine rotors are due primarily to the inertia of the rotor mass and depend, therefore, on the form of the rotor, i.e. on the disposition of the mass, and on the peripheral speed. In most cases, the stresses imposed are relatively

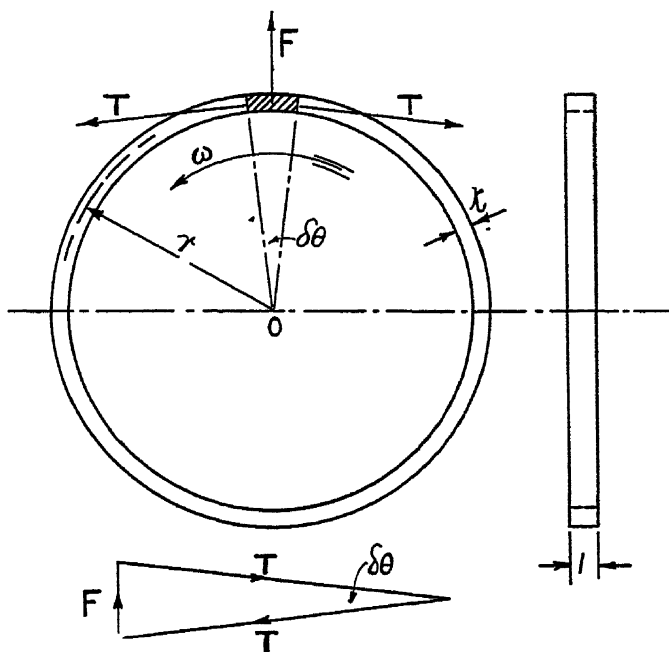


FIG 216

high and become a very definite limiting factor in the design. Consequently, the accurate estimation of rotor stresses is a matter of great importance. For this reason, a concise outline of this branch of strength of materials is considered essential in this book, if only to pave the way to a more extended study of the subject.

171. **Stress in Thin Rotating Ring.** Consider a thin ring, i.e. one in which the thickness t in the radial direction is small compared with the diameter, rotating about an axis through its centre with an angular velocity ω radians per second.

Let w = density of material.

f_{tr} = stress in thin ring. This is a tangential or hoop stress.

r = mean radius of ring.

Consider a ring whose length in the axial direction is unity. The cross-sectional area is then t units and the volume of the element subtending the angle $\delta\theta$ at the centre O is $t \cdot r \cdot \delta\theta$.

Hence mass of element = $w \cdot r \cdot t \cdot \delta\theta$.

$$\begin{aligned}\text{Centrifugal force } F &= \frac{wrt\delta\theta}{g} \times \omega^2 r \\ &= \frac{w \cdot \omega^2 r^2 \cdot t \cdot \delta\theta}{g}\end{aligned}$$

The element is in equilibrium under the action of the force F and the hoop tensions T . The angle between the two hoop tensions is $\delta\theta$ and their resultant, which is directed inwards, is $T \cdot \delta\theta$.

$$\therefore T \cdot \delta\theta = \frac{w \cdot \omega^2 r^2 \cdot t \cdot \delta\theta}{g}$$

Since the stress f_{tr} will be equal to the force T divided by the area of the ring—

$$f_{tr} = \frac{w \cdot \omega^2 r^2}{g} \quad . \quad . \quad . \quad (163)$$

or if u = peripheral velocity ($u = \omega r$)

$$f_{tr} = \frac{w \cdot u^2}{g} \quad . \quad . \quad . \quad (163a)$$

UNITS. If w = density in lb. per cub. in.

u = peripheral velocity, ft. per sec.

g = acceleration in ft. per sec.²

$$\text{then} \quad f_{tr} = \frac{1728 \cdot w \cdot u^2}{144 \cdot g} \text{ lb. per sq. in.}$$

$$\text{i.e.} \quad f_{tr} = \frac{12w \cdot u^2}{g} \text{ lb. per sq. in.}$$

For steel, $w = 0.283$; $f_{tr} = 0.1055u^2$ lb. per sq. in.

172. Stresses in Turbine Rotors of the Drum Type. The stresses in a turbine rotor of the drum type are greater than those in a thin ring rotating with the same peripheral velocity owing to the additional centrifugal force imposed by the blades.

In calculating the hoop stress, the component stresses may be obtained separately, that due to the blade load being obtained as follows—

In Fig. 217, let F_b be the centrifugal force on the blades per inch of periphery over an axial length equal to the axial pitch of the blades. Let A be the net area of rotor section per axial pitch in

square inches. Then on an arc subtending an angle $\delta\theta$ at the centre O, the total force F_1 due to the blade load $= r \cdot \delta\theta \cdot F_b$.

But
$$T_1 = \frac{F_1}{\delta\theta} = r \cdot F_b$$

$\therefore$ Stress in drum due to blade load

$$= \frac{r \cdot F_b}{A} \text{ lb. per sq. in.}$$

$\therefore$ Total hoop stress $= \frac{12wu^2}{g} + \frac{r \cdot F_b}{A} \text{ lb. per sq. in.}$

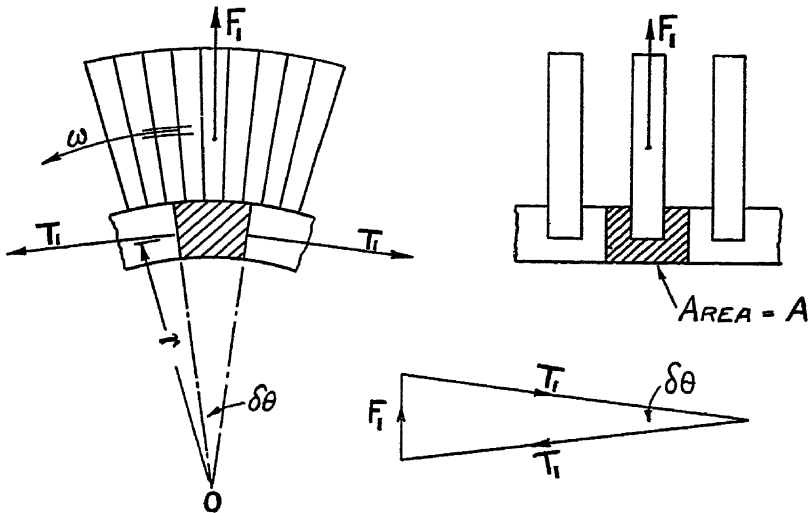


FIG 217

EXAMPLE 22. Calculate the intensity of hoop stress in a rotor of the drum type, given the following particulars—

External diameter of drum = 86 in.

Internal diameter of drum = $80\frac{1}{4}$ in.

Speed = 750 r.p.m.

Blades, 1 in. wide; $3\frac{1}{4}$ in. axial pitch; 19 in. high above rotor. Weight of blade per foot = 0.65 lb. Depth of groove, $1\frac{1}{4}$ in. Number of blades per foot of circumference, 22.2

Mean radius of drum = 41.56 in. = 3.47 ft.

$\therefore u = \frac{3.47 \times 2\pi \times 750}{60} = 272 \text{ ft. per sec.}$

$\therefore$ Stress in thin ring without blading $= 0.1055 \times (272)^2 = 7,800 \text{ lb. per sq. in.}$

Total length of one blade = $19 + 1\frac{1}{4} = 20\frac{1}{4}$ in.

Total weight of one blade = 1.097 lb. (W)

Distance of centre of gravity of blade from axis of rotation = 51.875 in.

$$\omega = \frac{2\pi \times 750}{60} = 78.5 \text{ rad. per sec.}$$

$$\therefore \text{Centrifugal force on each blade} = \frac{W\omega^2 r}{g} = 908 \text{ lb.}$$

$$\text{Force } F_b \text{ per inch} = \frac{908 \times 22.2}{12} = 1,680 \text{ lb.}$$

$$\text{Net area } A = (3.25 \times 2.875) - 1.25 = 8.1 \text{ sq. in.}$$

$$\therefore \text{Stress due to blade load} = \frac{43 \times 1680}{8.1} = 8,925 \text{ lb. per sq. in.}$$

$$\therefore \text{Combined drum stress} = 16,725 \text{ lb. per sq. in.}$$

173. **Stresses in Rotating Discs of Variable Thickness. Derivation of General Equations.** Consider an element of a turbine wheel,

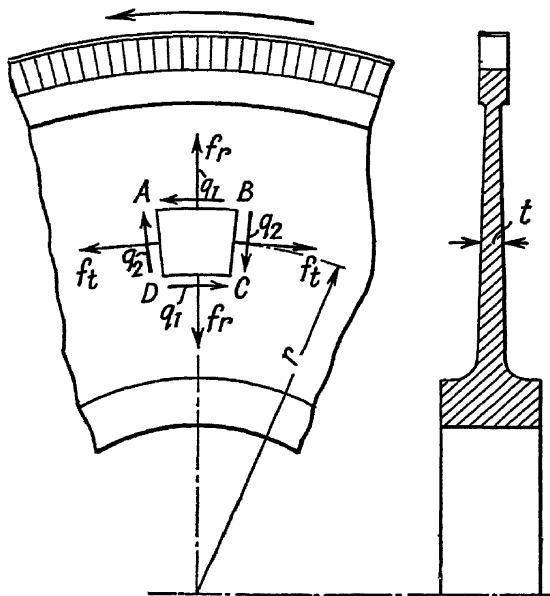


FIG. 218.—Stresses on Element of Turbine Disc

Fig. 218, bounded by radial lines AD, BC, and arcs AB, CD. The inertia forces due to the blades, the rim, and the disc itself will obviously produce direct stresses on the edges AB and CD of the element.

These are known as radial stresses. There will also be induced certain direct stresses on the radial edges AD and BC of the element in question. These are known as tangential, or hoop, stresses. Owing to the torque T applied by the blades to the rim of the wheel there will be shear stresses q_1 on the faces AB and CD (of magnitude

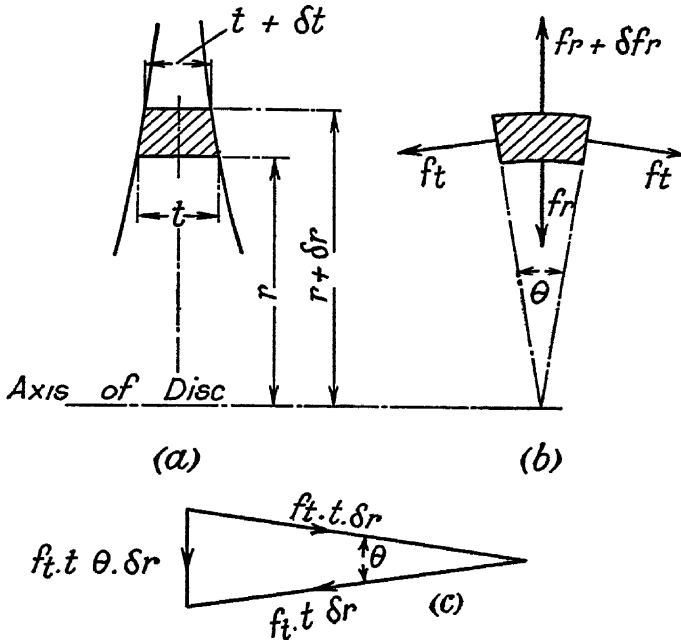


FIG 219

$q_1 = T/2\pi r^2 t$) and complementary shear stresses $q_2 = q_1$ on the faces AD and BC. In practically all cases, the shear stresses are negligibly small when compared with the direct stresses. Consequently, in any analysis where the wheel stresses are important, the shear stresses may be ignored. The radial and hoop stresses at any point may then be regarded as principal stresses

In the following expressions—

Let r = radius of disc at any point.

t = thickness of disc at radius r .

$$\mu = \frac{w}{g} \text{ (inch units).}$$

σ = Poisson's ratio = (strain at right angles to direction of stress) $\div$ (strain in direction of stress).

Consider an element of a disc of variable thickness, Fig. 219. (a) shows a section through the axis, while (b) shows the stresses in side view. The element is in equilibrium under the forces due to

stress and inertia. The forces due to hoop tension are each $f_t \cdot t \cdot \delta r$ and their resultant, as shown in Fig. 219 (c), is $f_t \cdot t \cdot \delta r \cdot \theta$, and is directed inwards.

The volume of the element is $r \cdot t \cdot \delta r \cdot \theta$, and the centrifugal force on it equals $\mu \omega^2 r^2 t \cdot \delta r \cdot \theta$.

$$\therefore (f_r + \delta f_r)(t + \delta t)(r + \delta r)\theta + \mu \omega^2 r^2 t \cdot \delta r \cdot \theta \\ = f_r \cdot t \cdot r \cdot \theta + f_t \cdot t \cdot \delta r \cdot \theta.$$

Working out, dividing by $\delta r \cdot \theta$, and proceeding to the limit, we have—

$$r \cdot t \cdot \frac{df_r}{dr} + r \cdot f_r \cdot \frac{dt}{dr} + t \cdot f_r - t \cdot f_t = -\mu \omega^2 r^2 t$$

$$\text{or} \quad \frac{d}{dr}(r \cdot t \cdot f_r) - t \cdot f_t = -\mu \omega^2 r^2 t. \quad (164)$$

Eq. (164) is a general equation for any rotating disc.

Suppose now that a point on the disc is at radius r when the disc is at rest (and unstressed) and that its radius is increased to $r + y$ when the disc is rotating and under stress; i.e. y is the radial displacement of a point on the disc at radius r . Then obviously the radial strain e_r will be $\frac{dy}{dr}$.

$$\text{Hoop strain } e_t = \frac{\text{Change in circumference}}{\text{Original circumference}} \\ = \frac{2\pi(r + y) - 2\pi r}{2\pi r} = \frac{y}{r}$$

$$\text{Then} \quad E \cdot \frac{dy}{dr} = f_r - \sigma \cdot f_t$$

$$\text{and} \quad E \cdot \frac{y}{r} = f_t - \sigma \cdot f_r$$

These are simultaneous equations in f_r and f_t , and give—

$$f_r = \frac{E}{1 - \sigma^2} \left(\sigma \cdot \frac{y}{r} + \frac{dy}{dr} \right). \quad (165)$$

$$f_t = \frac{E}{1 - \sigma^2} \left(\frac{y}{r} + \sigma \cdot \frac{dy}{dr} \right). \quad (166)$$

Introducing these values in eq. (164), differentiating, and multiplying throughout by $\frac{r(1 - \sigma^2)}{E \cdot t}$, we have—

$$r^2 \cdot \frac{d^2 y}{dr^2} + r \left(\frac{r}{t} \cdot \frac{dt}{dr} + 1 \right) \frac{dy}{dr} + \left(\frac{r \cdot \sigma}{t} \cdot \frac{dt}{dr} - 1 \right) y \\ = - \frac{\mu \omega^2 r^3 (1 - \sigma^2)}{E} \quad (167)$$

This equation is quite general and is independent of the manner in which the thickness varies with radius. As will be seen, it is a homogeneous linear differential equation

174. Disc of Constant Thickness. We shall now apply eq. (167) to find expressions for the stresses in a disc of constant thickness

Since t is a constant, $\frac{dt}{dr} = 0$, and hence—

$$r^2 \cdot \frac{d^2y}{dr^2} + r \cdot \frac{dy}{dr} - y = -\frac{\mu\omega^2 r^3 (1 - \sigma^2)}{E}$$

The solution consists of two parts, the complementary function and the particular integral. For the former, assume $y = b \cdot r^\psi$, where b and ψ are constants.

Then
$$r^2 \cdot \frac{d^2y}{dr^2} + r \cdot \frac{dy}{dr} - y = 0$$

Since
$$\frac{dy}{dr} = b \cdot \psi \cdot r^{\psi-1}, \text{ and } \frac{d^2y}{dr^2} = b (\psi^2 - \psi) r^{\psi-2}$$

$$b (\psi^2 - \psi) r^\psi + b \cdot \psi \cdot r^\psi - b r^\psi = 0$$

$$\therefore \psi^2 - 1 = 0$$

$$\text{i.e. } \psi = \pm 1$$

Since there are two roots, the solution is

$$y = b_1 r + \frac{b_2}{r}$$

where b_1 and b_2 are independent constants

For the particular integral, assume $y = ar^3$, where a is a constant.

Then
$$\frac{dy}{dr} = 3ar^2, \text{ and } \frac{d^2y}{dr^2} = 6ar$$

Substituting in the original equation, we have—

$$6ar^3 + 3ar^3 - ar^3 = -\frac{\mu\omega^2 r^3 (1 - \sigma^2)}{E}$$

from which
$$a = -\frac{\mu\omega^2 (1 - \sigma^2)}{8E} \quad . \quad . \quad . \quad (168)$$

a is a constant which depends on the angular velocity of the disc, the density of the material, and its elastic properties.

Hence the complete solution of the equation is

$$y = ar^3 + b_1 r + \frac{b_2}{r} \quad . \quad . \quad . \quad (169)$$

Inserting now the values of $\frac{y}{r}$ and $\frac{dy}{dr}$ derived from (169) in the general equations (165) and (166), we have—

$$f_r = \frac{E}{1 - \sigma^2} \left\{ ar^2(3 + \sigma) + b_1(1 + \sigma) - \frac{b_2}{r^2}(1 - \sigma) \right\}. \quad (170)$$

$$f_t = \frac{E}{1 - \sigma^2} \left\{ ar^2(1 + 3\sigma) + b_1(1 + \sigma) + \frac{b_2}{r^2}(1 - \sigma) \right\}. \quad (171)$$

The stresses f_r and f_t cannot be calculated by means of these expressions until the two constants b_1 and b_2 have been found. These must be determined with reference to the conditions as regards stress and strain at the inner and outer boundaries of the disc. Some examples will now be given.

175. Continuous Disc of Constant Thickness. Consider first a continuous disc, i.e. a disc without a hole at the centre. Assume, further, that the disc is not subjected to any edge loading, i.e. that there are no forces imposed due to masses attached to its periphery.

Then the constants b_1 and b_2 may be found from the following conditions—

(1) The radial displacement y must be zero at the centre of the disc.

Since
$$y = ar^3 + b_1r + \frac{b_2}{r}$$

it follows that b_2 must be zero; otherwise the displacement y would be infinite at the centre.

(2) The radial stress f_r must be zero at the periphery of the disc. Then if R_2 is the outer radius, by eq. (170)—

$$b_1(1 + \sigma) = -a \cdot R_2^2(3 + \sigma)$$

or
$$b_1 = -a \cdot R_2^2 \left(\frac{3 + \sigma}{1 + \sigma} \right)$$

Therefore, for any radius r

$$f_r = \left(\frac{3 + \sigma}{8} \right) \cdot \frac{w\omega^2}{g} \cdot (R_2^2 - r^2) \quad . \quad . \quad (172)$$

$$f_t = \frac{w\omega^2}{8g} \{ (3 + \sigma) R_2^2 - (1 + 3\sigma)r^2 \} \quad . \quad . \quad (173)$$

For steel discs $w = 0.283$ lb. per cub. in.

$\sigma = 0.3$ approx.

Let $N = R.p.m.$

$$f_r = 3.31 \left(\frac{N}{1000} \right)^2 (R_2^2 - r^2) \quad . \quad . \quad (174)$$

$$f_t = 1.91 \left(\frac{N}{1000} \right)^2 (1.74 R_2^2 - r^2) \quad . \quad . \quad (175)$$

If the disc had a radial stress imposed upon its periphery due to attached masses, then the constants may be found as before, except that f_r at the periphery would be made equal to the applied stress.

176. Disc of Constant Thickness with Central Hole.

Let R_1 = inner radius of disc.

R_2 = outer radius of disc.

Case 1. Disc free from constraints. Consider first of all a disc which is free from any applied stresses at the boundaries. Then the radial stress is zero when $r = R_1$ and also when $r = R_2$. By substituting in eq. (170) it is found that

$$b_1 = -a \left(\frac{3 + \sigma}{1 + \sigma} \right) (R_1^2 + R_2^2)$$

$$b_2 = -a R_1^2 R_2^2 \left(\frac{3 + \sigma}{1 - \sigma} \right)$$

Inserting these constants in (170) and (171), we have—

$$f_r = \frac{w \omega^2}{g} \left(\frac{3 + \sigma}{8} \right) \left(R_1^2 + R_2^2 - \frac{R_1^2 R_2^2}{r^2} - r^2 \right) \quad (176)$$

$$f_t = \frac{w \omega^2}{8g} \left\{ (3 + \sigma) \left(R_1^2 + R_2^2 + \frac{R_1^2 R_2^2}{r^2} \right) - (1 + 3\sigma)r^2 \right\}. \quad (177)$$

For steel discs—

$$f_r = 3.31 \left(\frac{N}{1000} \right)^2 \left(R_1^2 + R_2^2 - \frac{R_1^2 R_2^2}{r^2} - r^2 \right) \quad . \quad (178)$$

$$f_t = 1.91 \left(\frac{N}{1000} \right)^2 \left\{ 1.74 \left(R_1^2 + R_2^2 + \frac{R_1^2 R_2^2}{r^2} \right) - r^2 \right\} \quad . \quad (179)$$

In the continuous disc, the tangential and radial stresses must necessarily tend to the same value as the centre of the disc is approached. By writing $r = 0$ in eqs. (172) and (173), it is seen that

$$\begin{aligned} f_t = f_r &= \left(\frac{3 + \sigma}{8} \right) \frac{w \omega^2 R_2^2}{g} \\ &= \left(\frac{3 + \sigma}{8} \right) f_{tr} \end{aligned}$$

where f_{tr} is the hoop stress in a thin ring rotating with an equal peripheral velocity.

Consider now the disc with a central hole. The hoop stress given by eq. (177) is clearly the maximum when r is the least value possible, i.e. when $r = R_1$.

$$f_t (\text{max.}) = \frac{w \omega^2}{8g} \{ (3 + \sigma)(R_1^2 + 2R_2^2) - (1 + 3\sigma)R_1^2 \}$$

When R_1 is made very small

$$f_t (\text{max.}) = \left(\frac{3 + \sigma}{4} \right) \cdot \frac{w \cdot \omega^2 R_2^2}{g} = \left(\frac{3 + \sigma}{4} \right) \cdot f_{tr}$$

i.e. by drilling a small hole at the centre of a continuous disc free from constraints, *the maximum tangential stress is just doubled.*

The results given by eqs. (176) and (177) may be shown graphically in an interesting and instructive manner. We shall express the stress in the disc as a fraction of that in a thin ring of the same material and rotating with the same peripheral velocity.

Let $m_0 = \frac{R_1}{R_2}$, and $m = \frac{r}{R_2}$

Since $f_{tr} = \frac{w \omega^2 R_2^2}{g}$

$$\begin{aligned} \frac{f_r}{f_{tr}} &= \left(\frac{3 + \sigma}{8} \right) \left(\frac{R_1^2}{R_2^2} + \frac{R_2^2}{R_2^2} - \frac{R_1^2 R_2^2}{r^2 R_2^2} - \frac{r^2}{R_2^2} \right) \\ &= \left(\frac{3 + \sigma}{8} \right) \left(m_0^2 + 1 - \frac{m_0^2}{m^2} - m^2 \right) \quad . \quad . \quad (180) \\ &= 0.4125 \left(m_0^2 + 1 - \frac{m_0^2}{m^2} - m^2 \right) \text{ for steel} \end{aligned}$$

$$\begin{aligned} \frac{f_t}{f_{tr}} &= \left(\frac{3 + \sigma}{8} \right) \left(m_0^2 + 1 + \frac{m_0^2}{m^2} \right) - \left(\frac{1 + 3\sigma}{8} \right) m^2 \quad . \quad (181) \\ &= 0.4125 \left(m_0^2 + 1 + \frac{m_0^2}{m^2} \right) - 0.2375 m^2 \text{ for steel} \end{aligned}$$

Curves showing the values of these ratios as a function of m_0 and m and for steel discs are given in Fig. 220. It will be seen that in all cases the tangential stress is a maximum at the bore and diminishes, rapidly at first, towards the periphery. The influence of the size of hole should also be noticed; as the ratio m_0 is increased the maximum and minimum hoop stresses increase until, when m_0 approaches unity, the stress is equal to that in the thin ring, while the radial stress vanishes.

Case 2. Disc subjected to stresses at the inner and outer edges. When the disc is subjected to radial stresses at either the bore or the periphery, or both, the radial and hoop stresses within the disc will be different from those given by the foregoing equations and the graphs of Fig. 220. There are two methods of evaluating the stresses.

Let f_1 = radial stress applied at bore (Fig. 221).

f_2 = radial stress applied at periphery (Fig. 221).

In general, it is more convenient to consider tensile stresses as positive and compressive stresses as negative. Reverting to the general

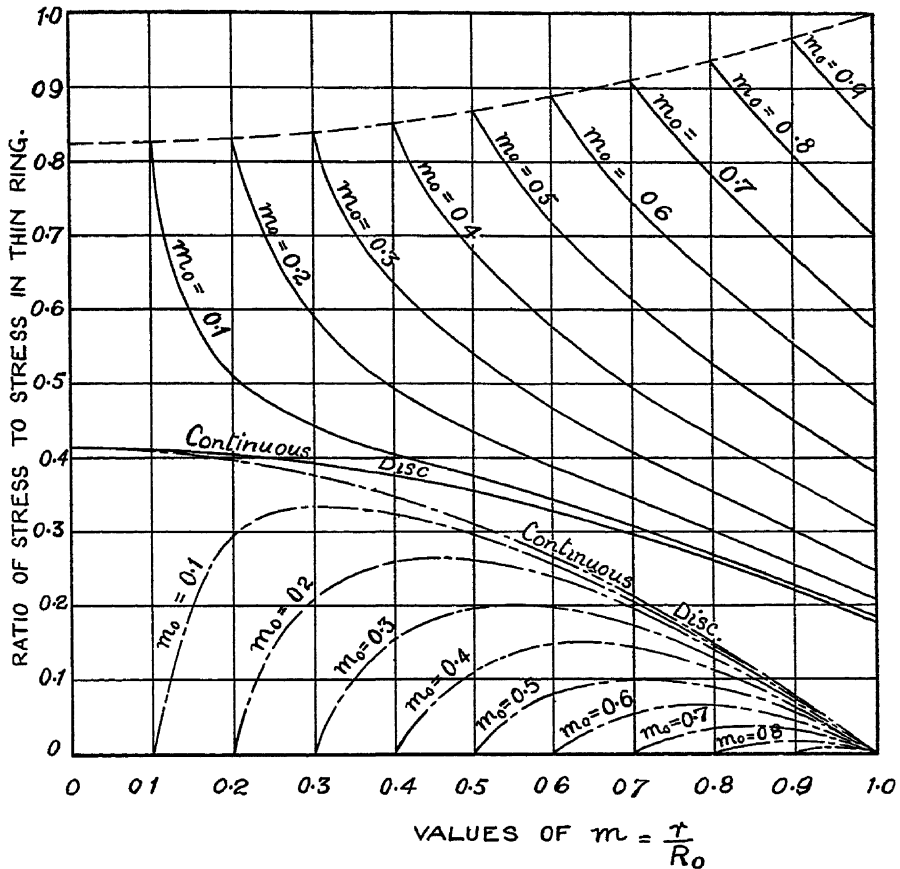


FIG. 220—Curves Showing the Ratio of the Stress in a Disc of Uniform Thickness to the Stress in a Thin Ring with an Equal Peripheral Velocity

$$m_0 = \frac{\text{Radius of bore}}{\text{External radius of disc}}$$

eqs. (170) and (171), the constants b_1 and b_2 may be found by writing in (170) $f_r = f_1$ when $r = R_1$, and $f_r = f_2$ when $r = R_2$. This gives the necessary expressions for f_r and f_t , also, if required, the radial displacement y , by inserting the values of b_1 and b_2 in eq. (169).

In the second method the stresses due to inertia and those due to the applied radial forces are calculated separately and then

added to give the resultant stress. The stresses due to inertia are calculated by using eqs. (176) to (179). The stresses in the disc due to the applied edge stresses may be found by using Lamé's

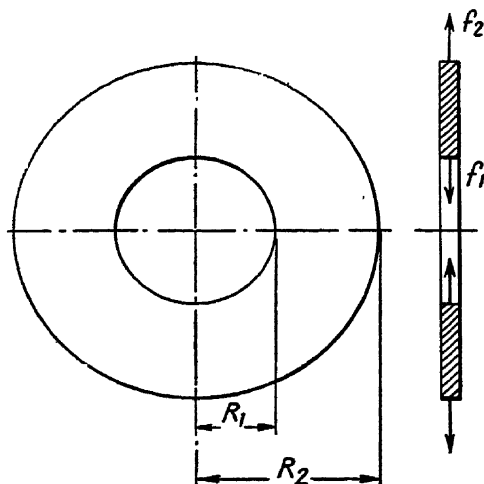


FIG. 221

equations for a thick cylinder, provided the thickness of the disc is relatively small compared with the diameter.

Then assuming that

$$f_r = A - \frac{B}{r^2}, \text{ and } f_t = A + \frac{B}{r^2}$$

where A and B are constants, and substituting $f_r = f_1$, when $r = R_1$, and $f_r = f_2$ when $r = R_2$, we have—

$$f_r = \frac{f_2 \cdot R_2^2 - f_1 \cdot R_1^2}{R_2^2 - R_1^2} - \left(\frac{f_2 - f_1}{R_2^2 - R_1^2} \right) \frac{R_1^2 R_2^2}{r^2} \quad . \quad (182)$$

$$f_t = \frac{f_2 R_2^2 - f_1 R_1^2}{R_2^2 - R_1^2} + \left(\frac{f_2 - f_1}{R_2^2 - R_1^2} \right) \frac{R_1^2 R_2^2}{r^2} \quad . \quad (183)$$

EXAMPLE 23. A disc 1 in. thick, 4 in. internal diameter, and 20 in. external diameter rotates at 4,000 r.p.m. Given that the density of the material is 0.284 lb. per cub. in., and that the value of σ is 0.3, calculate the following—

- (a) Stresses in free rotating disc.
- (b) Stresses due to an imposed radial stress (tensile) of 6,000 lb. per sq. in. at the periphery of the stationary disc.
- (c) Stresses due to rotation and edge load combined.

(a) Free Rotating Disc.

$R_1 = 2$ in., $R_2 = 10$ in. Substituting these values in eq. (176), we have—

$$f_r = 53.1 \left(104 - \frac{400}{r^2} - r^2 \right)$$

$$f_t = 53.1 \left\{ \left(104 + \frac{400}{r^2} \right) - 0.575r^2 \right\}$$

The calculated values are given in Table XII and are shown in Fig. 222.

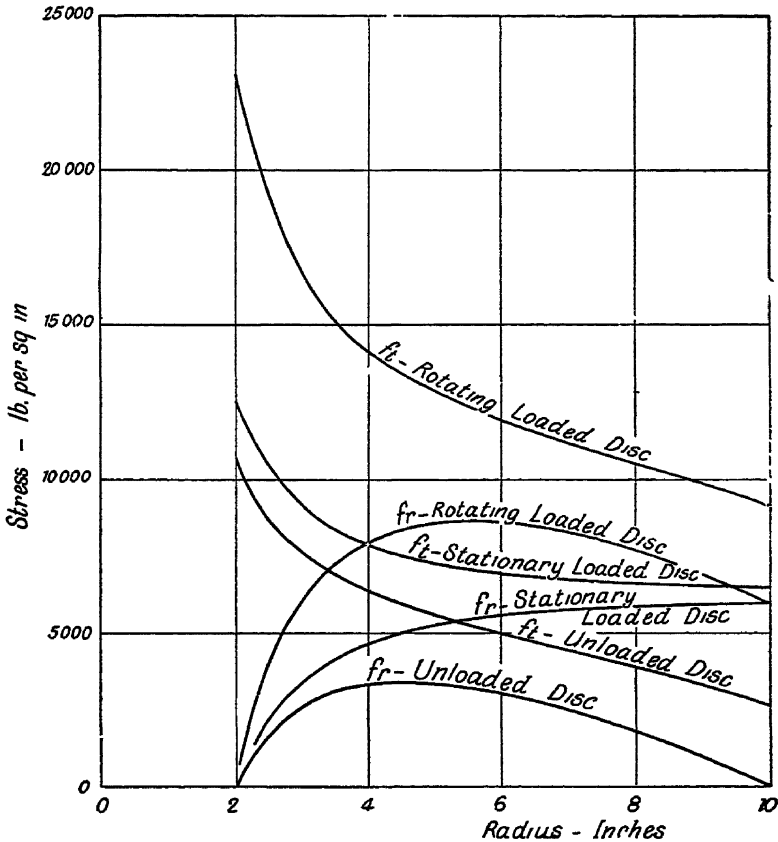


FIG. 222

(b) Here $f_1 = 0$, $f_2 = 6000$ Substituting in eqs. (182) and (183)—

$$f_r = 6250 - \frac{25,000}{r^2}$$

$$f_t = 6250 + \frac{25,000}{r^2}$$

The calculated values are again given in Table XII and Fig. 222.

(c) The resultant stresses in the loaded rotating disc are found by adding the values obtained in (a) and (b) above, or by the following method.

By eq. (170)—

$$f_r = \frac{E}{1-\sigma^2} \left\{ ar^2(3+\sigma) + b_1(1+\sigma) - \frac{b_2}{r^2}(1-\sigma) \right\}$$

and since

$$a = -\frac{w\omega^2(1-\sigma^2)}{8gE}$$

$$f_r = -53.1r^2 + E \left(1.43b_1 - 0.77 \frac{b_2}{r^2} \right)$$

When $r = 2$ in., $f_r = 0$; when $r = 10$ in., $f_r = 6000$.

$$\therefore b_1 = \frac{8230}{E}; b_2 = \frac{60,000}{E}$$

$$\therefore f_r = -53.1r^2 + 11,770 - \frac{46,100}{r^2}$$

and $f_i = -30.65r^2 + 11,770 + \frac{46,100}{r^2}$

Apart from negligible slide-rule errors, the values given by these expressions agree with those given in Table XII.

TABLE XII
STRESSES IN ROTATING DISC OF CONSTANT THICKNESS SUBJECTED
TO RADIAL STRESS AT PERIPHERY
(Example 23)

Radius r in.	Stresses in Unloaded Rotating Disc		Stationary Disc Considered as Thick Cylinder		Resultant Stresses in Loaded Rotating Disc	
	f_r	f_i	f_r	f_i	f_r	f_i
2	0	10,700	0	12,500	0	23,200
3	2,680	7,610	3,470	9,030	6,150	16,640
4	3,350	6,360	4,688	7,812	8,038	14,172
5	3,350	5,610	5,250	7,250	8,600	12,860
6	3,020	5,010	5,555	6,945	8,575	11,955
7	2,485	4,460	5,740	6,760	8,225	11,220
8	1,790	3,900	5,859	6,641	7,649	10,541
9	960	3,310	5,940	6,560	6,900	9,870
10	0	2,680	6,000	6,500	6 000	9,180

177. Stresses in Turbine Wheel of Constant Thickness. Fig. 223 shows a half-section through a turbine wheel forming part of a rotor turned out of a solid forging (see Fig. 200).

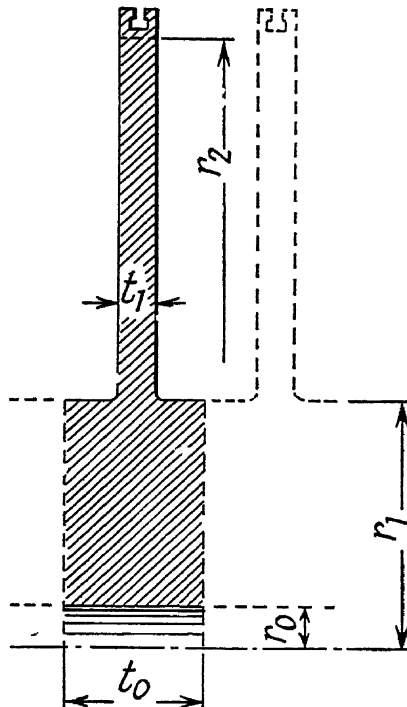


FIG 223

Let f_2 = stress due to blades, assumed to be uniformly distributed over the edge of the disc at radius r_2 .

f_r = radial stress in disc proper

f_t = hoop stress in disc proper.

f_r' = radial stress in shaft.

f_t' = hoop stress in shaft.

y = radial displacement of disc proper

y' = radial displacement of shaft.

Assume that the rotor has a hole (radius = r_0) bored through the centre, in accordance with usual practice.

We shall consider the shaft as a disc of constant thickness t_0 (equal to the axial distance between disc centres). If f_{r1} is the radial stress in the disc proper at radius r_1 , then, as a rough approximation, we may regard the shaft as being subjected to a

radial stress $f_{r1} = \frac{t_1 \cdot f_{r1}}{t_0}$ at its outer surface. The radial stress f_{r0} at radius r_0 is obviously zero.

Referring to eqs. (170) and (171) for the disc of constant thickness, and using capital letters for the constants for the shaft and small letters for the disc constants, we have—

$$\frac{E}{1-\sigma^2} \left\{ (3+\sigma)Ar_0^2 + B_1(1+\sigma) - \frac{B_2}{r_0^2}(1-\sigma) \right\} = f_{r0} = 0$$

$$\frac{E}{1-\sigma^2} \left\{ (3+\sigma)Ar_1^2 + B_1(1+\sigma) - \frac{B_2}{r_1^2}(1-\sigma) \right\} = f_{r1} = \frac{t_1 \cdot f_{r1}}{t_0}$$

These are simultaneous equations in B_1 and B_2 , and enable us to express B_1 and B_2 in terms of f_{r1} .

$$\text{But } f_{r1} = \frac{E}{1-\sigma^2} \left\{ ar_1^2(3+\sigma) + b_1(1+\sigma) - \frac{b_2}{r_1^2}(1-\sigma) \right\}$$

Hence we can express B_1 and B_2 in terms of b_1 and b_2 .

The radial displacement of the shaft at radius r_1 , when under stress, must be equal to that of the disc at the same radius.

$$\text{But } y_1' = Ar_1^3 + B_1r_1 + \frac{B_2}{r_1}$$

$\therefore y_1'$ may be expressed in terms of b_1 and b_2 .

$$\text{But } y_1 = ar_1^3 + b_1r_1 + \frac{b_2}{r_1}$$

Therefore, by equating y_1' and y_1 we obtain one equation connecting b_1 and b_2 .

Considering now the periphery of the disc, it will be clear that—

$$\frac{E}{1-\sigma^2} \left\{ ar_2^2(3+\sigma) + b_1(1+\sigma) - \frac{b_2}{r_2^2}(1-\sigma) \right\} = f_2$$

Since f_2 is known, we have a second equation connecting b_1 and b_2 . The solution for b_1 and b_2 enables B_1 and B_2 to be found and the stresses in the disc and shaft to be calculated.

r78. Discs of Hyperbolic Section. The foregoing examples on stresses in rotating discs of uniform thickness may be summarized in a diagrammatic form in Fig. 224. In (a) we have a continuous disc, in which the radial and tangential stresses are equal at the centre. The radial stress is zero at the periphery and the tangential stress diminishes from the centre outwards. In (b) we have a disc with a central hole, while in (c) the conditions are the same as in (b) except that a radial stress of uniform intensity is applied at the periphery. In each case it will be observed that the more important stress, i.e. the tangential stress, is much greater at the inner radius than at the periphery. In many cases it is important to

reduce the weight of the rotor to a minimum, for by so doing, the critical speed of the rotor may be raised or, alternatively, the shaft diameter for a given speed may be reduced. In order to attain this end, the distribution of stress should be as uniform as possible, and the greatest stress should be as high as possible. A better distribution of hoop stress may be obtained by reducing the thickness where the stress in the disc of constant thickness is low, and

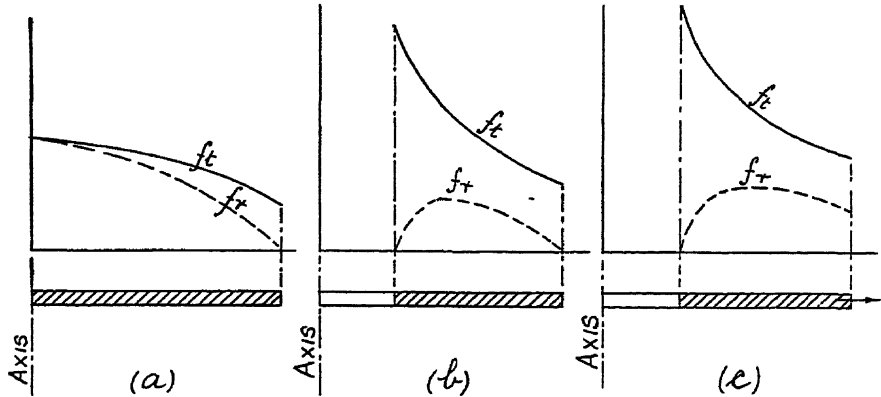


FIG 224

increasing the thickness where the stress is high; in other words, by using a disc of tapered section

The calculation of stresses in tapered discs of conical profile presents a mathematical problem of considerable difficulty—although solutions have been found. A much simpler form to deal with mathematically is the hyperbolic profile, in which

$$t = C \cdot r^{-\alpha}$$

where C and α are positive constants (see Fig 225). The solution for this type is due to Professor A. Stodola

$$\text{Since } t = C \cdot r^{-\alpha}, \quad \frac{1}{t} \cdot \frac{dt}{dr} = -\frac{\alpha}{r}$$

Substituting this value in the general equation (167), Art. 173, we have—

$$r^2 \cdot \frac{d^2 y}{dr^2} + r(1 - \alpha) \frac{dy}{dr} - (1 + \alpha\sigma)y = -\frac{\mu\omega^2 r^3(1 - \sigma^2)}{E}$$

For the complementary function, write $y = b \cdot r^\psi$.

$$r^2 \cdot \frac{d^2 y}{dr^2} + r(1 - \alpha) \frac{dy}{dr} - (1 + \alpha\sigma)y = 0$$

$$\therefore b(\psi^2 - \psi)r^\psi + (1 - \alpha)b \cdot \psi \cdot r^\psi - (1 + \alpha\sigma)b \cdot r^\psi = 0$$

$$\text{or} \quad \psi^2 - \alpha\psi - (1 + \alpha\sigma) = 0$$

from which
$$\psi_1 = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + \alpha\sigma + 1} . \quad . \quad . \quad (184)$$

$$\psi_2 = \frac{\alpha}{2} - \sqrt{\frac{\alpha^2}{4} + \alpha\sigma + 1} . \quad . \quad . \quad (185)$$

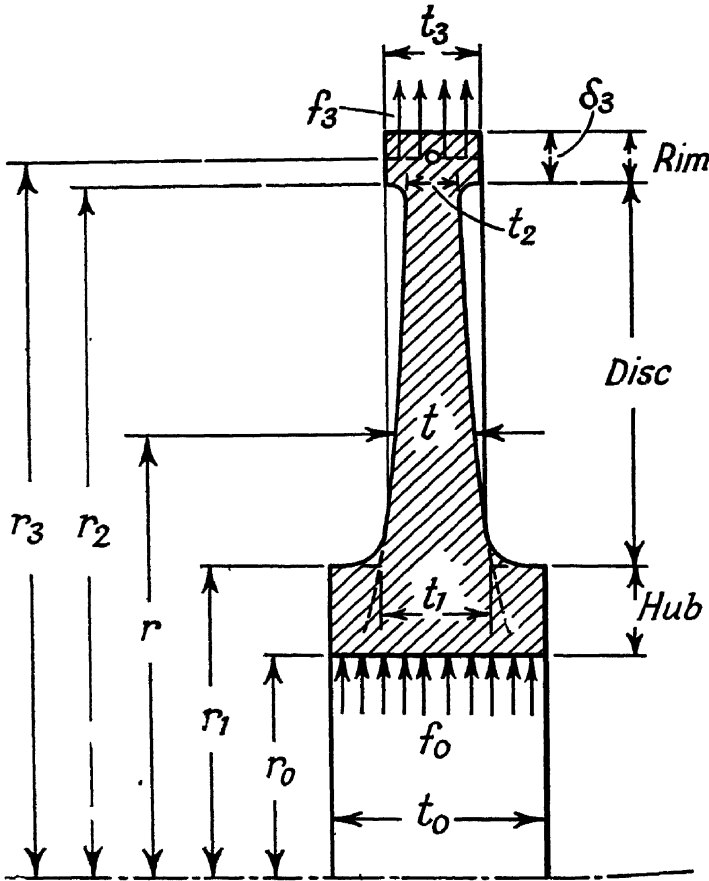


FIG. 225

Thus there are two constants b_1 and b_2 associated with the constants ψ_1 and ψ_2 , and

$$y = b_1 r^{\psi_1} + b_2 r^{\psi_2}$$

For a particular integral, write $y = ar^3$.

$$\therefore 6ar^3 + 3ar^3(1 - \alpha) - ar^3(1 + \alpha\sigma) = -\frac{\mu\omega^2 r^3(1 - \sigma^2)}{E}$$

from which
$$a = -\frac{\mu\omega^2(1 - \sigma^2)}{E\{8 - (3 + \sigma)\alpha\}} . \quad . \quad . \quad (186)$$

a is a constant depending on the speed and on the properties of the material. Then the complete solution for the radial deformation y is—

$$y = ar^3 + b_1 r^{\psi_1} + b_2 r^{\psi_2} \quad . \quad . \quad . \quad (187)$$

Inserting values of $\frac{y}{r}$ and $\frac{dy}{dr}$ in (165) and (166), we have—

$$f_r = \frac{E}{1 - \sigma^2} \left\{ (3 + \sigma) ar^2 + (\psi_1 + \sigma) b_1 r^{\psi_1-1} + (\psi_2 + \sigma) b_2 r^{\psi_2-1} \right\} \quad (188)$$

$$f_t = \frac{E}{1 - \sigma^2} \left\{ (1 + 3\sigma) ar^2 + (1 + \psi_1\sigma) b_1 r^{\psi_1-1} + (1 + \psi_2\sigma) b_2 r^{\psi_2-1} \right\} \quad . \quad . \quad . \quad (189)$$

Here again, b_1 and b_2 are constants which must be found by reference to the conditions at the inner and outer boundaries of the disc proper.

Consider the hub first, and suppose that the mutual pressure between the shaft and the inside of the hub due to force fit is f_0 lb per sq. in. at the running speed, or at the speed of rotation for which the stress calculations are being made. When the disc is stationary, this pressure will, of course, be much greater.

Considering the hub as a disc of constant thickness, then by eq (170) (using capital letters for the constants to distinguish them from the disc constants) we have—

$$f_r = \frac{E}{1 - \sigma^2} \left\{ Ar^2 (3 + \sigma) + B_1 (1 + \sigma) - \frac{B_2}{r^2} (1 - \sigma) \right\}$$

where
$$A = - \frac{\mu\omega^2 (1 - \sigma^2)}{8E}$$

The disc is pulling outwards on the hub over a width t_1 with a tensile radial stress f_{r1} . Then, approximately, the radial stress at the outer surface of the hub is—

$$f_{r1}' = \frac{t_1}{t_0} f_{r1}$$

This approximation demands a sudden change in radial stress at the junction of the disc and hub which is quite impossible. We accept the approximation, however, as a rough and ready means of solving a problem of almost insuperable difficulty. Then—

$$\frac{E}{1 - \sigma^2} \left\{ Ar_1^2 (3 + \sigma) + B_1 (1 + \sigma) - \frac{B_2}{r_1^2} (1 - \sigma) \right\} = \frac{t_1 \cdot f_{r1}}{t_0}$$

$$\frac{E}{1 - \sigma^2} \left\{ Ar_0^2 (3 + \sigma) + B_1 (1 + \sigma) - \frac{B_2}{r_0^2} (1 - \sigma) \right\} = -f_0$$

since f_0 is compressive. These equations enable us to express B_1

and B_2 in terms of the unknown stress f_{r1} , a value of f_0 having been assumed.

Let y represent radial deformation of disc.

Let y' represent radial deformation of hub or rim.

Then y_1' for the hub at radius r_1 is—

$$y_1' = Ar_1^3 + B_1r_1 + \frac{B_2}{r_1}$$

and may be expressed in terms of f_{r1} . But for the disc—

$$f_{r1} = \frac{E}{1 - \sigma^2} \left\{ (3 + \sigma) ar_1^2 + (\psi_1 + \sigma) b_1 r_1^{\psi_1 - 1} + (\psi_2 + \sigma) b_2 r_1^{\psi_2 - 1} \right\}$$

Hence y_1' may be expressed in terms of the disc constants b_1 and b_2 . But y_1 for the disc is $ar_1^3 + b_1 r_1^{\psi_1} + b_2 r_1^{\psi_2}$. Since the amount of radial deformation must necessarily be the same for the hub and the disc at r_1 , $y_1' = y_1$. Thus we obtain one equation connecting b_1 and b_2 .

In general, the rim is of comparatively small section, and we may treat it as a loaded thin rotating ring. Let r_3 be the radius to the centroid of the rim section. Due to the centrifugal force on the blades, etc., there will be a certain stress f_3 per sq. in. of periphery at radius r_3 , which may be calculated. The disc is pulling inwards at radius r_2 with a stress intensity f_{r2} over the thickness t_2 . Considering the forces across a diameter due, respectively, to the inertia of the rim, the stress f_3 and the stress f_{r2} , we have—

$$\text{Diametral force} = 2 (\mu\omega^2 t_3 \delta_3 r_3^2 + r_3 t_3 f_3 - r_2 t_2 f_{r2})$$

Let $f_3' =$ hoop stress in rim

$$= \frac{1}{t_3 \delta_3} (\mu\omega^2 t_3 \delta_3 r_3^2 + r_3 t_3 f_3 - r_2 t_2 f_{r2})$$

Then, ignoring the radial stress, the hoop strain in the rim will be—

$$\frac{f_3'}{E} = \frac{y_2'}{r_3}, \text{ or } y_2' = \frac{r_3 \cdot f_3'}{E}$$

Thus y_2' may be expressed in terms of f_{r2} , and hence in terms of b_1 and b_2 . But y_2 for the disc proper is—

$$y_2 = ar_2^3 + b_1 r_2^{\psi_1} + b_2 r_2^{\psi_2}$$

Since $y_2 = y_2'$, we obtain a second equation for b_1 and b_2 , enabling these constants, and in turn B_1 and B_2 , to be determined. Once these have been found, the stresses and strains in the complete rotor may be worked out.

EXAMPLE 24. Calculate the stresses in the turbine wheel shown in Fig. 226, given the following particulars—

Speed of rotation, 3000 r.p.m.; mean radius of blades = 20 in.;

mean circumferential pitch of blades = 0.503 in ; centrifugal force on each blade, its shrouding, and the blade root = 2210 lb., mutual force between wheel and shaft at 3000 r.p.m. = 500 lb per sq. in. Assume w for steel = 0.283 lb per cub in., σ for steel = 0.3

We first find the value of α for the wheel profile Tabulating r , t , $\log r$, and $\log t$ —

Radius r	Thickness t	Log r	Log t
6.5	1.5	0.8129	0.1761
8	1.2	0.9031	0.0792
10	0.96	1.0000	1.9823
13.75	0.7	1.1383	1.8451

For the hyperbolic profile, $t = C \cdot r^\alpha$, $\log t = \log C - \alpha \log r$,

$$\text{i.e.} \quad \alpha = \frac{\log \frac{t_1}{t_2}}{\log \frac{r_2}{r_1}}$$

Plotting the above values of $\log r$ and $\log t$, it will be found that $\alpha = 1.01$. For the purposes of calculation, α will be made equal to 1. Then, by eqs (184) and (185)—

$$\psi_1 = 1.744, \text{ and } \psi_2 = -0.744 \quad \text{Also, by eq 186, } a = -\frac{13.99}{E}$$

In order to obtain the constants b_1 and b_2 in the stress equations (188) and (189), we consider first the hub

$$\text{Hub.} \quad f_0 = -500. \quad A = -\frac{\mu\omega^2(1-\sigma^2)}{8E} = -\frac{8.22}{E}$$

Radial stress in hub—

$$\begin{aligned} f_r' &= \frac{E}{1-\sigma^2} \left\{ A r^2 (3+\sigma) + B_1 (1+\sigma) - \frac{B_2}{r^2} (1-\sigma) \right\} \\ &= -29.8 r^2 + \frac{E}{0.91} \left(1.3 B_1 - \frac{0.7 B_2}{r^2} \right) \end{aligned} \quad (a)$$

When $r = 5$ in, $f_r' = f_0 = -500$. Substituting in (a) we have—

$$B_1 - 0.0215 B_2 = \frac{171.2}{E} \quad (b)$$

Also, when $r = 6.5$ in.—

$$f_{r1}' = \frac{1.5 f_{r1}}{3.75} = 0.4 f_{r1}$$

where f_{r1} is the stress in the *disc* at 6.5 in. radius.

Substituting in eq. (188), we have—

$$f_{r1} = -2146 + 9.06Eb_1 - 0.01864Eb_2$$

$$\therefore f_{r1}' = -859 + 3.62Eb_1 - 0.00746Eb_2 \quad . \quad . \quad (c)$$

By eq. (a) above—

$$f_{r1}' = -1258 + 1.43EB_1 - 0.0182EB_2 \quad . \quad . \quad (d)$$

Equating (c) and (d), and simplifying—

$$B_1 - 0.0127B_2 = \frac{279}{E} + 2.53b_1 - 0.00522b_2 \quad . \quad (e)$$

From (b) and (e) we obtain the values of B_1 and B_2 , the hub constants.

$$B_1 = \frac{434.7}{E} + 6.19b_1 - 0.0128b_2 \quad . \quad . \quad (f)$$

$$B_2 = \frac{12,260}{E} + 288b_1 - 0.594b_2 \quad . \quad . \quad (g)$$

The radial deformation of the *hub* at radius r_1 is—

$$y_1' = Ar_1^3 + B_1r_1 + \frac{B_2}{r_1}, \text{ and as } r_1 = 6.5$$

$$y_1' = \frac{2450}{E} + 84.5b_1 - 0.1746b_2 \quad . \quad . \quad (h)$$

But for the *disc*, $y_1 = ar_1^3 + b_1r_1^{v_1} + b_2r_1^{v_2}$

$$\text{i.e.} \quad y_1 = -\frac{3840}{E} + 26.18b_1 + 0.248b_2 \quad . \quad . \quad (i)$$

Equating y_1' and y_1 , we find—

$$b_1 - 0.00726b_2 = -\frac{107.9}{E}$$

This is one of the two expressions relating b_1 and b_2 .

Considering now the rim, we have $r_3 = 14.7$ in. The area of the rim section is 2.84 sq. in., and since $t_3 = 2.125$ in., $\delta_3 = 1.335$ in. Since the circumferential distance at 14.7 in. radius, corresponding to one blade pitch, is 0.37 in., the stress f_3 due to the centrifugal force on the blades is—

$$f_3 = \frac{2210}{2.125 \times 0.37} = 2810 \text{ lb. per sq. in.}$$

Hoop stress in rim

$$\begin{aligned} &= \frac{1}{t_3\delta_3} \{ \mu\omega^2 t_3 \delta_3 r_3^2 + r_3 t_3 f_3 - r_2 t_2 f_{r2} \} \\ &= 46,600 - 3.39f_{r2} \end{aligned}$$

Also
$$y_2' = \frac{r_3 f}{E} = \frac{685,000}{E} - \frac{49.9f_{r_2}}{E}$$

Substituting for $r_2 = 13.75$ in eq (188) for the disc—

$$f_{r_2} = -9590 + 15.78Eb_1 - 0.00503Eb_2$$

$$\therefore y_2' = \frac{1,163,000}{E} - 787b_1 + 0.2505b_2$$

Also
$$y_2 = ar_2^3 + b_1r_2^{\psi_1} + b_2r_2^{\psi_2}$$

$$= -\frac{36,400}{E} + 97.05b_1 + 0.1423b_2$$

Since $y_2 = y_2'$,
$$b_1 = \frac{1380}{E}$$

and
$$b_2 = \frac{205,200}{E}$$

Substituting in eq. (188) and (189), we now obtain the general equations for the disc stresses—

$$f_r = -50.75r^2 + 3100r^{0.744} - \frac{100,000}{r^{1.744}}$$

$$f_t = -29.2r^2 + 2320 r^{0.744} + \frac{175,000}{r^{1.744}}$$

The hub constants are readily found to be—

$$B_1 = \frac{6348}{E}, \text{ and } B_2 = \frac{287,860}{E}$$

and the equations for the hub stresses—

$$f_r' = -29.8r^2 + 9070 - \frac{221,000}{r^2}$$

$$f_t' = -17.2r^2 + 9070 + \frac{221,000}{r^2}$$

The radial deformations of the disc are—

$$E \cdot y = -13.99r^3 + 1380 r^{1.744} + \frac{205,200}{r^{0.744}}$$

and for the hub—

$$Ey' = -8.22r^3 + 6348r + \frac{287,860}{r}$$

The calculated stresses and radial displacements are set out in Table XIII and in Fig. 226. The calculations were made with an ordinary 10-in slide rule

It is interesting to calculate the proportion (x , say) of the mass of the rim, the centrifugal force on which is directly applied to the disc.

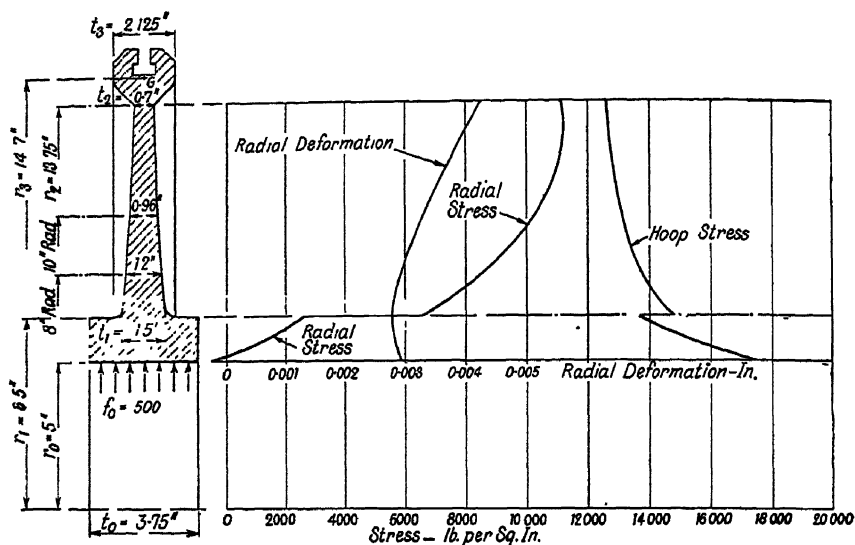


FIG. 226.—Illustrating Example 24

TABLE XIII
STRESSES AND RADIAL DEFORMATIONS IN TURBINE WHEEL
OF HYPERBOLIC SECTION

Radius Inches	Stresses—Lb. per sq. in.		Radial Deforma- tion γ in.
	f_r	f_t	
5	— 525	17,490	0.002 94
5.25	218	16,826	0.002 90
5.5	858	15,859	0.002 86
5.75	1,395	15,192	0.002 83
6	1,858	14,590	0.002 81
6.25	2,255	14,047	0.002 79
6.5	2,570	13,583	0.002 78
6.5	6,540	14,810	0.002 78
7	7,330	14,350	0.002 82
8	8,640	13,670	0.002 95
9	9,620	13,340	0.003 11
10	10,320	13,090	0.003 32
11	10,850	12,950	0.003 56
12	11,090	12,800	0.003 78
13	11,220	12,720	0.004 04
13.75	11,120	12,560	0.004 23

Consider an element of the rim corresponding to one blade. Then—

Volume of rim = $2.84 \times 0.37 = 1.05$ cub. in

Mass of rim = 0.297 lb

Centrifugal force on unsupported element = 1,115 lb.

Force on blade = 2,210 lb

$\therefore$ Total radial force = $2210 + 1115x$.

But this equals $0.346t_2 \cdot f_{r2}$, since 0.346 is the circumferential pitch corresponding to one blade, and as $t_2 = 0.7$, and $f_{r2} = 11,120$ —

$$2210 + 1115x = 2695$$

or

$$x = 0.435$$

This kind of information is useful in the initial stages of design, for if the permissible radial stress at the outer radius is known the thickness at the outer boundary of the disc may be readily estimated

179. The Disc of Constant Stress. If a turbine wheel can be constructed so as to be continuous through the centre, then it is possible to obtain a form which shall give a uniform stress, both radial and tangential, at all points. Such a disc is known as “the disc of constant strength.”

Let $f = f_r = f_t$ be the constant stress. Substituting in eq. (164) and noting that f is a constant—

$$\frac{d}{dr}(rt) - t = -\frac{\mu\omega^2 r^2 t}{f}$$

or

$$t + r \cdot \frac{dt}{dr} - t = -\frac{\mu\omega^2 r^2 t}{f}$$

or

$$\frac{dt}{dr} = -\frac{\mu\omega^2 r t}{f}$$

$\therefore$

$$\frac{\delta t}{t} = -\frac{w\omega^2}{gf} r \delta r$$

Integrating, we have—

$$\int_{t_2}^{t_1} \frac{1}{t} \cdot dt = -\frac{w\omega^2}{gf} \int_{r_2}^{r_1} r \cdot dr$$

$\therefore$

$$\log_e \frac{t_1}{t_2} = \frac{w\omega^2}{2gf} (r_2^2 - r_1^2) \quad . \quad . \quad . \quad (190)$$

If t is the thickness at any radius r , and t_0 is the thickness at the centre, then—

$$\log_e \frac{t_0}{t} = \frac{w\omega^2 r^2}{2gf} = \frac{wu^2}{2gf} \quad . \quad . \quad . \quad (191)$$

or

$$t = t_0 \cdot e^{-\frac{w\omega^2 r^2}{2gf}} \quad . \quad . \quad . \quad (192)$$

In employing these equations the same system of units must be used throughout.

The radial deformation y of the disc is given by the expression—

$$y = \frac{r}{E} (f_t - \sigma \cdot f_r) \\ = \frac{fr}{E} (1 - \sigma)$$

and since f is a constant, y varies directly as r .

EXAMPLE 25. Calculate the thickness of a de Laval steam turbine rotor for values of the radius increasing by 1 in., given the following data: Uniform stress, 25,000 lb. per sq. in.; 10,000 r.p.m., density of steel, 0.283 lb. per cub. in.; diameter of disc just under rim, 24 in.; thickness of disc at this point, $\frac{3}{8}$ in.

$$\omega = \frac{10,000 \times 2\pi}{60} = 1047 \text{ radians per sec.}$$

$$\therefore \frac{w}{2g} \cdot \frac{\omega^2 r^2}{f} = \frac{0.283}{64.4 \times 12} \cdot \frac{1047^2 \times r^2}{25,000} \\ = 0.01608 r^2$$

$$\therefore t = t_0 \cdot e^{-0.01608 r^2}$$

$$\therefore \log_{10} t = \log_{10} t_0 - 0.01608 r^2 \cdot \log_{10} 2.7183 \\ = \log_{10} t_0 - 0.00697 r^2$$

When $r = 12$ in., $t = 0.375$ in.

Hence $t_0 = 3.7758$.

$\therefore$ Equation for t is: $\log t = 0.577 - 0.00697 r^2$.

The work may be arranged as follows. A slide rule was used in constructing Table XIV.

TABLE XIV
CALCULATIONS FOR DISC OF CONSTANT STRENGTH

r	r^2	$0.00697 r^2$	$\log_{10} t$	t
0	0	0.0	0.5770	3.7758
1	1	0.00697	0.5700	3.7156
2	4	0.02785	0.5492	3.5412
3	9	0.0626	0.5144	3.2689
4	16	0.1113	0.4657	2.9222
5	25	0.174	0.403	2.5290
6	36	0.251	0.326	2.1180
7	49	0.341	0.236	1.7220
8	64	0.445	0.132	1.3552
9	81	0.564	0.013	1.0304
10	100	0.697	1.880	0.7586
11	121	0.844	1.733	0.5407
12	144	1.003	1.574	0.3750

A transverse section through one-half of the wheel is shown in Fig. 227. The method of attaching the spindle to the wheel while preserving the continuity of the disc is indicated by the dotted lines. See also Art. 160, and Fig. 194

180. **Effect of Holes in Discs.** The necessity of drilling a number of pressure-equalizing holes in the wheels of impulse turbines has

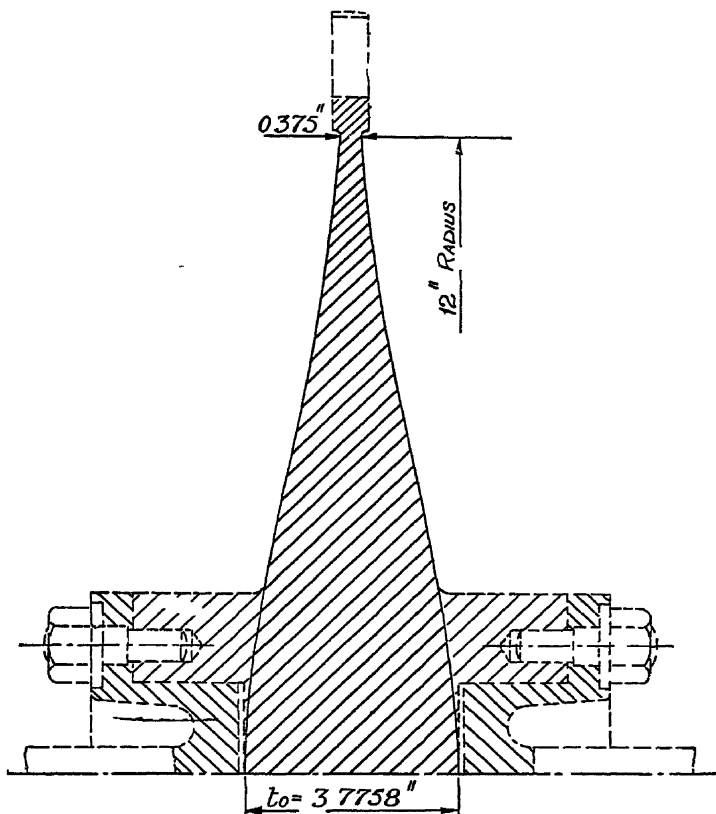


FIG 227.—Disc of Constant Strength

been pointed out in Art. 161. The influence of such holes on the maximum stresses in the disc or wheel may be approximately gauged from a consideration of the stresses at a hole in a flat plate subjected to a tensile stress. This problem has been studied mathematically and the results checked by photo-elastic and other methods. The results may be very briefly summarized as follows. If a flat plate (Fig. 228 (a)) be subjected to a tensile stress which, at some distance from the hole, has a sensibly uniform value f , then the stress across the sections AB and CD varies in the manner shown by the curves, the maximum values at A and C being $3f$.

The stresses at the edge of the hole are necessarily principal stresses, the radial stress (referred to hole) being zero. The tangential stress is tensile over the arcs EAF and GCH, steadily diminishing from A and C and vanishing at E, F, G, and H. Between E and G, and F and H, the tangential stress is compressive, reaching the maximum value, equal to f_t , at K and L.

Consider now an element of a turbine disc symmetrical about the hole ABCD, Fig. 228 (b), and a radial line through the centre of

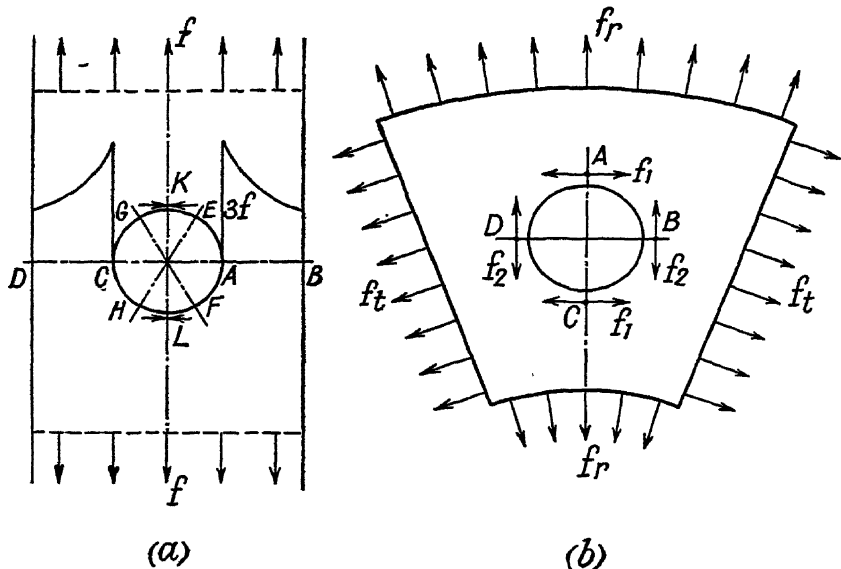


FIG. 228.—Stresses at Holes

the hole and of the disc, and suppose for the sake of illustration that at some little distance from the hole the radial and hoop stresses f_r and f_t in the disc are sensibly uniform. Then, by the principle of superposition, the stresses at A and C, acting in the direction shown by the arrows, due to the imposed disc stresses, will be approximately equal to—

$$f_1 = 3f_t - f_r$$

and the tensile stresses at B and D will be equal to—

$$f_2 = 3f_r - f_t$$

In the most favourable case, where f_r is approximately equal to f_t , the concentrated stress reaches the value $f_1 = f_2 = 2f_t$. The above analysis must be regarded as approximate only, giving the order of the greatest stress rather than its absolute magnitude.

181. Pre-stressing by Overspeeding. A more favourable distribution of stress in a turbine rotor may sometimes be obtained by intentionally overspeeding the rotor and causing the elastic limit

of the material to be exceeded in certain regions of the wheel. This sets up a state of stress in the stationary wheel, which, when superimposed on the stresses produced by rotation, results in a more uniform distribution of the hoop stress. The method is illustrated by Fig. 229, which shows stress curves for a disc 40 in. diameter with a central hole 8 in. diameter. At 3,000 r.p.m. ($u = 524$ ft per

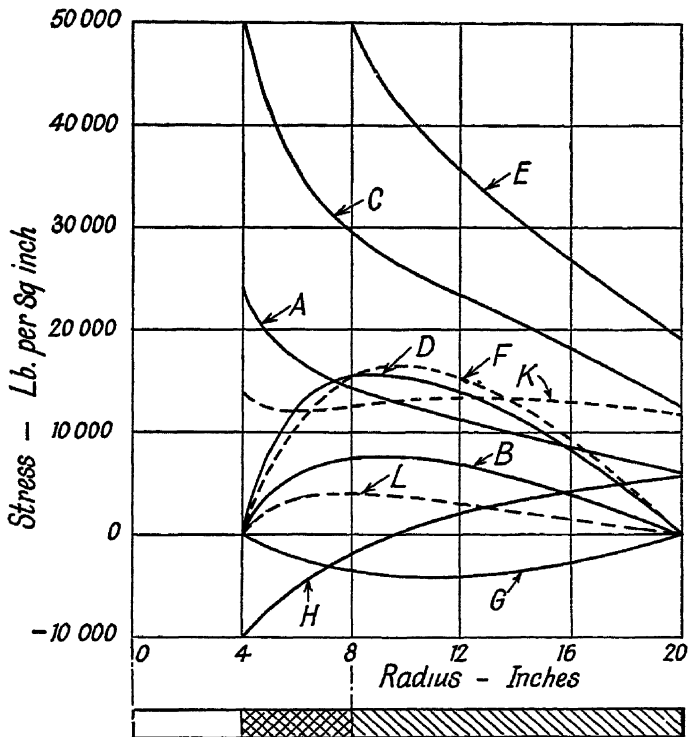


FIG 229

sec.) the curves of hoop stress and radial stress are A and B respectively. Suppose that the yield stress for the material is 50,000 lb per sq in. Then if the speed be increased to 4,320 r.p.m. the hoop stress curve is C and the radial stress curve is D. The maximum hoop stress is now just equal to the yield stress, and provided this stress is not exceeded, the disc on coming to rest will return to its former dimensions and will be quite free from stress. If now the speed is increased to, say, 5,000 r.p.m., the material near the centre of the disc will be in the plastic state and will yield at a stress equal to, or even lower than, the yield stress. The distribution of hoop stress will then be given by a curve such as E, and of radial stress by a curve such as F. If the disc is now allowed to come to rest, the outer part of the disc contracts elastically and puts the inner

part in compression. The radial stress is now entirely compressive, as shown by the curve G, while the hoop stress, curve H, is partly compressive and partly tensile. If now these stresses are superimposed on the original stress curves A and B, we obtain the hoop and radial stress curves, K and L, respectively.

Messrs. C. A. Parsons & Co., Ltd., have used this method of overspeeding turbine rotors since the year 1905. Particulars of the method have been published in a paper (1) by Mr. C. D. Gibb. Like most builders of large steam turbines, Messrs. Parsons have an overspeed test chamber constructed of strongly reinforced concrete 3 ft. thick, covered with earth to a depth of 15 to 18 ft. Rotors to be tested are set up in bearings in the test chamber and driven by an electric motor which is under remote control. The unbladed rough-machined shaft or disc is driven at a speed 1.5 times the normal speed. In the case of discs, the overspeed is carried on until the eye of the disc opens out and takes a slight permanent set.

182. Materials and Working Stresses. The type of material used will depend chiefly upon the estimated stress in the rotor and on the temperature to which the discs or wheels are submitted. The subject of materials and the heat treatment thereof is so vast that it is impossible to do more than give a few scanty facts on possible working stresses.

Before touching on the subject of materials, something may be said about the term "factor of safety." This, as is well known, is taken to mean the ratio of the strength of the material, based on test bars, to the estimated maximum stress in the part under consideration. In some fields of engineering work, the ultimate strength of the material is taken as the basis; in others, it is considered that the yield point (in place of the true elastic limit, which is difficult to determine) should also be considered in estimating working stresses. While avoiding a discussion of a somewhat difficult subject, it may be said that in dealing with materials which have a well-marked yield point there seem good reasons for taking this property into account. As already stated on page 333, Mr. K. Baumann recommends that the mean of the factors of safety referred to the yield point and the ultimate strength should not be less than four when the stresses have been calculated for the normal speed.

It should be pointed out that such a factor of safety is nominal, the real factor depending on the stress conditions. According to the strain-energy theory of elastic failure (which has received a certain amount of experimental support), if a material is subjected to two principal stresses f_1 and f_2 , and if the yield point of the material be f_y and Poisson's ratio be σ , then—

$$f_1^2 + f_2^2 - 2\sigma f_1 f_2 = f_y^2$$

Taking $\sigma = 0.3$ for steel—

$$f_1^2 + f_2^2 - 0.6f_1 f_2 = f_y^2$$

In many turbine discs, the radial and hoop stresses are approximately equal. In such a case, a stress equal to 0.845 times the yield stress would produce elastic failure. It will be seen, therefore, that in the case we have just considered, if the apparent factor of safety based on the yield stress of the material is 3, the real factor is not 3, but 2.55, while at an overspeed of 10 per cent the real factor of safety is only a little over 2.

For rotors subjected to moderate stresses and temperatures, a plain carbon steel may be used. The properties of such a steel depend on the percentage of carbon and other elements. The following data are taken from a paper by Dr. W. H. Hatfield. Steel No. 1 had the following analysis: C 0.25; Mn 0.58, Si 0.11; N₁ 0.13, while No. 2 gave: C 0.33, Mn 0.58; Si 0.21; and Ni 0.20

Steel No	1		2
Heat Treatment	Normalized 890° — 900° C	Annealed 660° C for 48 hr	Normalized 850° C
Limit of proportionality . .	17.5 tons/sq. in.	14.7 tons/sq. in.	19.5 tons/sq. in.
Yield point	20.6 "	16.5 "	23.5 "
Ultimate stress	33.9 "	29.8 "	38.2 "
% Elongation	30.0	32.0	28.0
% Red. of area	63.6	60.0	60.7
Ratio: $\frac{Y.P.}{U.S.}$	0.61	0.55	0.62

The safe working stresses for steels 1 and 2 would be about 16,000 and 18,000 lb. per sq. in. respectively.

The addition of nickel to a carbon steel increases the strength and also raises the yield point ratio. Typical properties of nickel steel are as follow—

Percentage of Nickel	3		5	
Condition	Annealed	Heat* Treated	Annealed	Heat* Treated
Yield point, tons/sq. in. .	27	47	28	47
Ultimate strength, tons/sq. in.	39	55	43	65
% Elongation	31	22	19	20
% Red. of area	64	57	42	60
Ratio: $\frac{Y.P.}{U.S.}$	0.69	0.85	0.65	0.72
Safe working stress, lb./sq. in.	18,500	28,500	20,000	31,000

* Oil quenched at 820° C.; tempered at 650° C., and again quenched in oil.

For high temperatures and high stresses, steels containing the element molybdenum are now used. Such steels may be hardened without quenching and are thus known as air-hardening steels. A typical material of which particulars have been published is Vickers' Vibrac steel. The approximate composition of Vibrac V30 is: C 0.3; Si 0.15; Mn 0.60; P 0.03; S 0.04; Ni 2.5; Cr 0.60; and Mo 0.60. Vibrac steel, oil hardened at 840° C. and tempered at 650° C., has the following mechanical properties: Yield point, 52 tons per sq. in.; ultimate strength, 60 tons per sq. in.; per cent elongation, 21.5; per cent reduction of area, 57. The probable safe working stress for such a material is about 31,000 lb. per sq. in.

For a full specification regarding the heat treatment of a turbine rotor forging, the reader is referred to the paper by Mr. C. D. Gibb (1).

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EXAMPLES XIX

1. Assuming that the elastic limit of nickel steel is 66,000 lb. per sq. in. and that the density is 0.283 lb. per cub. in., at what peripheral speed could a thin ring be rotated in order that the hoop stress would just equal the elastic limit? What is the alteration in diameter of a ring originally 40 in. diameter, if $E = 30 \times 10^6$ lb. per sq. in.?

2. The following particulars refer to a direct-coupled marine turbine of the reaction type—

Outside diameter of drum	113 in.
Thickness of drum	1.5 "
Height of blades (above drum)	10.5 "
Axial pitch of rotor blades	2 "
Circumferential pitch of blades	0.445 "
Depth of blade groove	0.625 "
Width of blade groove	0.625 "
Weight of blade material per ft. length	0.246 lb.
R.p.m.	320

Calculate the average hoop stress in the rim.

3. A plain continuous disc 20 in. diameter and of uniform thickness is rotating 5,000 r.p.m. Calculate the maximum intensity of stress in the disc and the increase in diameter. Assume that $w = 0.28$ lb. per cub. in., $E = 30 \times 10^6$ lb. per sq. in., and $\sigma = 0.3$.

4. If the disc in Question 3 is loaded with an edge tool of 3,000 lb. per sq. in. of peripheral area, calculate the maximum hoop stress in the disc and the increase in diameter.

5. If a disc of 20 in. diameter of uniform thickness, rotating 5,000 r.p.m.,

be pierced with a hole 4 in. bore for a shaft, and subjected to an edge load at the periphery of 3,000 lb per sq in., calculate—

- (a) The hoop stress at the bore.
- (b) The hoop stress at the periphery.
- (c) The increase in diameter at the bore
- (d) The increase in diameter at the periphery

Assume that the radial stress at the bore is zero

6 A De Laval rotor of constant strength is to be run at 16,500 r p m. The disc is 10 in diameter under the rim and the thickness at that point is 0.3 in. The allowable stress is 20,000 lb per sq in. and the weight of the material may be assumed to be 0.283 lb per cub. in. Calculate the thickness of the disc (a) at the centre, (b) at a radius of 3 in.

7 A disc of uniform thickness has external and internal diameters of 20 and 4 in. respectively, and turns at 4,000 r p m. The disc carries at its periphery a ring of weights which apply to the periphery a centrifugal force of 5,000 lb per in. of circumference. If the maximum stress in the disc is limited to 20,000 lb per sq in., what must be the thickness of the disc? Assume $E = 30 \times 10^6$ lb per sq. in., $\sigma = 0.3$, and $w = 0.283$ lb. per cub. in.

8. A series of discs 40 in diameter and 2 in thick are arranged close together on a hollow shaft which has external and internal diameters of 10 in and 4 in respectively. When the discs are turning at 2,000 r p m they are subjected to an edge load of 6,000 lb per in. of circumference and an internal pressure of 1,000 lb. per sq. in. surface due to being shrunk on the shaft. What is the maximum stress in the disc?

Calculate also the amount of shrink fit on the diameter, the actual intensity of pressure between the bore of the discs and the surface of the shaft when the rotor is at rest, and the maximum stress in the stationary disc. Assume $E = 30 \times 10^6$ lb per sq in., $\sigma = 0.3$, and $w = 0.283$ lb. per cub. in.

THE CRITICAL SPEED OF TURBINE ROTORS

183. Definition. Consider a symmetrical body such as a turbine rotor whose mass-centre, for one reason or another, does not quite coincide with its geometrical axis. Then as the body is rotated, there will be a certain transverse force (whose direction rotates at the same speed as the shaft) tending to deflect the shaft. It is found by experience that as the speed is increased the deflection of the shaft from its rest position also increases. As a particular speed is approached, the deflection increases very rapidly and may be so great that the shaft becomes permanently bent. If the speed be increased so rapidly that there is insufficient time for the deflection to grow, then it is found that after the above-mentioned speed is passed, the deflection begins to diminish and smoother running is obtained. The speed at which the shaft displacement tends to be very large is known as the critical speed. As the mass-centre rotates about the rest axis, the shaft behaves like a very taut skipping rope; hence the phenomenon is known as "whirling," and the critical speed is often referred to as the whirling speed. In certain cases, if the speed is increased still further a second resonant speed is encountered, then a third, and so on.

184. Balancing. Theoretically, a turbine rotor is a balanced body, but actually, errors of balance are introduced by various causes, such as—

- (1) Lack of homogeneity of the material.
- (2) Slight errors in machining.
- (3) Differences in the pitch of the blades and also in their individual masses.

Thus it is always necessary to test the balance of a completed turbine rotor and make any adjustments necessary to ensure that the balance is as good as it is possible to make it. There are two chief methods of balancing turbine rotors, namely, the static and dynamic methods.

(1) *Static Balancing.* The simplest method of static balancing is to place the rotor with its journals on two dead level metal surfaces, which are commonly termed "knife edges," but which have a narrow flat surface. The rotor is placed in various positions and allowed to roll until it comes to rest. If rolling friction were absent the rotor would finally come to rest with its mass-centre vertically below the geometrical axis—assuming the rotor to be out-of-balance. Actually, rolling friction does not permit the mass-centre to reach this position, but if the rotor be rolled to rest in

both directions the correct direction of the mass-centre will bisect the angle between the apparent directions. Once the heavy side has been found, trial balance weights are added to the light side and varied by trial and error until the rotor will remain at rest in any position on the knife edges. The rotor may now be balanced either by attaching or removing mass from the rotor in such a way that the moment of added mass is equal to that of the trial balance weight. It is generally preferable to remove the mass by chipping and filing or by drilling holes, according to the circumstances. Some turbine builders machine special grooves in the end discs or wheels for the insertion of balancing masses which are finally secured by screws. Experiments made by the author show that the limit of accuracy possible by this method depends upon the material used for the knife edges, and may be expressed by the final displacement δ of the mass-centre from the axis of rotation.

For mild steel journals on cast-iron knife edges, $\delta = 0.00113$ in.

For mild steel journals on hardened steel knife edges, $\delta = 0.0004$ in.

Such a close approach of the mass-centre to the axis of rotation as 0.0004 in. may seem at first sight to be a very satisfactory result, but a simple analysis will show that the ratio of centrifugal force to the weight of the rotor is about 10 per cent at 3,000 r.p.m. and no less than 40 per cent at 6,000 r.p.m.

The following is a simple method of static balancing which gives a greater refinement than the rough method described above. Assuming that the rotor is set up on "knife edges" or rolling ways, the periphery of the wheel at one end is divided into, say, eight equal parts and these are numbered in order. To overcome rolling friction and produce an appreciable amplitude of roll, a constant mass, the "rolling weight," is attached to the rotor at each of the eight stations in turn. In each case the rolling weight is brought into the horizontal plane and the rotor then carefully released. The distance which the journal rolls along the rolling ways before coming momentarily to rest is measured for each of the eight positions. This is repeated by rolling the rotor in the opposite direction and the mean amplitude of roll obtained for each station. These values are plotted against station numbers and a curve resembling a sine curve obtained. The heavy side is indicated by the greatest amplitude. Trial masses are now added on the light side and the process repeated until the amplitudes are practically identical. By the adoption of such a method the residual displacement δ of the mass-centre should not exceed about 0.0002 in. if the balancing is done on steel knife edges.

(2) *Dynamic Balancing.* That the "static" method of balancing does not necessarily lead to good balance in the case of a complete rotor will be at once evident from Fig. 230. Let W_1 and W_2 be two out-of-balance masses in a plane containing the axis and at radii r_1

and r_2 respectively, such that $W_1 r_1$ is equal to $W_2 r_2$. Then, so far as it is possible to detect want of balance by "static" methods, the rotor is in perfect balance, since the moment of W_2 about the axis

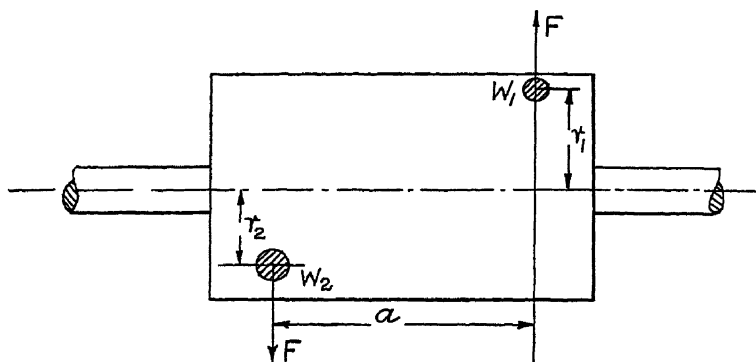


FIG. 230

will balance that of W_1 in any position. If now the rotor is made to rotate, centrifugal forces F , which are equal, act on the mass of rotor, producing a couple Fa and causing "rough" running.

Various dynamic balancing machines have been devised and employed, but one of the most successful is the Lawaczek-Heymann

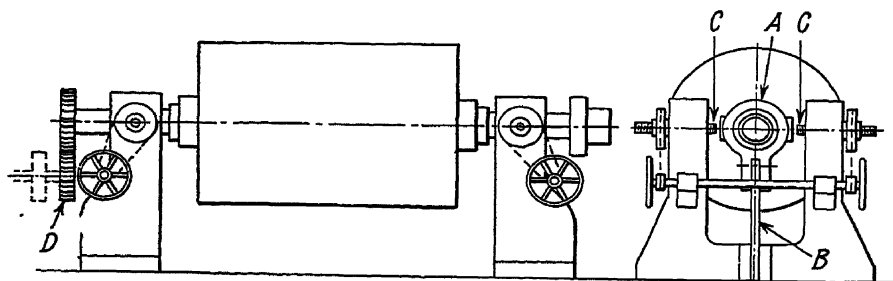


FIG. 231.—Lawaczek-Heymann Balancing Machine

machine constructed by Messrs. C. Schenk, of Darmstadt. The principle of this machine is illustrated diagrammatically in Fig. 231. The rotor is carried in bearings A which are supported by a vertical spring or reed B , made of steel. This reed is rigidly fixed at its lower end in a heavy bedplate, but is free to move laterally when the screwed stops C are withdrawn. It will be clear that for any given reed and any given weight of rotor there will be a particular speed at which, if the rotor is out of balance, the inertia forces will produce resonance. The reeds are so designed that the resonant speeds vary

from 200 to 500 r.p.m. The rotor is driven by an electric motor either through a magnetic clutch or a spur gear D, which may be withdrawn. In making a balance, both ends of the rotor are locked and the rotor run up to a speed above the resonant speed. After disconnecting the driving motor, the stops are withdrawn from one of the bearings, so that it is free to oscillate, the other bearing remaining fixed. As the speed falls the oscillations increase as the resonant speed is approached. The amplitude of oscillation, magnified ten times, is recorded on a drum. Various methods are employed to detect the heavy side of the rotor. In one method a small constant mass is attached to the rotor at each of eight points equally spaced round the end wheel, and the amplitude measured. The point for which the amplitude is least shows the light side, and balance weights are added to this side until one end of the rotor appears to be balanced. The bearings at that end are now locked and the others freed. The process is repeated for the other end and a temporary balance effected. That end is next locked and a re-balance made at the end which was first balanced. This process is repeated until a correct running balance is obtained with both bearings free. The sensitivity of the Lawaczek-Heymann machine is stated to be such that the final displacement δ of the mass-centre from the axis is of the order 0.00008 to 0.0001 in. The American Westinghouse Co. have designed and constructed balancing machines operating on a similar principle. The maximum out-of-balance obtained with these machines is stated to be 1 in.-oz. per 1,000 lb. rotor weight, i.e. δ is about 0.00006 in.

However carefully a rotor may be balanced, it is essential that there should be a sufficient margin of speed between the normal speed of rotation and the critical speed. The latter must be calculated in the design stage, and the proportions of the shaft or rotor must be so chosen that the necessary margin can be guaranteed. When the critical speed is higher than the running speed, the shaft is said to be "stiff", when it is lower than the running speed the shaft is said to be "flexible". Either type of shaft may be used for constant-speed turbines such as those driving alternators, etc., but in marine work the rotor shaft should always be stiff.

A number of special cases arise for consideration.

r85. Case I. *Small-diameter Shaft Supporting a Single Wheel.* Fig. 232 shows a very light spindle freely supported in spherical bearings and supporting a single wheel of mass W lb. All deflections are much exaggerated for the sake of clearness.

Note that Δ is the *static* deflection of the shaft at the wheel centre due to the weight of the wheel only. By considering a vertical shaft we can eliminate gravity, the rest position being the straight line through the bearings.

It will be assumed that imperfections of workmanship cause a

displacement δ of the mass-centre from the geometrical centre of the wheel. At low speeds the conditions are somewhat as represented in Fig. 232 (1) and (2). There is a force due to inertia which causes an increased deflection y .

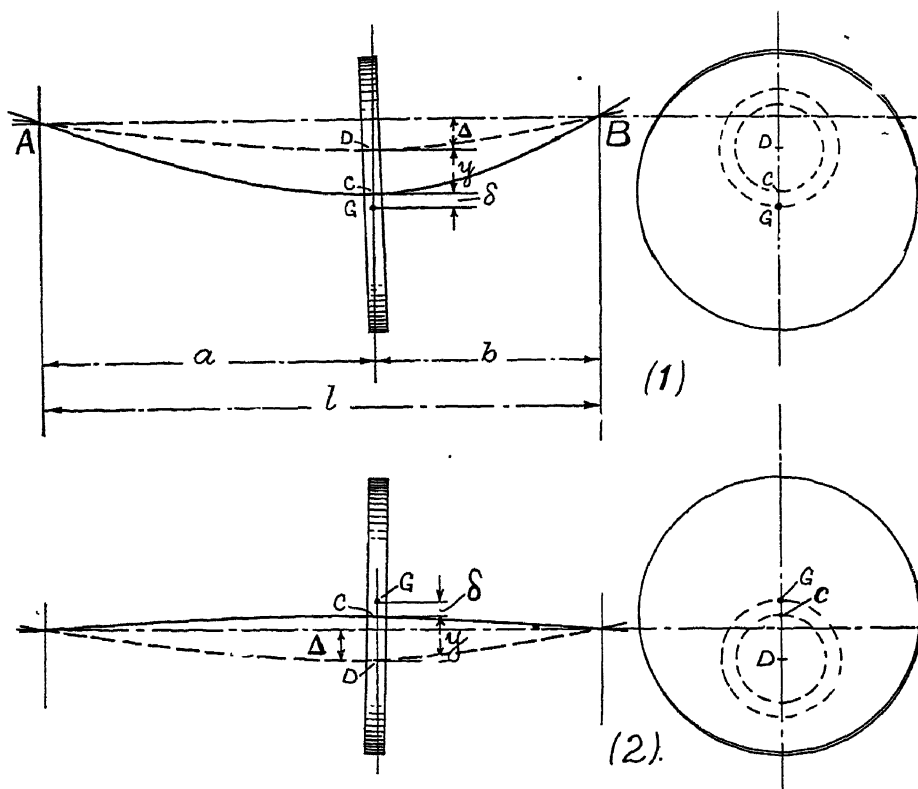


FIG. 232

In fact, the centre of gravity G rotates around the static shaft centre D in a circle whose radius is $y + \delta$, and everywhere the centrifugal force is equal to—

$$\frac{W\omega^2}{g} (y + \delta)$$

Similarly, the centre C rotates around the centre D in a circle whose radius is y ; and if the stiffness of the shaft is F lb. per in. deflection at the point where the wheel is attached, i.e. if it requires F lb. to produce a deflection of unity at the point where

the wheel is attached to the spindle, then the force of restraint which just balances the centrifugal force will be—

Fy lb weight.

Hence
$$Fy = \frac{W\omega^2}{g} (y + \delta)$$

$$\therefore y \left(F - \frac{W\omega^2}{g} \right) = \frac{W\omega^2}{g} \delta$$

and
$$y = \frac{\omega^2 \cdot \delta}{\left(\frac{Fg}{W} - \omega^2 \right)}$$

For any given accidental deflection δ , we may consider how y varies when ω is given different values

(1) When ω^2 is less than $\frac{Fg}{W}$, y and δ have the same sign, and hence G is always situated from D on the other side of C . Moreover, as ω is increased, the denominator decreases and y gradually increases, until when ω^2 approaches $\frac{Fg}{W}$ the value of y approaches infinity.

(2) When ω^2 equals $\frac{Fg}{W}$, the denominator is reduced to zero and the value of y becomes infinite. This value of ω is the *critical*. It may be shown that the natural frequency of transverse vibration of the above system is—

$$\frac{1}{2\pi} \sqrt{\frac{Fg}{W}} \text{ vibrations per sec.}$$

Hence the critical speed in revs. per second is the same as the frequency of natural vibrations.

In all cases where the radii of gyration of the rotating masses are small compared with the length of the shaft, the critical speed coincides with the natural frequency of vibration. The correction required for “rotational inertia” is usually only a matter of 1 or 2 per cent, and as we usually require a figure with a margin of some 15 to 25 per cent, the effect may be neglected, and the question of critical speeds may be approached as the problem of finding the fundamental frequency of natural vibrations

(3) When ω^2 is greater than $\frac{Fg}{W}$, i.e. when the running speed is greater than the critical, the value of the denominator increases and y is reduced. Moreover, the value of y is negative, which means that the centre of gravity G now lies between the static deflected centre D and the wheel centre C , as shown in Fig. 233, (1) and (2).

In this position the effective radius of the centre of gravity G is $y - \delta$ and, as y is further reduced, the effective radius $y - \delta$ is still further reduced. This goes on, until at a very high speed the value of $\frac{Fg}{W}$ becomes so small compared with ω^2 that y approaches the value $-\delta$ and then the wheel commences to rotate around its centre of gravity G , with great stability.

This is the principle of the *ingenious flexible shaft of Dr. de Laval*.

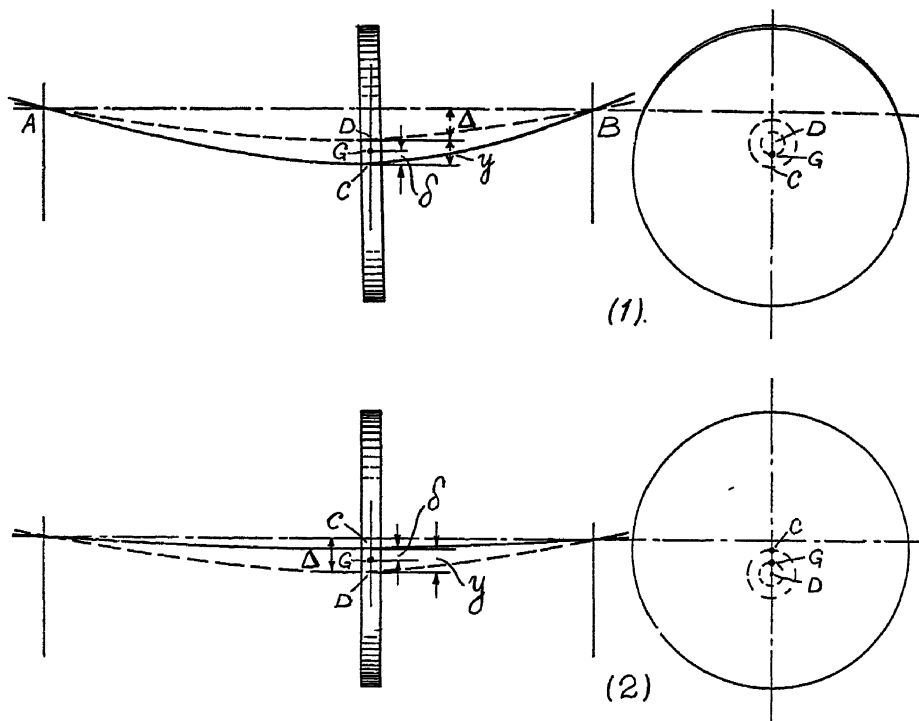


FIG. 233

In the above theory it has been shown that the whirling speed is equal to the natural frequency of transverse vibration of the same system. It has also been assumed that the centre of mass lies in the plane of bending of the shaft, i.e. y and δ are in the same straight line. At the critical speed, however, it can be shown experimentally that the centre of mass is at right angles to the plane of bending, i.e. y and δ are at right angles, while at speeds above and below the critical the truth of the previous deductions regarding the position of the mass-centre can be demonstrated.

In 1919 the late Dr. Jeffcott (1) formulated a new theory, dealing with the problem as a forced vibration. He maintained that the

shaft did not primarily rotate about its rest position, as we have assumed, but about its own centre line, i.e. that G does not rotate primarily about D at radius $y + \delta$, but about the wheel centre C, the centrifugal force being thus $\frac{W}{g}\omega^2\delta$. This force projected on to two planes at right angles introduces two similar simple harmonic forces in these planes. Two similar transverse vibrations of the same frequency ω as the disturbance and of amplitude y are thus forced, and the resultant of two such simple harmonic motions is a circular vibration of speed ω which, combined with the rotation of the shaft about its own axis, will produce rotation in a fixed bent shape. When the problem is treated in this way the same formula will be obtained as before but, in addition, all the observed phenomena can be explained.

$$\text{Hence the critical angular velocity} = \sqrt{\frac{Fg}{W}} \quad . \quad (193)$$

Let ω_c = critical angular velocity in radians per sec.

n_c = critical angular velocity in revolutions per sec.

N_c = critical angular velocity in revolutions per min.

g = acceleration of gravity (32.2×12 in. units).

$$\text{Since} \quad \omega_c = \sqrt{\frac{Fg}{W}}, \quad n_c = \frac{1}{2\pi} \sqrt{\frac{Fg}{W}}$$

Let l = span of bearings in inches

I = second moment of area of shaft section (in⁴)

E = Young's Modulus of Elasticity.

$$\text{Then static deflection } \Delta \text{ at wheel centre} = \frac{cWl^3}{EI}$$

where c is a constant depending on the method of supporting the shaft.

$$\text{Similarly, unit deflection } 1 = \frac{cFl^3}{EI}$$

$$\therefore \quad F = \frac{EI}{cl^3}$$

$$\text{Hence} \quad n_c = \frac{1}{2\pi} \sqrt{\frac{EIg}{cWl^3}} = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta}}$$

$$\therefore \quad N_c = \frac{30}{\pi} \sqrt{\frac{g}{\Delta}} = \frac{187.7}{\sqrt{\Delta}} \text{ r.p.m.} \quad . \quad . \quad . \quad (194)$$

Values of c —

(1) Both ends of shaft simply supported $c = \frac{a^2b^2}{3l^4}$

(2) Both ends of shaft fixed $c = \frac{a^3b^3}{3l^6}$

(3) End A fixed, end B simply supported—

$$c = \frac{a^3b^2(3a + 4b)}{12l^6}$$

EXAMPLE 26. The rotor of a De Laval turbine weighs 4 lb., the shaft is $\frac{3}{8}$ in. diameter, and the span of the bearings is 14 in. The

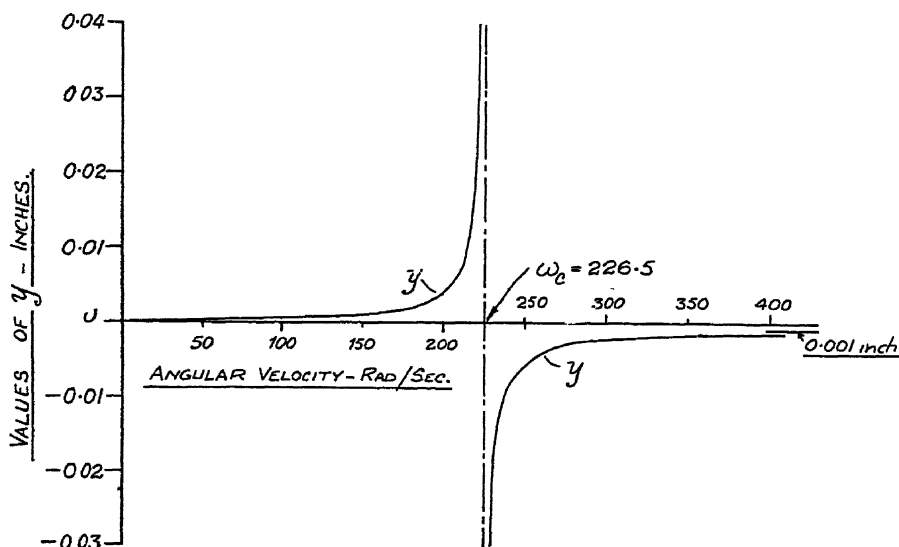


FIG. 234

wheel is 8 in. from one bearing. Calculate the critical speed, assuming both ends free and $E = 30 \times 10^6$ lb. per sq. in.

If $\delta = 0.001$ in., calculate and plot values of y for various values of ω .

$$\begin{aligned} \text{Static deflection } \Delta &= \frac{Wa^2b^2}{3EI} \quad \left(I = \frac{\pi d^4}{64} \right) \\ &= \frac{4 \times 8^2 \times 6^2}{3 \times 30 \times 10^6 \times 0.0009705 \times 14} \\ &= 0.007539 \text{ in.} \end{aligned}$$

$$\text{Critical speed } N_c = \frac{187.7}{\sqrt{0.007539}} = 2162 \text{ r.p.m.}$$

Since 4 lb. produces a deflection 0.007539 in., the force to produce unit deflection must be—

$$\frac{4}{0.007539} = F$$

$$\therefore \frac{Fg}{W} = \frac{32.2 \times 12 \times 4}{4 \times 0.007539} = 51,240$$

$$\text{Hence } y = \frac{0.001\omega^2}{51,240 - \omega^2}$$

A few values of y are given below for corresponding values of ω —

ω	0	100	200	224	226	226.5	230	2,000
y	0	.00024	.00356	.0483	.365	∞	-.032	-.001013

A graph showing these results is given in Fig. 234.

186. Rayleigh's Approximate Solution for the Fundamental Frequency of a Single-span Beam or Shaft. During a free vibration the total energy of the system remains constant. Thus, considering a shaft vibrating transversely, the maximum kinetic energy as the shaft passes through its mid-position will be equal to the maximum strain energy at the end of a vibration. To evaluate these terms the displacement of every point in the shaft during vibration must be known, i.e. the shape of the vibration curve must be known.

Lord Rayleigh has shown that different assumptions regarding the shape, provided they are reasonably like the actual shape, make very little difference to the calculated frequencies, the frequency obtained always being higher than the actual, since a wrong shape means greater constraint and therefore stiffening of the shaft.

A simple assumption is to consider that the shape taken by the shaft during vibration is that of the static deflection curve when the shaft is supported horizontally, i.e. if any one point in the shaft suffers a deflection from its rest position proportional to its static deflection then the deflection of every other point is the same proportion of its static deflection. But since the amplitude does not affect the frequency, we may assume that the amplitude of vibration of any point is equal to its static deflection. This gives a very simple solution for the strain energy since the force causing this deflection is equal to the loading.

187. Case II. Uniform Shaft with Uniformly-distributed Load. Ends freely supported. (Fig. 235.)

Let l = length of shaft between bearings (inches).

w = load per in. length in lb.

I = second moment of area of shaft (inch units).

y = static deflection of shaft at a point x in. from one end.

Consider an element of the shaft δx units long. Then mass of element $= w \cdot \delta x$.

Strain energy for element for deflection $y = \frac{w \cdot \delta x}{2} \times y$.

$\therefore$ Total strain energy for shaft $= \frac{w}{2} \int_0^l y \cdot dx$

Let ω_c = angular velocity of shaft (critical).

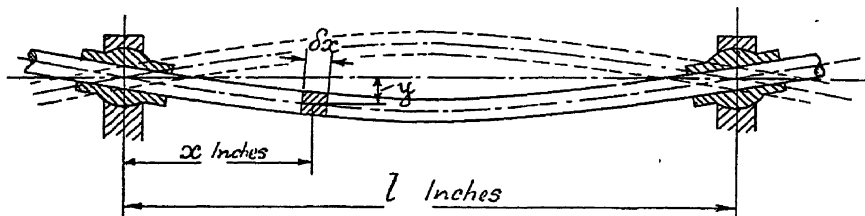


FIG. 235

Then maximum velocity of element $= \omega_c y$

and maximum kinetic energy of element $= \frac{w \cdot \delta x \cdot \omega_c^2 y^2}{2g}$

Hence total kinetic energy $= \frac{w \omega_c^2}{2g} \int_0^l y^2 \cdot dx$

Equating strain energy to kinetic energy, we have—

$$\frac{w \cdot \omega_c^2}{2g} \int_0^l y^2 \cdot dx = \frac{w}{2} \int_0^l y \cdot dx$$

$$\therefore \omega_c^2 = \frac{g \cdot \int_0^l y \cdot dx}{\int_0^l y^2 \cdot dx}$$

It may be shown that the deflection y is given by the expression

$$y = \frac{w}{24EI} (x^4 - 2lx^3 + l^3x)$$

By substituting this value for y in the above equation and integrating, we have—

$$\omega_c^2 = \frac{97.55 gEI}{wl^4} \quad . \quad . \quad . \quad (195)$$

Let Δ = static deflection of shaft at the centre of its length (inches).

$$\text{Then} \quad \Delta = \frac{5}{384} \frac{wl^4}{EI}$$

$$\text{i.e. } \frac{EI}{wl^4} = \frac{5}{384\Delta} \quad \text{Hence } \omega_c^2 = \frac{97.55 \times 32.2 \times 12 \times 5}{384\Delta}$$

$$\therefore \omega_c^2 = \frac{490}{\Delta} \quad \omega_c = \frac{22.12}{\sqrt{\Delta}}$$

$$\text{Hence } N_c = \frac{211.5}{\sqrt{\Delta}} \text{ r.p.m.} \quad . \quad . \quad . \quad (196)$$

188. Other Critical Speeds. At speeds higher than the first critical speed the shaft settles down until the second critical speed is approached, when it commences to bend in the curve shown in Fig. 236 (a) with a nodal point at the centre of its length.

Half the length of the shaft is now under the same conditions as the whole shaft when passing through its first critical speed.

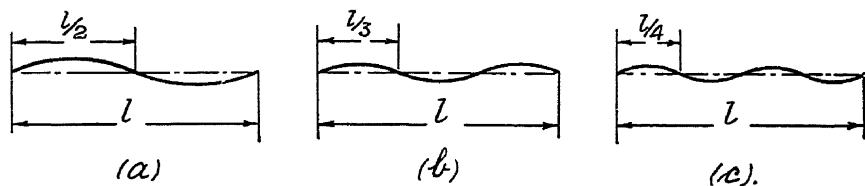


FIG. 236.—Form of Whirling Shaft at Second, Third, and Fourth Critical Speeds

Since deflection is directly proportional to (length)⁴, the corresponding static deflection is now $= (\frac{1}{2})^4 = \frac{1}{16}$ of the static deflection for the fundamental critical speed. But N_c is inversely proportional to the square root of the deflection, hence N_c is 4 times the value given by the above equation. Hence the second, third, and fourth critical speeds are to the first as

$$\begin{aligned} 1^2 : 2^2 : 3^2 : 4^2 & \text{ or as} \\ 1 : 4 : 9 : 16 \end{aligned}$$

Hence, in this particular case, there is a very wide range of speeds between the first and second critical speeds. This is not always the case, however, as, for example, in a shaft freely supported on *three* bearings of equal span, where the second critical speed is only 1.56 times the first. In such a case, great care must be taken to ensure that, if the running speed should lie between the fundamental and second critical speeds, it is sufficiently far removed from either, being about 30 per cent in excess of the fundamental critical speed. Various factors contribute to make the practical running of flexible "three bearing" shafts a difficult proposition. The hubs of the wheels virtually increase the diameter of the shaft owing to the fact that they are forced on; hence the probable critical speed is higher than that calculated on the bare shaft; and

in a "three bearing" design in which the actual speed of rotation lies between the first and second critical speeds, there is some risk of the running speed being dangerously near the first critical speed.

189. Case III. *Uniform Shaft with Uniformly-distributed Load. Ends Fixed.* Let y equal the deflection of the shaft centre at any

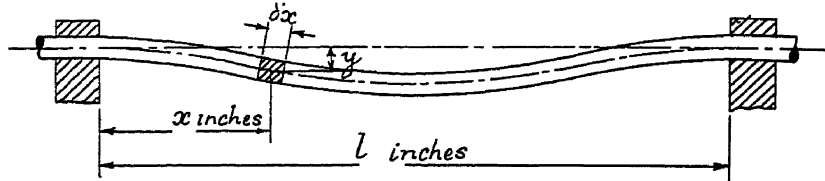


FIG. 237

distance x from the left-hand bearing. It is shown in textbooks on "Strength of Materials" that

$$y = \frac{w}{24EI} (x^2l^2 - 2lx^3 + x^4)$$

As shown in Art. 187, the critical angular velocity =

$$\omega_c^2 = \frac{g \int_0^l y \cdot dx}{\int_0^l y^2 \cdot dx}$$

By substituting for y and integrating, we have—

$$\omega_c^2 = \frac{504 g EI}{wl^4} \quad . \quad . \quad . \quad (197)$$

Let Δ = static deflection of shaft at the centre of its length measured in inches

$$\text{Then } \Delta = \frac{wl^4}{384EI}$$

$$\text{i.e. } \frac{EI}{wl^4} = \frac{1}{384 \Delta} \quad . \quad \text{Hence } \omega_c^2 = \frac{504 \times 32.2 \times 12}{384 \Delta}$$

$$\therefore \omega_c^2 = \frac{507}{\Delta} \quad \omega_c = \frac{22.52}{\sqrt{\Delta}}$$

$$\text{Hence } N_c = \frac{215.1}{\sqrt{\Delta}} \text{ r.p.m.} \quad . \quad . \quad . \quad (198)$$

190. Comparison of "Free" and "Fixed" Ends of Shafts. It is interesting to note the effect of fixing the ends of the shaft. When the ends are fixed, the deflection y at any point due to a given force applied at that point is much less than when the ends

are free. Hence for a constant centrifugal force $\frac{w \cdot \omega^2 y}{g}$, since y is reduced, ω^2 is increased and the critical speed is raised.

Load uniformly distributed { Thus if ω_1 = critical speed with "free" ends
 distributed { „ ω_2 = „ „ „ "fixed" „

$$\left(\frac{\omega_2}{\omega_1}\right)^2 = \frac{504gEI}{wl^4} \times \frac{wl^4}{97.55gEI} = 5.17$$

$$\therefore \omega_2 = 2.27\omega_1$$

Whilst perfect constraint at the bearings will raise the critical speed to more than twice the value obtained with free ends, the mere fixing of the bearing brasses in the bearing block or housing could not produce such an effect unless the brasses gripped the shaft tightly. Such a condition is, of course, quite contrary to the requirements of practice. A fairly large radial clearance between the journal and the "brass" is required for lubrication purposes

Referring to the worked example at the end of this chapter, the maximum slope at the bearings under gravity alone is 0.000127 radian; and since the bearing is 3 in. long, the total deflection of the journal at one end of the bearing, assuming the other end to act as a fulcrum, will be—

$$3 \times 0.000127 = 0.000381 \text{ in.}$$

when the increase of deflection due to whirling is everywhere equal to that produced by gravity. The actual oil clearance is 0.02, or

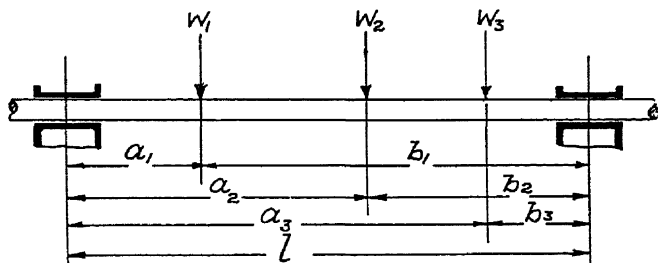


FIG 238

50 times this amount. In other words, the brasses would not begin to exert constraint until the deflection produced at any point at the first whirling speed reached a value 50 times the normal deflection produced by gravity.

A few results for the most simple cases are summarized in Table XV.

191. **Case IV.** *Dunkerley's Semi-empirical Formula for the First Critical Speed of a Uniform Shaft Carrying a Number of Concentrated*

TABLE XV
CRITICAL SPEED OF SHAFTS IN SIMPLE CASES

$$\text{Critical speed in revs. per min.} = \frac{C}{\sqrt{\Delta}}$$

where C is a constant given in table below ;

„ Δ is the maximum static deflection in inches.

Conditions of Support	Type of Loading	C	Max. Deflection Δ Inches.
One end fixed	W concentrated at free end	187.7	$\frac{Wl^3}{3 EI}$
One end fixed	wl uniformly distributed	235.0	$\frac{wl^4}{8 EI}$
Two ends free	W concentrated at centre of span	187.7	$\frac{Wl^3}{48 EI}$
Two ends free	wl uniformly distributed	211.5	$\frac{5 wl^4}{384 EI}$
Two ends fixed	W concentrated at centre of span	187.7	$\frac{Wl^3}{192 EI}$
Two ends fixed	wl uniformly distributed	215.1	$\frac{wl^4}{384 EI}$

Loads (in the form of wheels). The following formula is of a semi-empirical nature, and is based upon a number of valuable experiments carried out by the late Professor Dunkerley (2).

If it is assumed that the shaft is of uniform diameter or very nearly so, the formula gives results which are sufficiently accurate for most purposes.

Let N_s = critical speed of shaft alone—revs. per min.

N_1 = „ „ when supporting W_1 only and when the influence of the shaft is neglected. (Case I.)

N_2 = critical speed of shaft when supporting W_2 only.

N_3 = „ „ „ „ W_3 „

N_c = „ of complete system.

$$\text{Then} \quad \frac{1}{N_c^2} = \frac{1}{N_s^2} + \frac{1}{N_1^2} + \frac{1}{N_2^2} + \frac{1}{N_3^2} \quad (199)$$

EXAMPLE 27. The rotor of a De Laval turbine weighs 3.45 lb. (wheel only), and the spindle is $\frac{3}{8}$ in. diameter. The span of the bearings is 11 in., and the centre of the wheel is 6.2 in. from the centre of one of the bearings. Calculate—

- (1) The static deflection of the shaft at the wheel centre.
- (2) The critical speed of the rotor, assuming a weightless shaft.
- (3) The critical speed by Dunkerley's formula.

$$(1) \quad I = \frac{\pi d^4}{64} = \frac{\pi \times 3^4}{64 \times 8^4} = 0.0009705 \text{ ins.}^4$$

$$\Delta = \frac{W a^2 b^2}{3EI \cdot l} = \frac{3.45 \times 38.44 \times 23.04}{3 \times 30 \times 10^6 \times 0.0009705 \times 11}$$

$$\therefore \Delta = 0.00318 \text{ in.} \quad \text{Ans. (1)}$$

$$(2) \quad N_1 = \frac{187.7}{\sqrt{\Delta}} = \frac{187.7}{\sqrt{0.00318}} = 3329 \text{ r.p.m.} \quad \text{Ans. (2)}$$

- (3) Deflection (maximum) of shaft due to its own weight

$$= \frac{5wl^4}{384EI} = \Delta_s$$

$$w = (\text{wt. per cub. inch of steel}) \times (\text{Vol. of shaft per in. length}) \\ = 0.283 \times 7854 \times \left(\frac{3}{8}\right)^2 = 0.0312 \text{ lb. per in.}$$

$$\therefore \Delta_s = \frac{5 \times 0.0312 \times 11^4}{384 \times 30 \times 10^6 \times 0.0009705} = 0.0002044 \text{ in.}$$

$$\text{Hence, critical speed of shaft alone} = N_s = \frac{211.5}{\sqrt{\Delta_s}}$$

$$\text{,, ,, ,,} = \frac{211.5}{\sqrt{0.0002044}} = 14,760$$

By Dunkerley's equation—

$$\begin{aligned} \frac{1}{N_c^2} &= \frac{1}{N_s^2} + \frac{1}{N_1^2} \\ &= \frac{1}{(14,760)^2} + \frac{1}{(3329)^2} \end{aligned}$$

$$\text{from which} \quad N_c = 3250 \text{ r.p.m.} \quad \text{Ans. (3)}$$

Hence the correction in the above example for the mass of the shaft amounts to about 80 r.p.m. in 3,250 or about $2\frac{1}{2}$ per cent.

The number of practical examples of turbine rotors to which Dunkerley's equation can be applied is, indeed, very small. Generally, the diameter of the shaft is very variable, being much smaller at the journals than at the middle; and the problem is

often further complicated by the fact that at the middle portion where the wheels are attached, the shaft is stepped so as to facilitate the attachment and removal of wheels.

A more exact method which is applicable to the general case of non-uniform loading and a shaft of variable cross-section is given in the following article.

192. Case V. *Critical Speed of a Shaft of Variable Cross-section and Loaded by a Number of Concentrated Loads.* Assume that the

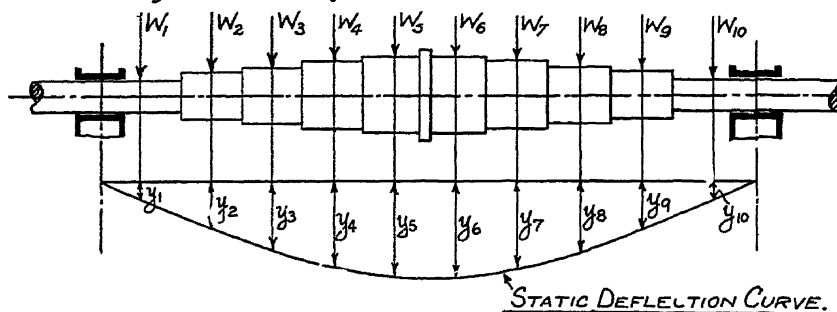


FIG. 239

non-uniform shaft in Fig. 239 is loaded with a number of concentrated loads, W_1 , W_2 , W_3 , etc., which are arranged to include appropriate portions of the weight of the shaft. If "fixing" at the bearings is ignored, then the shaft may be treated as a beam of variable cross-section, freely supported at each end. The curve of static deflection (shown exaggerated) may then be drawn.

Let y_1 , y_2 , y_3 , etc., represent the static deflection of the shaft at the points where the loads W_1 , W_2 , W_3 , etc., are applied.

Then by the method of Art. 186, the total strain energy when the system is at its extreme position of vibration

$$\begin{aligned} &= \frac{W_1 y_1}{2} + \frac{W_2 y_2}{2} + \frac{W_3 y_3}{2} + \dots \\ &= \frac{1}{2} \Sigma W y \end{aligned}$$

Similarly, if ω_c = critical angular velocity, maximum kinetic energy of system

$$\begin{aligned} &= \frac{\omega_c^2}{2g} (W_1 y_1^2 + W_2 y_2^2 + W_3 y_3^2 + \dots) \\ &= \frac{\omega_c^2}{2g} \Sigma W y^2 \end{aligned}$$

$$\text{Hence } \omega_c^2 = \frac{g \Sigma W y}{\Sigma W y^2}$$

$$\therefore N_c = \frac{60}{2\pi} \sqrt{\left(\frac{g \Sigma W y}{\Sigma W y^2} \right)} \text{ r.p.m.} \quad . \quad . \quad . \quad . \quad (200)$$

The derivation of the static deflection curve is a tedious process and has sometimes to be repeated two or three times during design. One method of obtaining it is to draw two funicular polygons, one giving the bending moment and the second giving the deflection. This method is described in textbooks on "Graphics" and is suitable for drawing office work.

A second method is to obtain the shearing force curve, and the bending moment, slope, and deflection curves successively, by a process of graphic integration, which may be effected largely by means of an ordinary planimeter. The latter method is illustrated by the following example—

EXAMPLE 28 To determine the critical speed of the turbine rotor shown in Fig. 240, assuming that it is freely supported by the bearings.

It is first necessary to calculate the weights which are to produce the static deflection. These include the weights of various portions of the shaft and also of the wheels. The various weights are clearly indicated in the upper portion of Fig. 240.

By taking moments of the weights about the right-hand bearing, the upward reaction at A is found to be 301 lb. The total load is 621 lb, so that the reaction at the right-hand bearing is 320 lb. With this information, the shearing force diagram (F) can be drawn.

Scale for shearing force on original 1 in. = 100 lb.

Length scale ,, 1,, = 4 in.

The bending moment curve may now be drawn by commencing with a zero bending moment at the extreme left hand, and integrating the shearing force diagram graphically. Thus, if any section be chosen, as at D, and A_F represents the area (positive) of the shearing force diagram from 0 to D, then since 1 sq in. area of the shearing force diagram must represent—

$$4 \times 100 = 400 \text{ lbs. in.}$$

the bending moment M_D at D must be—

$$400 A_F \text{ lb.-in.}$$

This may be set down to scale, giving the M curve as shown by the broken line in Fig. 240.

The scale for the M curve in the original was—

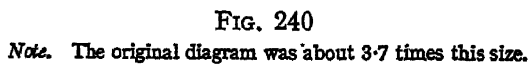
$$1 \text{ in.} = 1000 \text{ lb.-in.}$$

$\therefore$ Actual height or ordinate M_D on diagram

$$= \frac{400 A_F}{1000} \text{ in.}$$

It is easy to see that such work may be done by means of a planimeter and profitably set out in tabular form.

Before obtaining the slope curve, it is necessary to correct the



Note. The original diagram was about 3.7 times this size.

bending moment curve for the varying cross-section of the shaft by dividing the values of the bending moment by the moment of inertia at each section. The derived or corrected $\frac{M}{I}$ curve is thus obtained.

$$\text{Scale on original} \quad 1 \text{ in.} = 100 \frac{\text{lb}}{(\text{inches})^3}$$

It may be shown that the change of slope or inclination of a beam between any two points on it is equal to—

$$\frac{1}{E} \left(\text{Area of } \frac{M}{I} \text{ curve between those points} \right)$$

where E = Young's Modulus of Elasticity

The $\frac{M}{I}$ curve is split up into a number of convenient sections, the areas are found, divided by E , and set up to scale on an assumed base line XX . For example, let A_m be the actual area of the $\frac{M}{I}$ curve between A and D .

$$1 \text{ sq. in. on original} = 100 \times 4 = 400 \frac{\text{lb.}}{(\text{inches})^2}$$

Then change of slope between A and D is—

$$i_D - i_A = \frac{1}{E} \times 400 A_m \cdot (\text{radian})$$

The end ordinate of this curve gives the total change of slope between A and B , which in this example works out to be 0 00024767 radian. The slope curve was set out on the original to a scale of
1 in. = 0.00005 radian

To find the true base of the slope curve, planimeter the area above XX , and divide it by the base XX , thus obtaining the mean-ordinate which in this case is also the slope or inclination of the shaft at A ; setting up the mean-ordinate above XX , the true base line ZZ is obtained.

The deflection curve may now be obtained by integrating graphically the slope curve (above and below ZZ , *not* XX).

$$\begin{aligned} 1 \text{ sq. in. area of slope curve} &= 4 \times 0.00005 \\ &= 0.0002 \text{ in deflection} \end{aligned}$$

Hence, if A_s = area of slope curve between A and D , deflection of D below A equals—

$$0.0002 \times A_s \text{ in.} = y_D$$

In this way, the curve of deflection may be quickly obtained and the deflections read off at the points of application of the concentrated loads (imaginary and real). Maximum deflection in this example occurs at F (slope is zero), and equals 0.001364 in.

The information now available enables us to construct the following table—

TABLE XVI
CALCULATION OF CRITICAL SPEED OF TURBINE ROTOR

Section	W	y	Wy	Wy^2
1 .	9.8	0.000 26	0.002 55	0.000 000 663
2 .	2.0	0.000 45	0.000 9	0.000 000 405
3 .	43.1	0.001 046	0.045 1	0.000 045 000
4 .	7.0	0.001 325	0.009 27	0.000 012 280
5 .	157.0	0.001 355	0.213	0.000 289 000
6 .	154.0	0.001 339	0.206	0.000 276 000
7 .	147.0	0.001 223	0.180	0.000 220 200
8 .	14.9	0.001 120	0.016 7	0.000 018 700
9 .	26.2	0.000 845	0.022 1	0.000 018 680
10 .	2.0	0.000 480	0.000 96	0.000 000 461
11 .	9.8	0.000 25	0.002 45	0.000 000 612
Totals . .			0.699 03	0.000 882 001

$$\text{Hence } \frac{g \Sigma Wy}{\Sigma Wy^2} = \frac{32.2 \times 12 \times 0.69903}{0.000882001} \\ = 306,240$$

$$\therefore N_c = \frac{60}{2\pi} \cdot \sqrt{306,240} = 5,285 \text{ r.p.m.}$$

193. Relationship Between Critical Speed and Maximum Static Deflection. It will be apparent that, from the nature of the expression

$$N_c = \frac{60}{2\pi} \sqrt{\left(\frac{g \cdot \Sigma Wy}{\Sigma Wy^2} \right)}$$

it will be generally possible to write out an expression of the form

$$N_c = \frac{C}{\sqrt{\Delta}}$$

where Δ is the maximum value of y . The exact value of C will depend on the form of the deflection curve and on the distribution of the masses. This has already been shown in connection with the simpler cases, and values of C are given in Table XV. In the example worked out above—

$$C = N_c \cdot \sqrt{\Delta} \\ = 5285 \sqrt{0.001364} \\ = 195$$

This value is intermediate between the value 187.7 for a single concentrated load and 211.5 for a uniformly distributed load.

Where lightness and compactness are of importance, the value of K is large. In merchant turbines, where allowances are pretty liberal, the value of K appears to be about 3,500,000; whereas in naval impulse and reaction turbines fitted with disc wheels, the value increases to about 5,000,000. In land practice, where the disc construction is much lighter than in marine turbines and where the ratio r is larger, the value of K for the examples tested appears to exceed 6,000,000.

The method may be extended slightly by the elimination of the ratio r , i.e. by estimating the diameter d required to give a certain critical speed N_c .

Then
$$d = l \sqrt{\frac{N_c D}{K_1}} \quad . \quad . \quad . \quad (201a)$$

For a few impulse turbines for which N_c has been calculated by the graphical method, K_1 varies from 8,000,000 to 8,500,000.

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2. DUNKERLEY. "On the Whirling and Vibration of Shafts." *Phil. Trans. R.S.*, 1895; 185; 279.

EXAMPLES XX

1. The weight of a De Laval turbine wheel is 6 lb., the spindle is 0.42 in. diameter, the span of the bearings is 13 in., and the wheel centre is 7.5 in. from one end. Calculate—

- (a) The critical speed of the rotor, assuming a weightless shaft.
- (b) The critical speed, using Dunkerley's formula.

2. A shaft 2 in. diameter is freely supported in bearings 15 ft. apart. Calculate the first and third critical speeds.

3. A shaft of uniform diameter projects a distance l through a bearing which may be assumed to fix its direction. The weight per unit length of the shaft is w lb.; and if x be any distance measured from the centre of the bearing, it may be shown that the deflection of the shaft due to gravity is—

$$y = \frac{w}{24EI} (6l^2x^2 - 4lx^3 + x^4)$$

Prove that the first critical speed is—

$$N_c = \frac{60}{2\pi} \sqrt{\frac{162 E \cdot I \cdot g}{13wl^4}} \quad (\text{revs. per min.})$$

4. A shaft 30 in. between bearings is to rotate at 10,000 r.p.m. If the running speed must not exceed 60 per cent of the critical speed and the bearings exert no fixing moment on the shaft, calculate the least diameter of the shaft.

5. Calculate the diameter required in Example 4 if the shaft is to run at a speed midway between the first and second critical speeds.

6. A shaft 3 in. diameter is freely supported in bearings which are 40 in. apart. The shaft supports three wheels, which are each 100 lb. weight.

THE CRITICAL SPEED OF TURBINE ROTORS 415

The centre wheel is midway between the bearings and the other two are placed 6 in. away. Calculate the critical speed of the system.

7 A shaft of variable cross-section supports a number of concentrated loads, which are arranged to take into account the weights of the various portions of the shaft. The following table gives the weights and the corresponding static deflection of the shaft at the points of application of the loads. Calculate the critical speed.

Sect.	1	2	3	4	5	6	7	8
W	20	44	130	134	138	144	44	20
y	0.0003	0.00075	0.00115	0.00138	0.0014	0.00117	0.0008	0.0003

8 The wheel of a Riedler-Stumpf turbine is 2,000 lb. weight, and is supported at the end of an overhung shaft, which is assumed to be fixed by its bearing. The distance from the edge of the bearing to the centre of gravity of the wheel is 6 in. and the speed of rotation is 3,000 r.p.m. If the critical speed is to be 30 per cent in excess of the running speed, calculate the least diameter of the shaft, neglecting the stiffening effect of flanges, etc.

CONSTRUCTION OF CYLINDERS

195. **Form of Cylinder.** The form of the cylinder should be as simple as possible so that distortion may be avoided. In impulse turbines of all sizes, the diaphragms are usually fitted into grooves in the cylinder, provision being made in certain cases to maintain the bore of the diaphragm in correct alignment with the shaft, and yet allow for expansion. Similarly, in reaction turbines it is usually

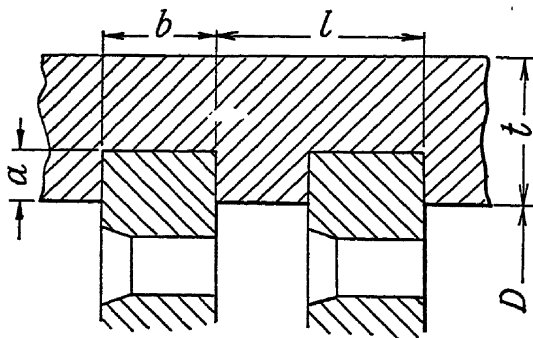


FIG. 241

the practice to fit the stationary blades direct into grooves machined in the cylinder.

Where the thickness of the cylinder is determined by its ability to withstand internal pressure, it may be calculated by the simple formula—

$$t = \frac{\phi D}{2f}$$

where t and D are in inches, and ϕ is the gauge pressure and f the allowable stress in lb. per sq. in. If allowance be made for the diaphragm grooves (Fig. 241), then the thickness

$$t = \frac{\phi D}{2f} + \frac{ab}{l}$$

For very high pressures, the thickness is preferably calculated by the formula—

$$n = \sqrt{\frac{f + \phi}{f - \phi}}$$

based on Lamé's equations for thick cylinders. Here n is the ratio of the external to the internal diameter of the cylinder.

In a single-cylinder turbine, the casting thickness can generally be reduced towards the L. P. end. At that end, the cylinder, under normal operating conditions, tends to collapse, and it is practically impossible to calculate the thickness from considerations of strength alone. Rigidity is essential, and for cylinders having diameters of

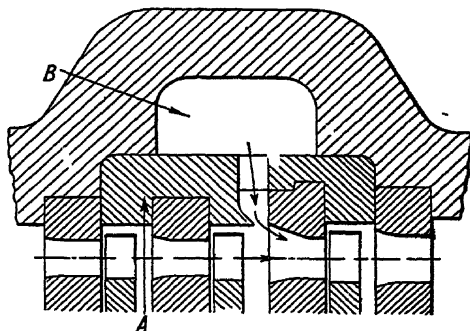


FIG 242 — Admission of By-pass Steam in Impulse Turbine

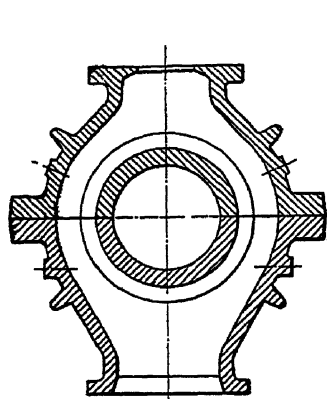
30 in. or more, the thickness of the L.P. end may be calculated from the relation—

$$t = \frac{D}{100} + 0.8 \text{ in}$$

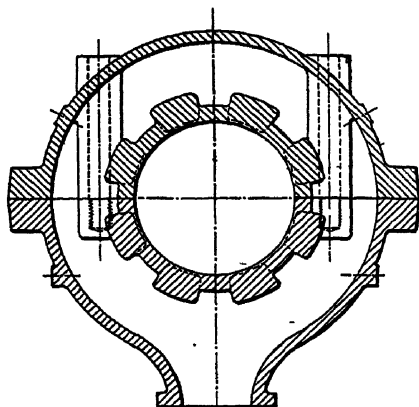
The admission of by-pass steam calls for some modification in the design of the H.P. cylinder. The by-pass steam should be admitted around the complete circumference, and in such a manner as to cause the minimum disturbance to the steam flow. Fig 242 shows an excellent arrangement. A short inner cylinder A is fitted into a recess in the main cylinder casting, forming a steam belt B to which the by-pass steam is admitted. From this belt the steam flows through a passage and is deflected so that it enters the nozzles with the least disturbance to the steam flow from the preceding row of blades.

Interesting comments on reaction turbine cylinder design are made in a paper (see Ref. 1, Chapter XIX) by Mr. C. D. Gibb. Fig. 243, taken from the paper, shows an earlier design of reaction turbine cylinder. The area which is cross-hatched represents a part of the cylinder which will be at the full steam temperature, whereas the outer wall will be at a lower temperature, owing to heat losses. The shaded area tends to expand outwards, but is partially prevented by the outer wall. As a result, the parts marked X distort in the direction shown by the arrows. The improved design is shown in Fig. 244. Here the inner cylindrical portions are connected by heavy cast longitudinal links and the internal part of the cylinder is made more rigid than the external part. To give

greater flexibility to the horizontal joint flange, slots are cut from the bolt holes to the outside of the flange. Further, to assist in maintaining a steam-tight joint and to increase the internal rigidity,



SECTION THRO' A-A



SECTION THRO' B-B

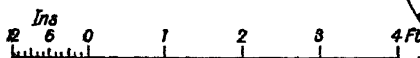
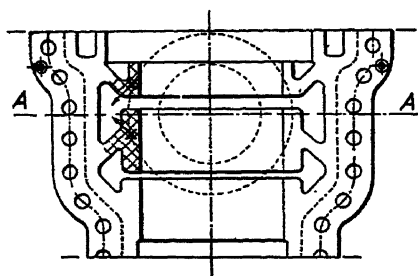


FIG. 243.—Design of Turbine Cylinder Liable to Distortion

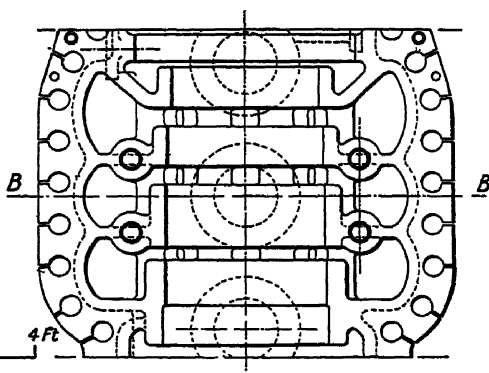


FIG. 244.—Modern Cylinder Designed to Obviate Distortion

(Reproduced, by permission, from Proc. I. Mech. E.)

long studs are carried from the lower half of the cylinder right through to the top of the upper half.

196. Joints and Bolting. All multi-stage turbine cylinders, excepting those of a few turbines designed for very high pressures, are divided into two parts, upper and lower. In almost every case, the diaphragms are similarly divided. The advantages of this construction in connection with the assembly, erection, and inspection of the turbine are too obvious to require explanation.

The flange for the horizontal joint must have considerable rigidity in the vertical plane, and its thickness should be 1.5 to 1.75 times that of the cylinder. The diameter of the bolts must be estimated

from the magnitude of the bursting forces and may be about 0.75 times the flange thickness. Fig. 245 shows the section through a typical horizontal joint. The bolt is driven from underneath and secured by a cap nut. The cylinder tends to open out at the joint when under pressure, and to counteract such a tendency the bolts should be placed as near to the metal of the cylinder as possible. In way of branches, etc., where it is impossible to insert a bolt from

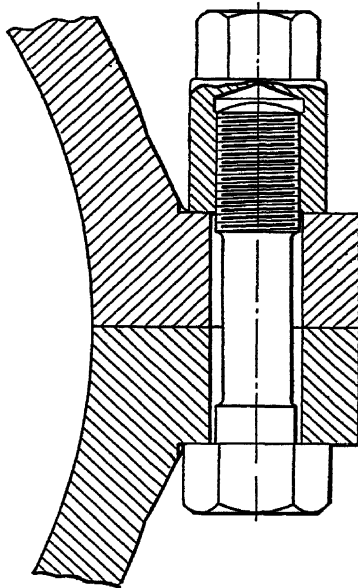


FIG 245

the underside, studs are used. In cylinders subjected to very high steam pressures it is sometimes necessary to employ two rows of staggered bolts.

The method of tightening the bolts at the horizontal joint of turbine cylinders, especially those for high steam pressures and temperatures, has undergone some modification in recent years. It was formerly the practice to hammer up the nuts with a heavy hammer. This undoubtedly puts a severe strain on the threads. In the improved method, the amount of tightening necessary to produce a certain safe stress in the bolt—and at the same time to ensure a steam-tight joint—is calculated beforehand. In closing up the cylinder, the nuts are all first tightened up by hand with the aid of a spanner. The bolts are then heated in pairs, either electrically or by a gas flame. During the heating process the bolts expand slightly and the nuts are gradually tightened up by hand. When the prescribed rotation of the nut has been obtained the heating of

the bolt is stopped, and as the bolt cools it contracts and tightens the joint. The methods of heating the bolts are shown in Fig. 246. For gas heating, a hole is drilled right through the bolt and cap nut. The electrical method has been developed by Messrs.

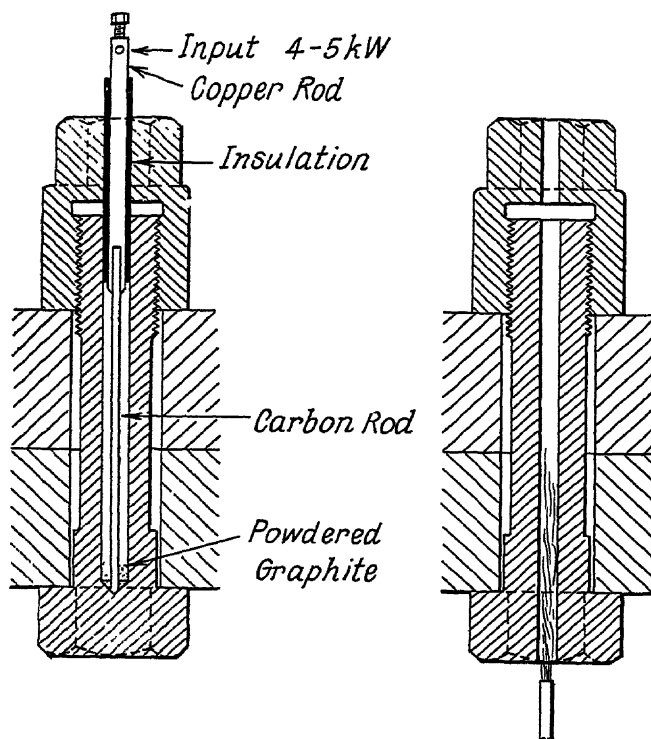


FIG. 246

C. A. Parsons & Co., Ltd. Two bolts are heated at a time. Either alternating or direct current at 100 to 200 volts is suitable.

The amount of rotation to be given to the nut after it first becomes tight may be calculated as follows. Suppose E_1 and E_2 are Young's Moduli for the bolt and flange materials respectively, and that A_1 is the area of the bolt under a final safe tensile stress f_1 , and A_2 is the area of the flange under a compressive stress f_2 . Since the stress f_2 is by no means uniform, the area A_2 is somewhat difficult to estimate. With closely pitched bolts, A_2 may be taken as the product of the flange width and bolt pitch. This will err on the high side and discretion must be used in selecting the value of A_2 . Suppose that l is the combined flange thickness.

Since the bolt increases in length by the amount $\frac{f_1 l}{E_1}$ and the

flange contracts by the amount $\frac{f_2 l}{E_2}$, and as $f_1 A_1 = f_2 A_2$, the total shortening of the bolt which must be produced by screwing up is—

$$x = f_1 l \left(\frac{1}{E_1} + \frac{A_1}{A_2 E_2} \right)$$

If p is the pitch of the threads, then the angle through which the nut must be turned is—

$$\frac{x}{p} \times 360 \text{ degrees}$$

When turbines have been operating for long periods at high temperatures the bolts and nuts are apt to seize when an attempt

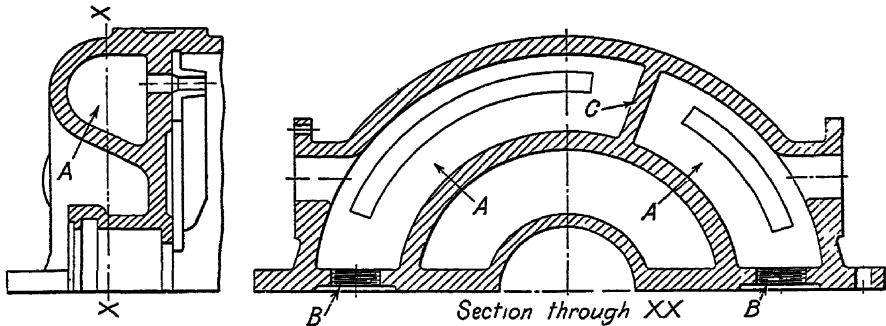


FIG 247

is made to unscrew the nuts. To avoid this trouble, the following practice is adopted. The bolt threads are made slightly under size and afterwards sprayed with copper or aluminum by the Schoop process. In opening up the turbine cylinder, the bolts are first heated and the nuts slacked back. If seizing should begin, a little of the sprayed metal is broken off and the nut is freed.

Plain carbon steels are used for bolts, but 3 per cent Ni steel is used in high-pressure turbines. This steel has a minimum yield point of 30 and a minimum ultimate strength of 45 tons per sq. in.

197. Nozzle-box Construction. A form of construction commonly used in impulse turbines is shown in Fig 247. The nozzle boxes or steam chests A are cored out, and may extend completely round the circumference, or only partially, as shown in Fig. 247. In the latter case the core holes at B are tapped and plugged. With nozzle-control governing (hand or automatic) the nozzle box is divided by walls C, as shown.

A simplified construction suitable for by-pass governing is shown in Fig. 248. The core for the nozzle box can be built up as part of the cylinder core. The first-stage nozzles are built up in a plate

which covers the front of the nozzle box. Fig. 249 shows a similar construction (General Electric Co., U.S.A.), the object here being to provide a steam belt of sufficiently large area. The diaphragm A is in halves.

It sometimes happens that although the live steam temperature exceeds 450°F. , the limiting temperature for the employment of

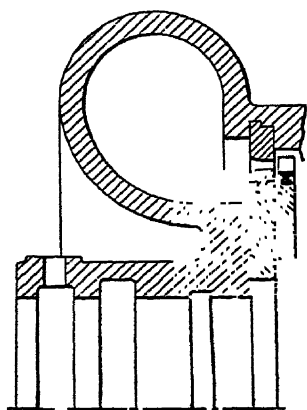


FIG. 248

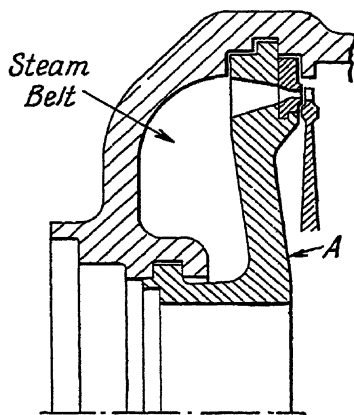


FIG. 249

cast iron, the temperature of the steam after expansion through the first-stage nozzles falls below 450°F. This is especially the case when the first stage of the turbine is velocity-compounded. In such a case the cylinder may be made of cast iron and the nozzle box a steel casting inserted through an opening in the side, top, or bottom of the cylinder. Fig. 250 shows such a nozzle box as formerly used in Metropolitan-Vickers steam turbines. The part lying within the cylinder is in the form of a curved pipe of gradually diminishing flow area. The nozzle segment is bolted to a machined face on the side. A further example, from a Brown-Curtis marine turbine, is illustrated in Fig. 251.

198. Construction of Exhaust End. In all condensing turbines excepting those of small output and special design, the condenser is placed underneath the turbine. This position has a number of advantages, chief among which are that the moisture formed during expansion in the turbine drains naturally into the condenser, and that the turbine may be opened up for inspection without the removal of exhaust branches. In general, the velocity of steam flow through the condenser inlet branch is less than the absolute velocity of the steam as it leaves the last wheel. The exhaust hood should be designed so that, so far as possible, the change of steam velocity will be accompanied by a small rise in pressure, but it is doubtful if such a result is ever obtained, because changes in the direction

of flow, and the resulting losses, are inevitable. The shape of the upper part of the exhaust hood should be such that the flow area

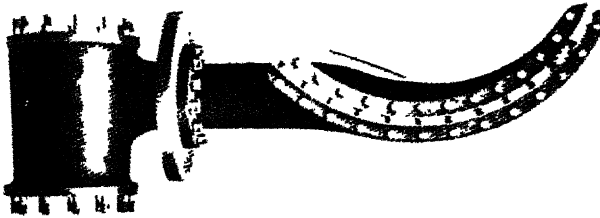


FIG 250

grows uniformly from the vertical centre-line downwards. Many manufacturers now cast large deflector blades in the lower half, one on each side, to deflect downwards the steam from the lower

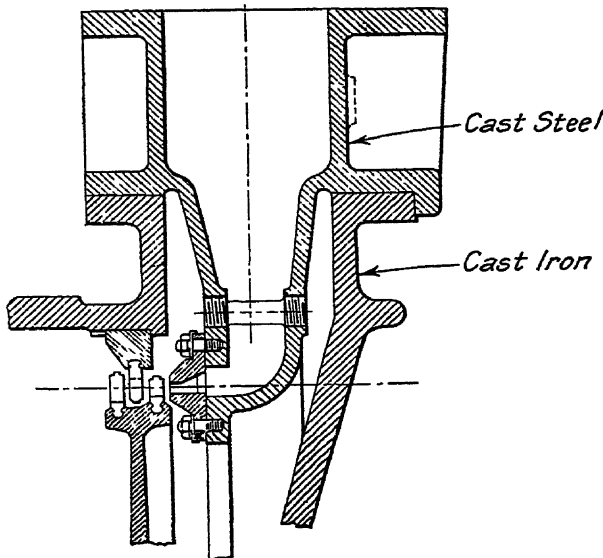


FIG 251

half of the wheel and so prevent it from disturbing the flow of that coming from the upper half.

The exhaust-end bearing, including sometimes the generator bearing, is frequently supported from the exhaust hood, so both rigidity and strength are essential. The lower half should be stiffened by means of cast stay plates (vertical so as not to interfere with the steam flow), steel stay plates bolted to cast brackets, or

by means of stay bolts, for in addition to such a load the exhaust hood has to withstand a certain collapsing pressure and an even higher test pressure.

199. Cylinder Supports. Consideration must be given to the differential expansion which takes place in the turbine and its supports in order that—

(1) The turbine cylinder and rotor may remain co-axial, so

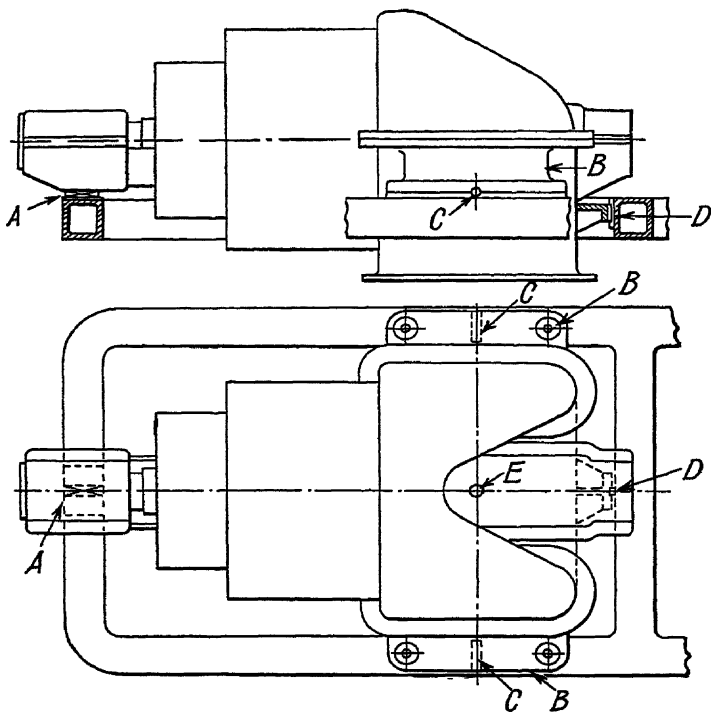


FIG. 252

maintaining the small radial clearances in the various labyrinth packings.

(2) The cylinder and rotor may expand freely without inducing heavy forces and consequent distortion.

(3) The axial clearances between the stationary and rotating parts of the turbine may not, under any conditions of operation, be reduced below certain safe limits.

Such considerations assume a special importance in turbines operating at high temperatures.

Fig. 252 shows a method commonly employed for supporting a single-cylinder condensing turbine operating at moderate pressures and temperatures. The H.P. bearing block and the L.P. end of

the turbine are cast in one piece, the bearing block being supported on the bedplate at A. The L.P. end is supported by two feet or brackets B cast on the side of the cylinder and also resting on the bedplate. When the turbine has been finally lined-up, holes are drilled at C for circular keys. The cylinder is not rigidly held down at the exhaust end, but is free to slide outwards or inwards on the keys. Correct transverse alignment is maintained by a key D, fitted between a bracket on the cylinder and a recess in the bedplate, and another key at A. Thus the only fixed point is at E, which is usually arranged at the centre of the exhaust branch. All expansions must be measured from E and that at the H.P. end must be borne in mind when arranging steam connections to that end, otherwise severe strains might be thrown on the cylinder.

The temperature fall along the cylinder will be nearly linear, and if t_1 and t_2 are the temperatures at the steam and exhaust ends respectively, α the coefficient of expansion, and l the distance from the fixed point to the H.P. end, then the movement of the cylinder towards that end will be—

$$\frac{\alpha l}{2} (t_1 + t_2)$$

For cast iron, $\alpha = 0.0000057$ per $^{\circ}\text{F}$.

For forged steel, $\alpha = 0.0000061$ per $^{\circ}\text{F}$.

Thus if $t_1 = 600^{\circ}\text{F}$ and $t_2 = 80^{\circ}\text{F}$, and $l = 50$ in., the expansion will be 0.097 in.

Whilst the method of support described above is generally suitable for moderate pressures and temperatures, it is quite unsuitable for high temperatures. In the first place, the steam-end bearing block and the cylinder should not be cast together, otherwise a large amount of heat will be conducted to the bearing pedestal and severe strains may be brought to bear upon it. Furthermore, if the cylinder is supported from the bedplate, it is doubtful whether the cylinder and rotor will be kept truly coaxial. This is most important as the clearances in the labyrinth packings will be affected.

When the pedestal is a separate casting, the method of support shown in Fig. 253 may be employed. A half-round projecting flange A is cast on the lower half of the cylinder and is attached to the pedestal by studs or bolts in clearance holes. Keys B, C, and D are half sunk in the flange and pedestal. In spite of differential expansion of the cylinder and pedestal the heights of the spindle and gland centres are maintained very nearly the same by means of the keys B and D, while the transverse alignment of the cylinder and pedestal is secured by the key C. Correct alignment with axial movement is obtained by means of the key E.

A method of support which ensures good alignment is shown

in Fig. 254 (a) and (b). The bearing blocks carry a support A at each side with a machined surface which lies nearly in the horizontal plane of the axis. Brackets B, cast integrally with the lower

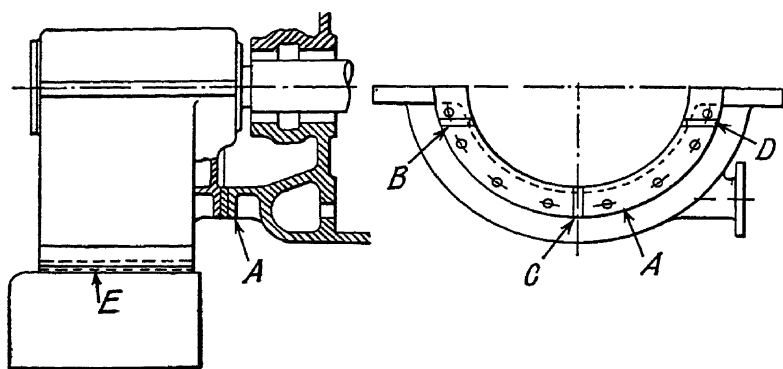


FIG. 253

half of the cylinder, rest on these faces and transmit to the bearing block part of the weight of the cylinder and the reaction torque. It is clear that these supports will maintain the cylinder axis at *nearly* the correct height, however the temperature of the cylinder

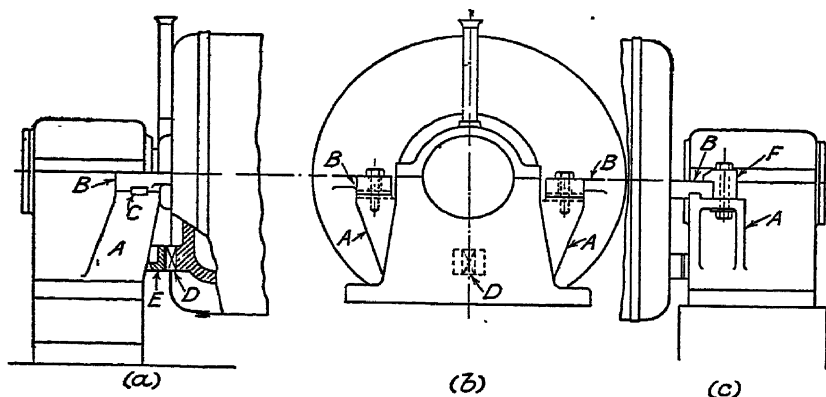


FIG. 254.—Method of Supporting High-pressure Turbine to Maintain Co-axial Alignment of Turbine Cylinder and Rotor

may change. Keys C permit transverse sliding, yet keep the cylinder and pedestal in their correct relative positions. A key D, sunk partly in the bracket E and partly in a facing on the cylinder, allows for the downward expansion of the cylinder and yet maintains the transverse alignment of the cylinder axis. A slight modification of the construction is shown in Fig. 254 (c), in that the cylinder

bracket B is held down by a wrought-steel keep F, bolted to the support A. There is, of course, a small clearance between F and B to permit sliding.

Perfection is not achieved by the method just described. If, for example, the bracket B has a thickness of 3 in., and suppose the steam temperature is 850°F . The temperature of B, due to conducted heat, might be about 750°F . Then, assuming the bearing block to be at 100°F ., the upward expansion of the cylinder relative to the rotor would be about twelve-thousandths of an inch.

An improvement is shown in Fig. 255, in which the supporting brackets for the cylinder are formed by an extension, at each end,

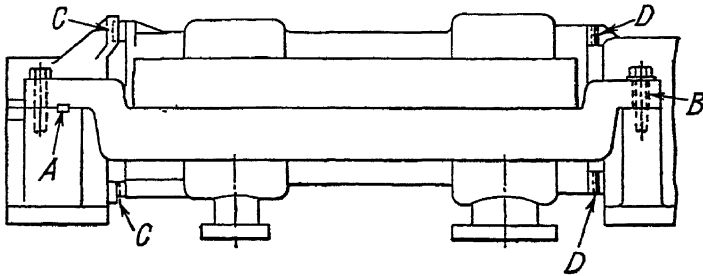


FIG 255

of the flange for the lower half of the cylinder. The supporting faces are arranged exactly in the horizontal plane passing through the turbine axis. The longitudinal position of the cylinder may be fixed at either end by the keys A (which permit transverse movement), while a clearance is left in the bolt holes B at the other end. Transverse alignment is maintained by the pairs of keys C and D.

An alternative method of support employed by the British Thomson-Houston Co., Ltd., for the H.P. end of single-cylinder turbines and the H.P. casing and inlet end of the L.P. casing of two-cylinder turbines is shown diagrammatically in Fig. 256. The cylinders are supported by brackets A resting on sloping pads B forming part of the bearing housing. The planes of the pad surfaces pass through the cylinder axis and are preferably at 45° to the horizontal, although the angle may be varied to suit individual cases. The longitudinal position of the cylinder is fixed by dowel pins C, fitted in holes which are drilled and reamed after the assembly and lining-up of the cylinders. There is a small clearance between the brackets and pads to facilitate adjustment, and actually the brackets rest on the dowel pins. The cylinders are definitely held down by bolts D, which, however, being in elongated holes, do not prevent sliding.

200. Materials. The material most commonly used for cylinders

is cast iron. Owing to the phenomenon known as "growth" of cast iron, which is an increase in volume caused by a change in the internal structure of the cast iron at high temperatures, the employment of cast iron at temperatures above 450° F. is not advisable. It has been found that the tendency to growth is increased by a relatively large amount of silicon in the iron. The composition of a typical cast iron for turbine work is: C 3.3; Si 1.2; Mn 0.8; P 0.2; and S 0.1. The ultimate strength of such iron would be about

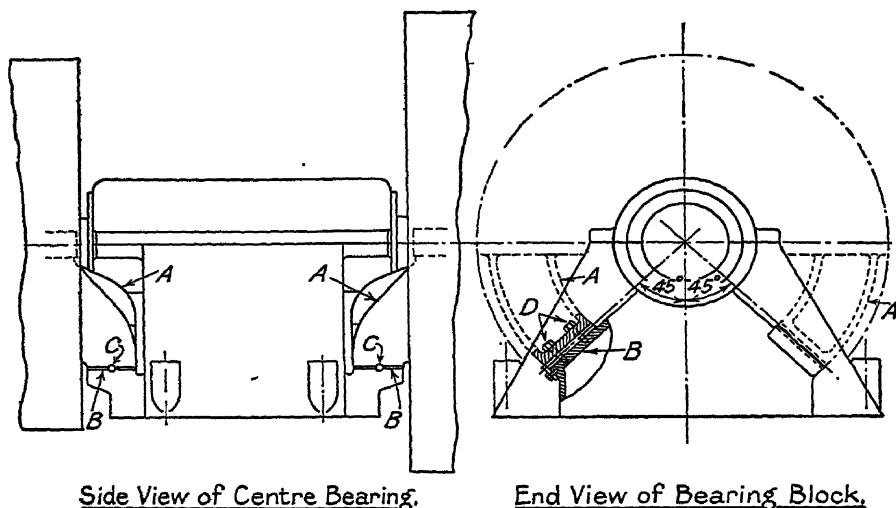


FIG. 256.—Method of Supporting Turbine Cylinders so as to Maintain Co-axial Alignment of Turbine Cylinder and Rotor

16–18 tons per sq. in., and the transverse rupture stress 24–30 tons per sq. in. The addition of small percentages of nickel (less than 2 per cent) adds to the strength of cast iron, gives it good machinability, and makes it less porous. Thus a nickel cast iron containing 3.1 C, 1.2 Si, and 1.5 Ni has a tensile strength of 17–22 tons per sq. in. and a modulus of rupture of 32–40 tons per sq. in.

For moderate steam pressures and temperatures between 450° F. and 800° F. the cylinders are made of cast steel. In single-cylinder turbines in which the initial temperature exceeds 450° F., the H.P. end of the cylinder would be made of cast steel and the L.P. end of cast iron. In two- and three-cylinder turbines, the H.P. or H.P. and I.P. cylinders would be of cast steel and the I.P. and L.P., or the L.P. cylinder only, of cast iron, according to the conditions.

For steam temperatures exceeding about 800° F. it is usual to employ an alloy steel. Such steels contain molybdenum up to 0.5–0.75 per cent and small amounts of nickel and chromium. Steel castings are cleaned, then annealed at temperatures varying between

920° and 950° F. and cooled in the furnace. After rough machining the casting may be further treated by heating it in a furnace to a temperature somewhat higher than the temperature in service.

In one or two special cases of high pressure combined with high temperature, the H.P. cylinders have been machined out of solid blocks of forged steel. Such a construction, though expensive, gives a guarantee of a sound and reliable cylinder

GLANDS AND PACKING DEVICES

201. **Diaphragm Glands.** Labyrinth glands are used almost exclusively for the purpose of minimizing the steam leakage through the diaphragm gland in impulse turbines. Various forms of diaphragm

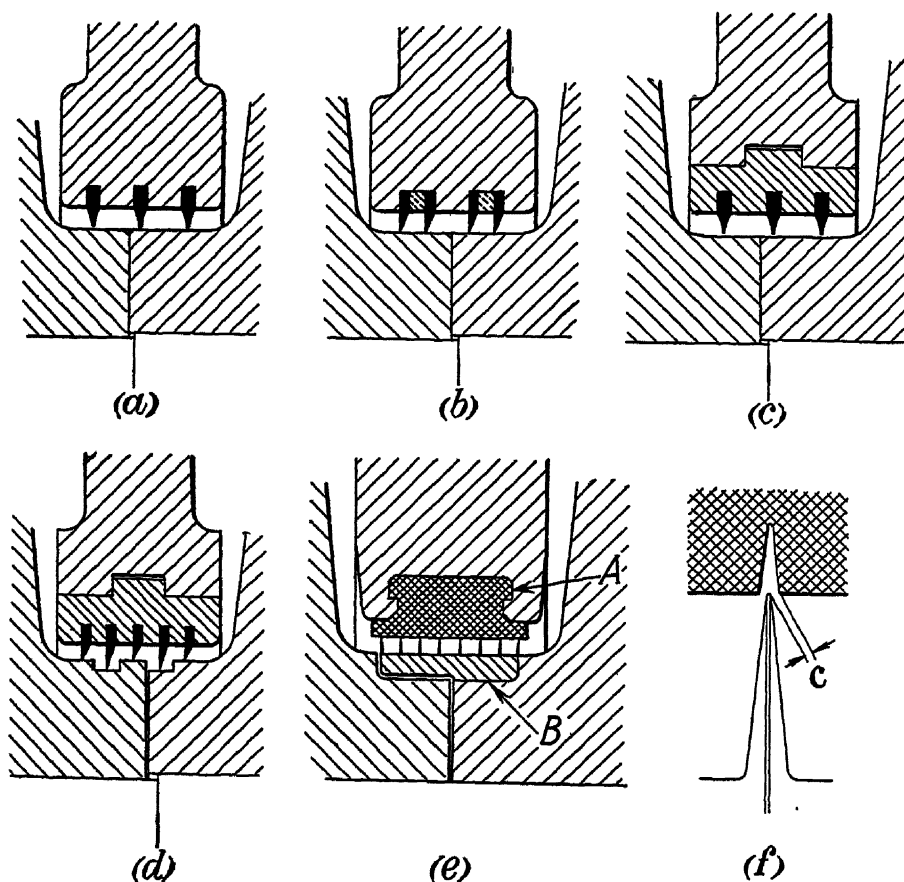


FIG. 257

gland are shown in Fig. 257. In (a), (b), (c), and (d) the packing strips are of brass or other non-ferrous alloy and are caulked or rolled into the diaphragm or bush. The edge of the strip near the shaft surface is made very thin so that if rubbing should accidentally take place the thin edge of the packing strip will be quickly worn away without the generation of much local heat. Hard rubbing with the consequent heating is a possible cause of a bent shaft. Fig 257 (a) shows

the simplest construction; (c) is essentially the same except that the packing rings are carried in a separate bush. In (a), (b), and (c) the wheel surface is smooth and the baffling action of the rings somewhat imperfect. This is improved in Fig. 257 (d), grooves being turned in the wheel hub and alternate long and short packing rings used. Fig. 257 (e) shows a novel design of gland by Messrs. Escher Wyss Engineering Works, Ltd. A ring A of graphitic carbon is fitted in halves into the bore of the diaphragm. A steel sleeve B pressed on

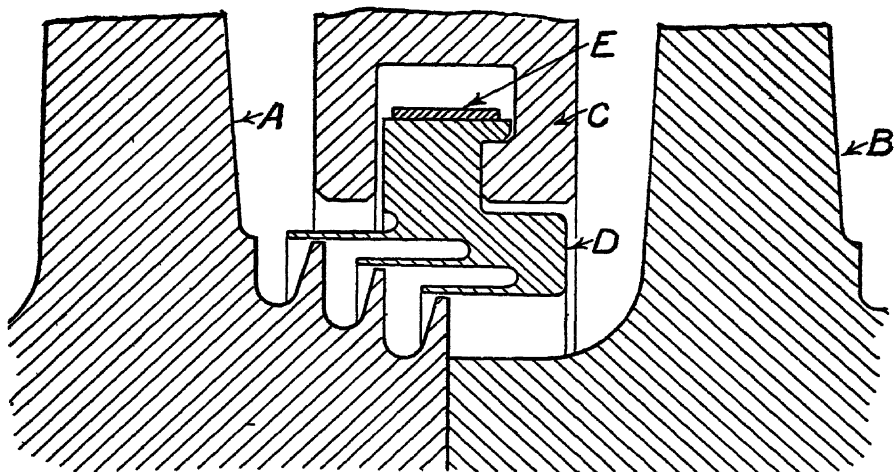


FIG. 258—Diaphragm Gland for High-pressure Steam

to one of the wheel hubs is provided with a number of sharp fins on its outer surface. The initial clearance is practically zero and normally remains very small. If the shaft should be accidentally displaced then very narrow grooves are cut in the carbon ring without damage to the fins. The clearance then controlling leakage is the small amount *c*.

An effective type of diaphragm gland employed by the Metropolitan-Vickers Electrical Co., Ltd., for turbines operating with very high steam pressures is shown in Fig. 258. A and B represent the discs of two adjacent wheels, A being on the high-pressure side of the diaphragm C. The latter has a slot bored in it which carries the packing ring D and keeps it in the central position. D is made in segments and is furnished with three thin cylindrical projecting strips. Each of the segments D is backed up by a plate spring E. Three projections are machined on the wheel hubs, and the diameters of these are such that the clearance between them and the overhanging strips is practically nil. If accidental rubbing should occur and heating take place then the rings will expand slightly and relieve the rubbing pressure. The fins on the rotor would also be heated, but here the expansion is extremely small because of the

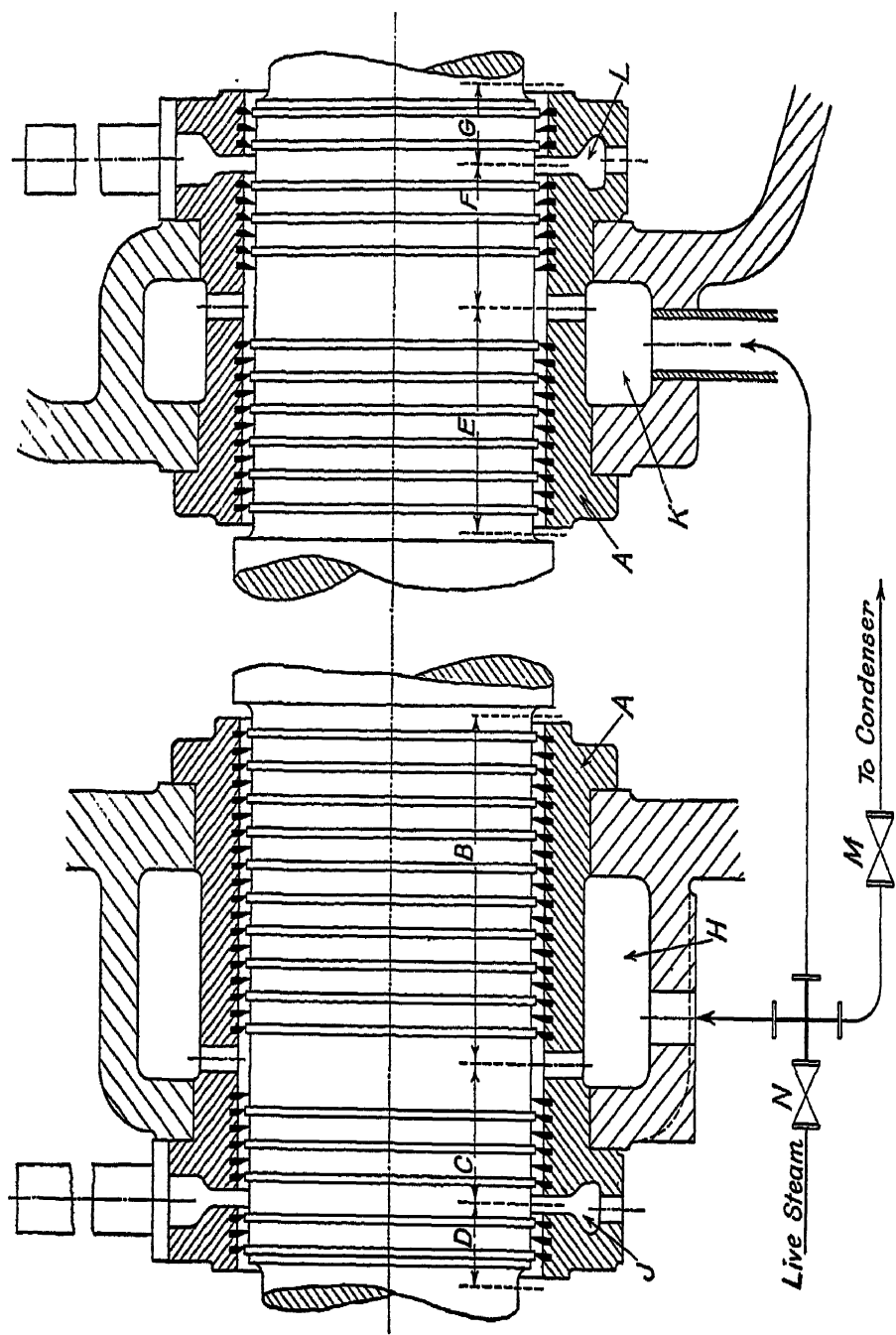


FIG. 259.—Arrangement of H.P. and L.P. Glands for Condensing Turbine

restraint exerted by the more massive hub. As soon as the cause of the disturbance is removed, the fins D cool and contract and the clearance is once again restored to its original value.

202. External Glands of the Labyrinth Type. Fig. 259 shows in general outline the H.P. and L.P. external glands for a single-cylinder condensing turbine. Each gland comprises a bush A which is in halves, bolted together. These bushes are located by machined surfaces at each end of the cylinder and are provided with some form of labyrinth packing. In the type of gland illustrated the H.P. gland is furnished with three separate groups of packing, namely, B situated between the first-stage and the leak-off pocket H, C between H and the vapour hood J, and D between J and the atmosphere. There are three groups of packing in the L.P. gland, namely, E between the steam pocket K and the exhaust space, F between K and the vapour hood L, and G between L and the atmosphere. In general, group B contains the largest number of packing rings, while group E contains the next largest. Groups C and F are made similar, also D and G; but the number of constrictions in C and F will usually be greater than those in the outermost sections. The steam connections to such glands are shown diagrammatically in Fig. 259, and in the operation of such connections it may be said at once that the steam pressure in pocket K must be continuously maintained at a value slightly above atmospheric. There is a free connection between pockets H and K, and under normal load conditions, most of the steam which leaks past group B enters pocket K of the L.P. gland. Some of this steam flows into the turbine through group E and the rest flows outwards, so preventing an inleakage of air. If the steam pressure in the pocket H is excessive, e.g. under full load or overload conditions, then the valve M may be opened just sufficiently to maintain a slight flow of steam through the vapour pipes. At light loads the steam pressure in the first stage will fall to such an extent that there will be insufficient steam available to seal the L.P. gland. Under these conditions, live steam is admitted to both glands through the valve N.

In some cases, the H.P. gland is provided with an additional pocket on the cylinder side (Fig. 260). This is connected by an external pipe to a suitable stage of the turbine where the pressure is normally well above atmospheric. Thus a part of the leaking steam is able to do work in the L.P. stages of the turbine. It will be seen that there are now four separate groups of packing in the H.P. gland. The steam connections are shown and do not call for much explanation. Live steam is supplied continuously to the L.P. gland so that its performance is not affected by load changes. The pressure in the pocket K is then controlled by opening the valves P or Q, according as the turbine load is heavy or light.

In tandem turbines the H.P. glands are usually provided with two or even three tappings, the steam being led either to later

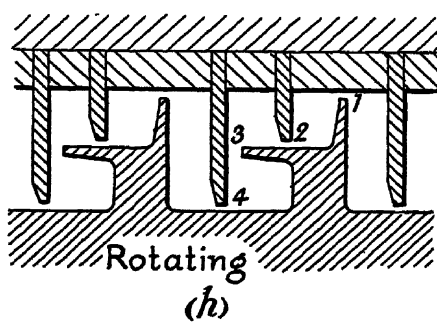
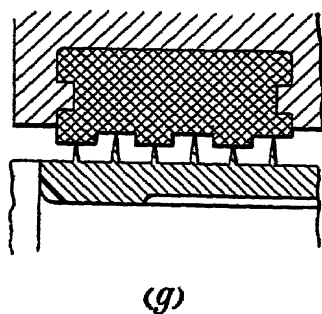
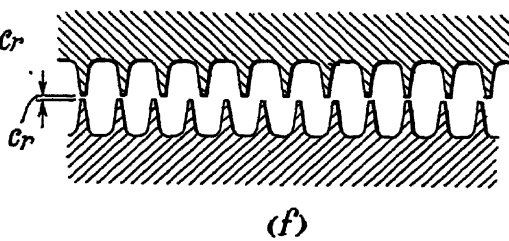
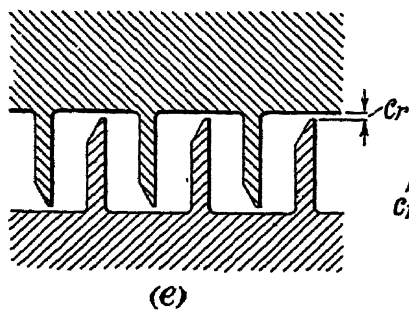
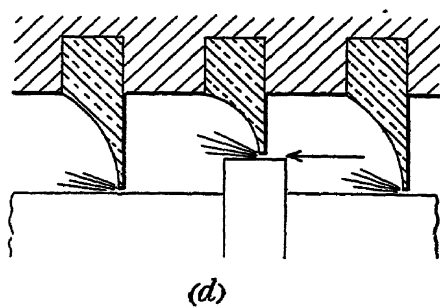
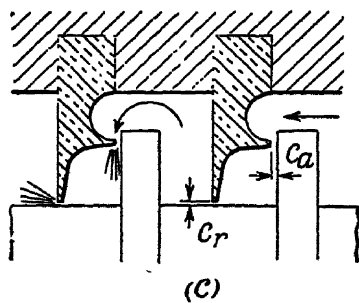
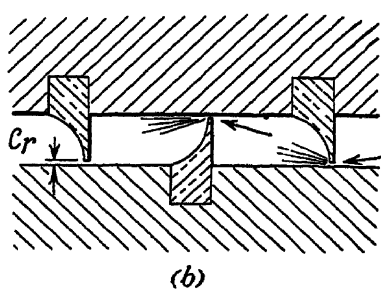
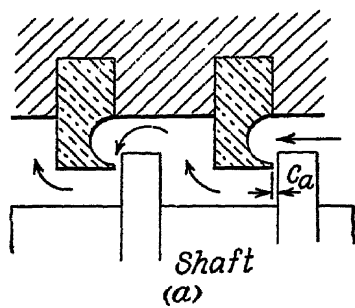


FIG. 261.—Forms of Labyrinth Packing

All the packings shown in Fig. 261 are of the rigid or non-yielding type, and therefore suffer from the disadvantage that if the shaft should be accidentally displaced, the packing strips will be permanently damaged and the leakage loss increased, to say nothing of the risk of a bent spindle. An example of yielding labyrinth packing is given in Fig. 262, showing the L.P. gland of a large

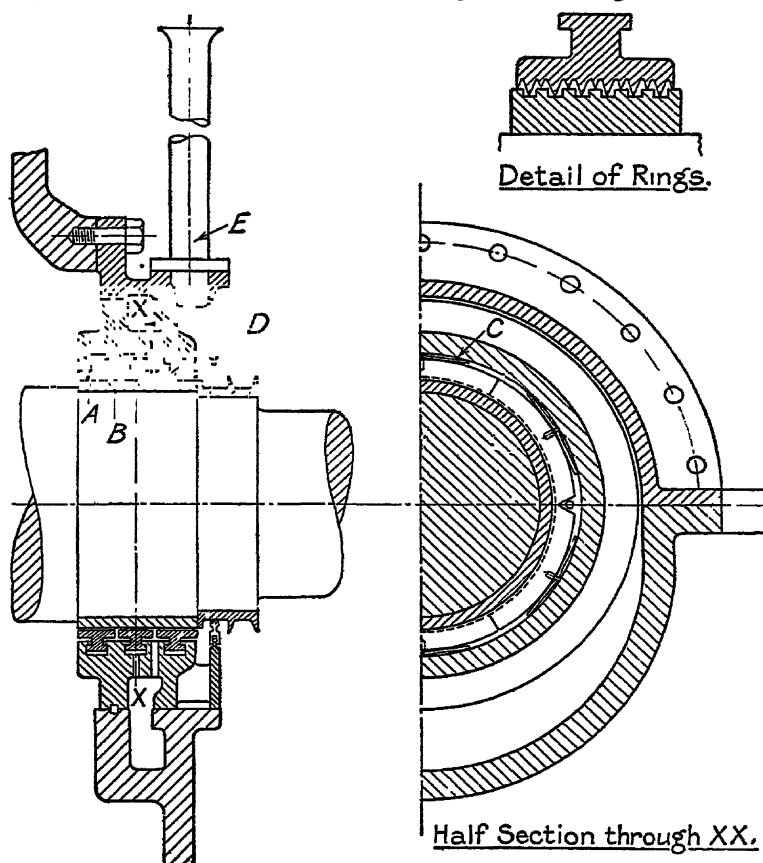


FIG 262.—L P Labyrinth Gland of B T -H Turbine

British Thomson-Houston turbine. A number of rings A, each formed in six segments, touch, or almost touch, the sleeve B on the shaft. The rings are held up to their work by flat plate springs C. As originally assembled, the rings are tight on the shaft, but after the turbine has been run at a low speed for a short time the fine edges of the fins ($\frac{1}{8}$ in. wide) are worn away, and the rings finally become centralized by the shoulders in the gland housing. Sealing steam is supplied to the pocket D, and leaking vapour is carried away by the pipe E.

There is another form of yielding packing in which, if rubbing between the rotating and stationary elements should take place, the stationary element is so shaped that the expansion which occurs

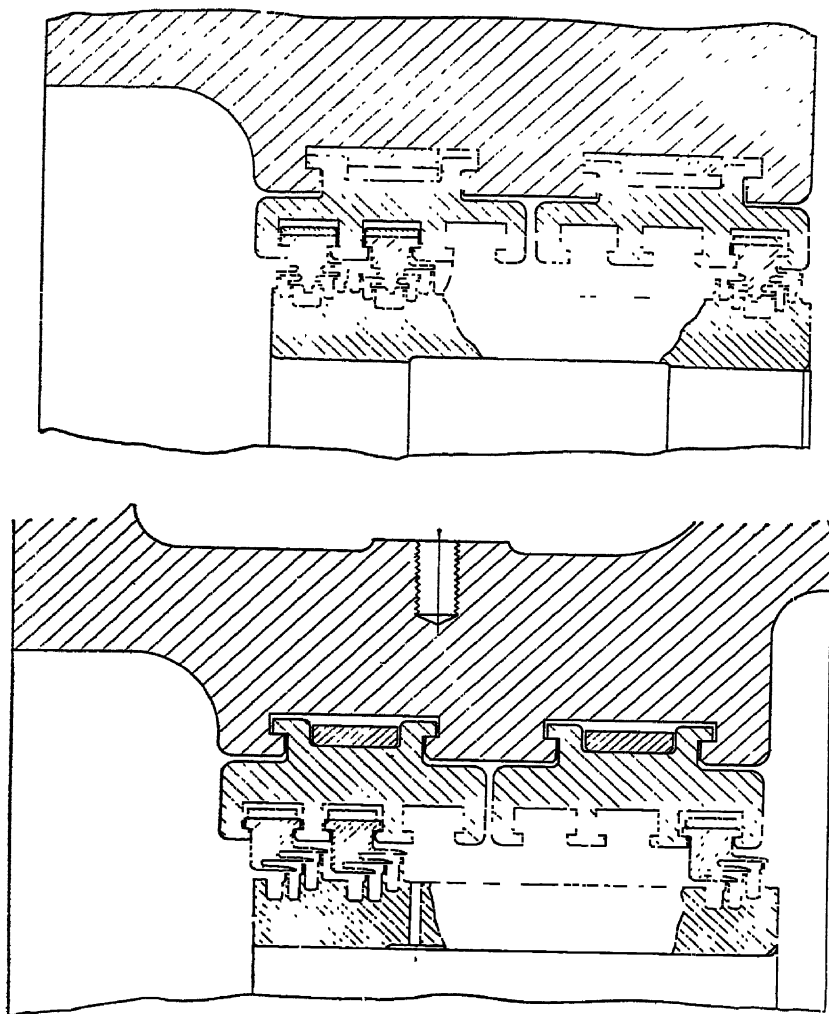


FIG. 263.—Metropolitan-Vickers Type of External Labyrinth Gland

due to heating causes that element to move outwards and so relieve the rubbing pressure. This principle appears to have been first employed in the labyrinth packings of the Ljungström turbine about the year 1912. An example of such a yielding packing has

already been given in Fig. 258. Further examples by the Metropolitan-Vickers Co. are shown in Fig. 263. Thin radial fins on the shaft sleeve mate with projecting cylindrical fins on the segmental stationary rings, which are centred by the larger rings and backed by plate springs. In the upper part of the figure, the gland has six elements providing, in all, thirty throttlings. The lower illustration shows the single type of gland which is used at the outlet end of the H.P. cylinder of tandem turbines so as to allow for the greater axial expansion at that end, the axial position of both the H.P.

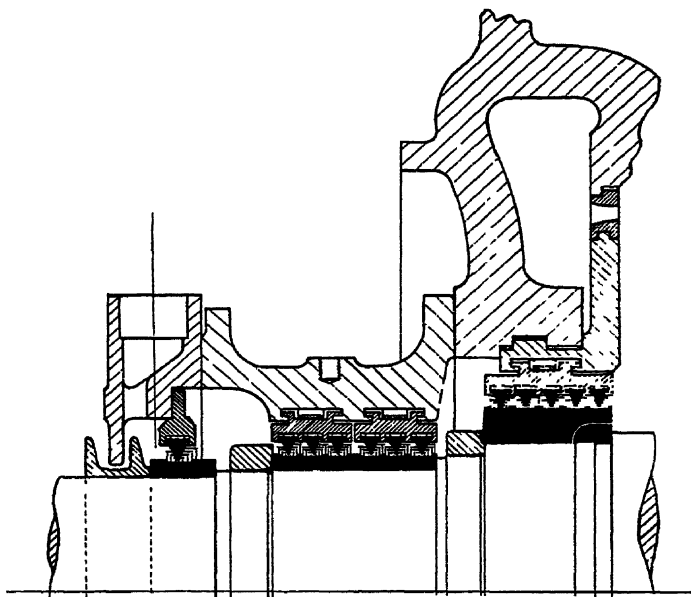


FIG. 264

and L.P. turbines in such tandem sets being fixed by a thrust block fitted at the inlet end of each turbine.

The application of such glands to the H.P. cylinder of a Metropolitan-Vickers turbine is shown in Fig. 264. The labyrinth packing is divided into two main sections, the pocket between being connected to a stage in the H.P. turbine. Leakage from the outer pocket is taken to a special feed-heater utilizing gland steam. A single packing element prevents steam from leaking into the turbine room.

The leakage of steam into the turbine room is often a nuisance because the steam condenses on roof girders, windows, and the like. A hydraulic seal is sometimes arranged on the atmospheric side of labyrinth glands in order to stop steam leakage.

204. Carbon-ring Glands. This type of gland comprises a number

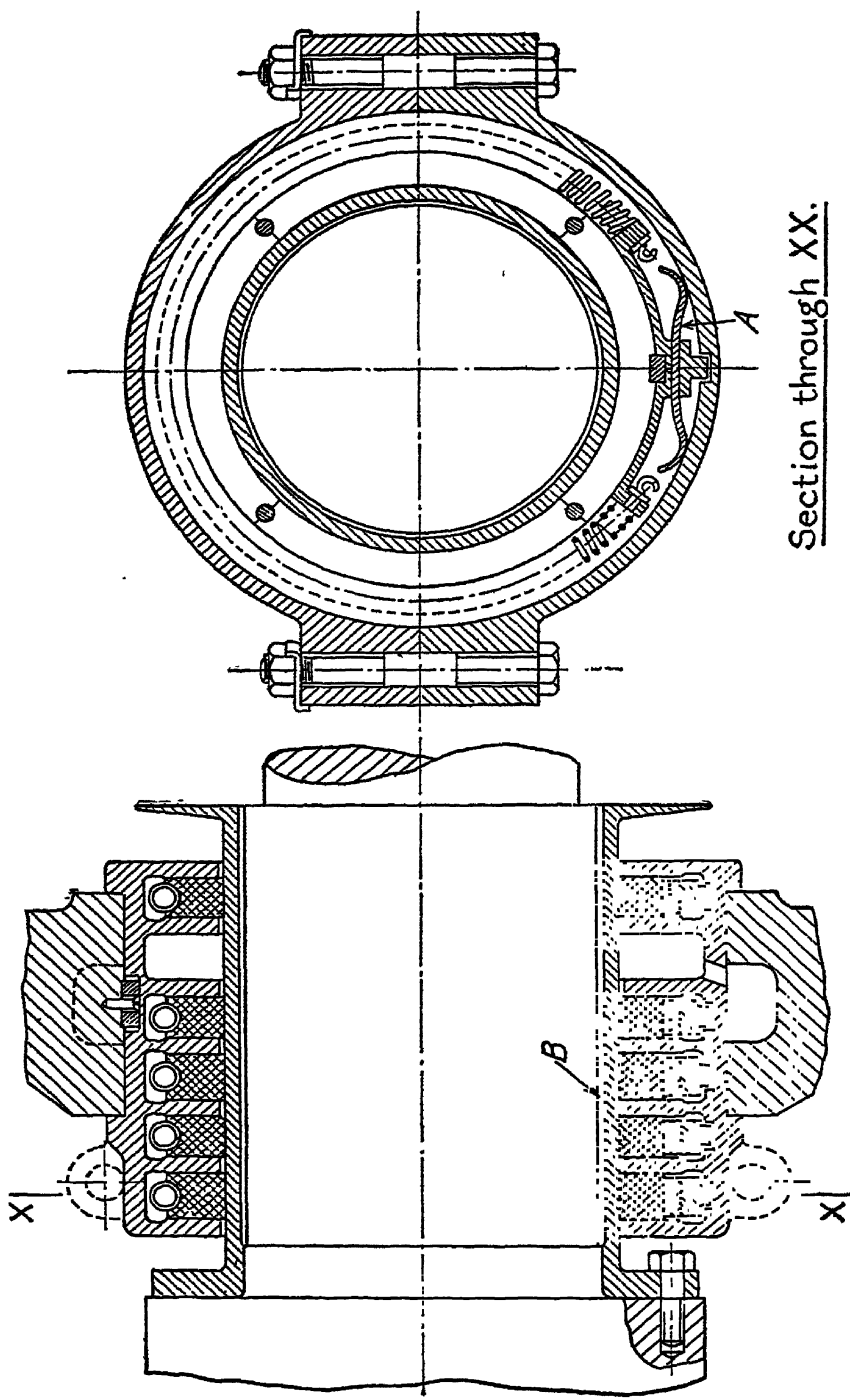


FIG. 265.—Carbon-ring Gland

of segmental rings of graphitic carbon. This material is self-lubricating. The rings are enclosed in a suitable housing. In some cases, the carbon rings are actually in contact with the spindle or sleeve thereon, but in others the lengths of the segments are such that there is a small radial clearance between the shaft and the inner surface of the rings. Such rings are said to be "arch-bound."

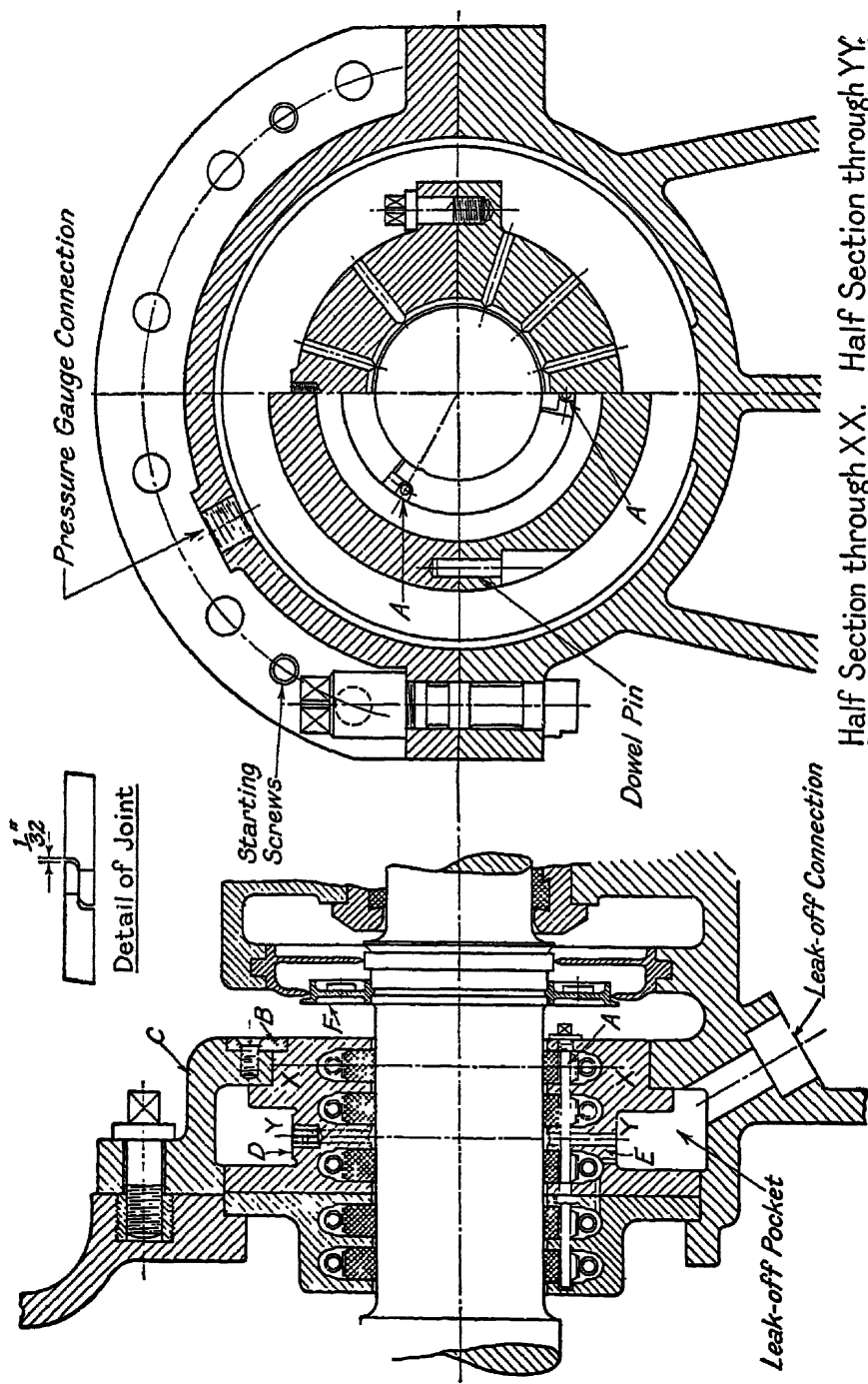
In all cases, the rings are held close to the shaft by a garter spring which is wrapped round the outside of the gland rings. When the rings are actually rubbing on the shaft (or sleeve) the tension of the spring should be such that the rings exert only a slight pressure on the shaft, e.g. about $1\frac{1}{2}$ to 2 lb. per sq. in. of ring surface.

In the arch-bound type of ring, the segments are supported, in addition, by a small coach spring (or springs) which keeps the rings centred with the shaft and yet permits freedom of movement if necessary. The radial clearance in this type should be sufficient to allow for expansion of the shaft due to temperature increase. Further, it is essential that the joints at the ends of the segments should be quite square and radial.

A typical carbon-ring gland is illustrated in Fig. 265. There are four rings between the inside of the turbine and the leak-off pocket, and one between the pocket and the atmosphere. The rings are arch-bound and are supported centrally by the coach spring A. This is attached to a light casting to which the garter springs are attached. Rotation of the rings is prevented by a key sunk into the lowest segment, and by a second key in a shallow recess at the bottom of the gland housing. The outer end of the sleeve B is fin-shaped and forms a deflector which prevents steam blowing out and overheating the bearing.

The design of gland used in the well-known Brown-Curtis turbine is shown in Fig. 266. The carbon rings are arranged in three segments, each with overlapping, or scarf, joints. There is a clearance at the joint so that the rings are in contact with the shaft. Rotation of the rings is prevented by the pins A. The rings are kept in contact with the shaft by the usual garter spring, but the outer surface of the carbon is protected by a thin strip of sheet brass. The outer half of the housing is prevented from possible rotation by a sunk key plate B. A number of radial holes drilled in the housing communicate with the leak-off (or steam supply) pocket. The rotating baffle F, which is a gunmetal casting in halves, serves the dual purpose of preventing leaking steam from blowing on to the bearing, and oil from creeping along the shaft and entering the gland.

Labyrinth rings and carbon rings are sometimes combined in the same gland, the labyrinth gland being on the steam side. It is claimed that the leakage with this arrangement is lower than with an all-labyrinth gland.



Half Section through XX. Half Section through YY.

FIG. 266.—Carbon-packed Glands of Brown-Curtis Marine Turbine

205. Packing of Balance Pistons in Parsons Turbines. Labyrinth packing of some form or other is invariably used. The types shown in Fig. 261 (*a*) and (*c*) are commonly employed in land turbines, since the balance pistons are fairly close to the thrust block, and the axial clearance may be kept reasonably small (say about 0.015 to 0.02 in.). If the relative axial movement between the rotating and stationary elements is likely to be appreciable, as in marine astern turbines for example, the radial type of packing shown in Fig. 261 (*b*) would be employed.

BEARINGS AND LUBRICATION

206. Introduction. When a lubricant such as a mineral oil is smeared over a metal surface and the surplus lubricant is wiped off it is found that there remains an exceedingly thin film on the surface. It appears that there is a bond between the molecules of the

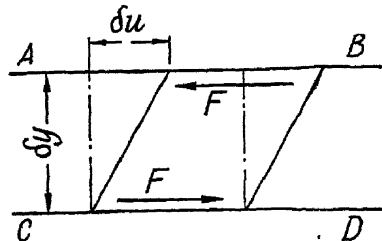


FIG. 267

lubricant and those of the metal surface which causes the film to adhere strongly to the metal. Such a film is said to be adsorbed and is only a few molecules thick. When metal surfaces are separated by adsorbed oil films the friction is described as *boundary friction* and the lubrication as *boundary lubrication*. The force of friction with

boundary lubrication is lower than the friction of dry surfaces. Owing to the imperfections of actual metal surfaces, true boundary lubrication occurs only at the "high spots" which are in contact.

If, by any means, a film of oil which is still quite thin (say 5 to 10 thousandths of an inch in thickness) but which is hundreds of thousands of molecules in thickness can be introduced and maintained between the two moving surfaces in a bearing, then the resistance to motion is due to the sliding of layers of oil over one another and is very much smaller than the resistance experienced in boundary lubrication. Moreover, the maintenance of a full and continuous film of oil between the bearing surfaces prevents any contact between the metals and thus prevents wear. Lubrication of this type is termed *film lubrication*, and the object of bearing design is to ensure complete film lubrication over the bearing surface with the minimum friction consistent with reliability and safety.

The most important property of a lubricant in film lubrication is its viscosity. Consider a fluid moving with stream line or viscous flow, i.e. with a motion free from eddies. Let AB and CD, Fig. 267, represent two imaginary parallel layers of fluid particles situated a small distance apart δy , and let δu be the velocity of AB relative to that of CD. Then the movement of AB will be opposed by a viscous drag F per unit area.

Then

$$F = Z \left(\frac{\delta u}{\delta y} \right)$$

where Z is defined as the coefficient of viscosity (or simply the viscosity) of the fluid.

In the c.g.s. units, $F = \frac{\text{dynes}}{\text{cm}^2}$, $\delta u = \frac{\text{cm.}}{\text{sec.}}$; and $\delta y = \text{cm.}$ Hence $Z = \frac{\text{dynes} \cdot \text{sec.}}{\text{cm}^2}$. This unit is known as the *poise* (after Poisieulle) The centi-poise is $\frac{1}{100}$ poise. Since $1 \text{ dyne} = \frac{\text{gr. cm.}}{\text{sec.}^2}$, the units of Z are also $\frac{\text{gr.}}{\text{cm. sec.}} \left(\frac{M}{LT} \right)$. To convert to British units—

$$\begin{aligned} 1 \text{ Poise} &= 0.00209 \frac{\text{lb. wt. sec.}}{\text{ft.}^2} \\ &= 0.0000145 \frac{\text{lb. wt. sec.}}{\text{in.}^2} \\ &= 0.0672 \frac{\text{lb.}}{\text{ft. sec.}} \left(\frac{M}{LT} \right) \end{aligned}$$

Typical temp.-viscosity curves for turbine lubricating oils are shown in Fig. 268, which shows clearly the influence of temperature

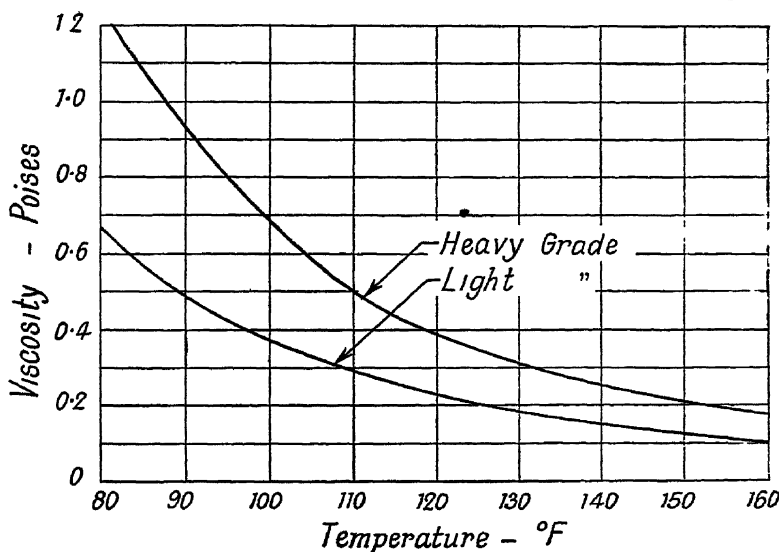


FIG. 268.—Range of Viscosity of Turbine Lubricating Oils

on viscosity. The normal range of temperature is from 110° F. to 140° F., and the tendency is to use the lighter grades of oil except where the bearing temperature is high or where the pressure intensity in the bearing is high.

207. Beauchamp Tower's Experiments. Little was known about the lubrication of journal bearings prior to the year 1883, when Beauchamp Tower published the *First Report of the Research*

Committee on Friction for the Institution of Mechanical Engineers. Subsequent reports were published in 1884, 1885, 1888, and 1891.

In most of his experiments on journal bearings, Tower used a bearing 4 in. diameter by 6 in. long, and subtending an angle of 155° . The brass rested on top of the shaft, and an oil bath was so arranged that the shaft could dip into it (see Fig. 269). Thus the oil was carried up into the brass as the shaft rotated. It was found that the brass was completely supported by an oil film. At one

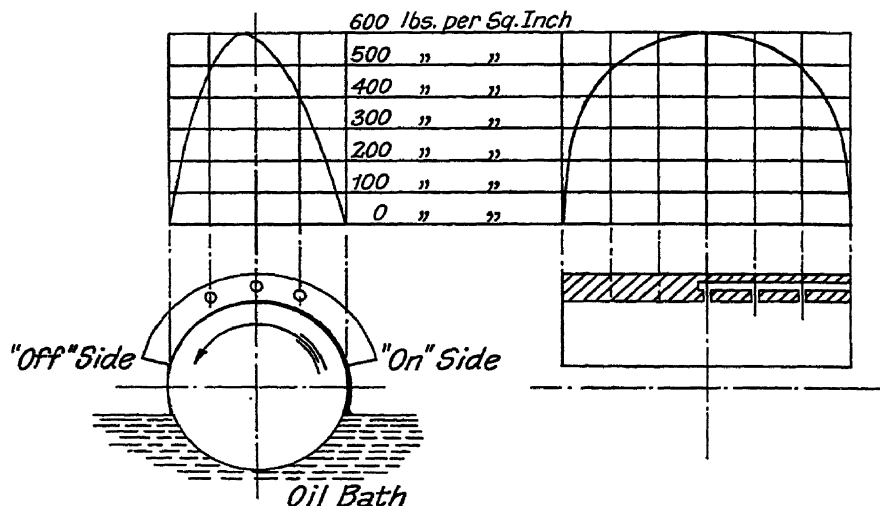


FIG. 269

point in the experiments, the brass had been dismantled and a $\frac{1}{8}$ -in. hole was drilled in the crown for a lubricator. On running the journal without the lubricator it was found that oil was emerging from the hole. The hole was accordingly plugged with wood, but it was found that, owing to the pressure of the oil, the plug was gradually being forced out. A series of holes was now drilled in the brass at different points around its arc of contact with the oil film and along its length. By means of these holes the pressures in the bearing could be measured and curves such as those in Fig. 269 obtained. The point of maximum pressure occurs a little on the "off" side of the centre of the brass, and the brass sets itself slightly eccentric to the journal, the oil film being thicker at the "on" side than at the "off" side.

Professor Osborne Reynolds studied Tower's results and came to the conclusion that the problem could be considered from the standpoint of hydrodynamics. He deduced certain equations which have formed the basis of lubrication theory ever since. These are set out, together with analyses of Tower's results, in a classical paper (2).

208. **Conditions Essential for Film Lubrication.** Tower's experiments show quite clearly that in a journal bearing the pressure in the oil film is atmospheric at inlet and outlet, and that in the central plane it increases to a certain maximum and then falls away again. The same remark applies to all other transverse planes except those coinciding with the sides of the bearing, where the pressure necessarily falls to atmospheric. Certain conditions are essential for the formation of such a pressure film. These will first be considered for a bearing of infinite width, i.e. without side leakage.

Imagine that Fig. 270 represents the cross-section of two plane surfaces A and B, both of which are infinitely wide in a direction perpendicular to the plane of the paper. A is assumed to be fixed while B is moving with constant speed. Assume, in the first case, that the two surfaces are separated by a continuous film of oil, and that there is no force tending to cause one surface to move

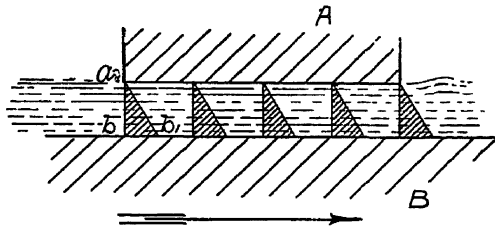


FIG 270

towards the other. Now the oil will cling to both surfaces and hence the particles of oil adhering to A will be stationary, while those adhering to B will have the same velocity as B. The oil film is being subjected to a continuous shearing and particles lying between A and B will have a velocity proportional to their distance from A. Hence, if bb_1 represents the velocity of B, the triangles abb_1 are velocity triangles, and the area cross-hatched represents the volume of oil passing any section per unit width perpendicular to the plane of the paper. Hence the same volume of oil flows past each section, and the arrangement simply constitutes a pump operating without any pressure difference.

Assume now that both surfaces A and B are at rest, but that a force is applied which will gradually cause the two surfaces to approach each other and so squeeze out the oil film. If the dotted lines represent the initial conditions (Fig. 271) and the full lines the conditions at a later instant, it is clear that the areas a_1 and a_2 of the bulges will be equal respectively to $x_1 \cdot \delta t$ and $x_2 \cdot \delta t$. It will be understood that the curved lines represent successions of oil particles which were originally represented by the straight dotted lines. The existence of the bulging lines is evidence of an internal pressure in the oil, which is momentarily balancing the external forces applied to A and B.

Suppose now that the conditions assumed in the foregoing considerations exist simultaneously, i.e. that B is moving and that a force is applied to A and B tending to bring them together (Fig. 272). It is to be further assumed that the surfaces are so constrained as to remain parallel. From what has been said before, it will be

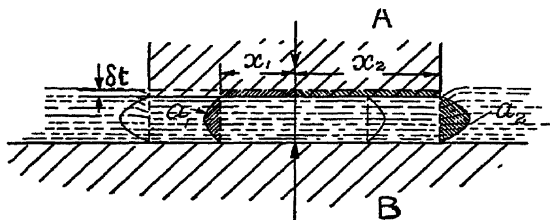


FIG. 271

at once apparent that the cross-hatched areas in Fig. 272 represent the volume of oil passing any section of unit width in unit time. But these areas are not equal, and it will be seen that more oil is passing out from between A and B than is entering, and there-

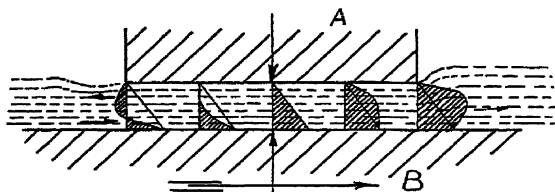


FIG. 272

fore that in a short time no oil will separate the surfaces and metallic contact will ensue.

It may be pointed out in passing that these are the conditions which exist in that type of thrust block in which one or more collars turned on the shaft abut against fixed rings or shoes. The latter are usually lined with white-metal and the oil is fed into the space between the shoe and collar at various points. In practice, the performance of this type of thrust bearing has never been satisfactory, and the maximum allowable pressure under a steady load does not exceed 30 lb. per sq. in.

Imagine now that the block A is freed from constraint (Fig. 273) and allowed to tilt relative to the surface B. Then if the tilt be such that the effective cross-hatched areas are equal at all points, a constant quantity of oil will pass between the two spaces. Moreover, the oil film is capable of sustaining and, therefore, of generating a certain pressure. This simple analysis shows that the essential condition for a pressure film is that the space between the two surfaces should be tapered or wedge-shaped.

This requirement may be shown more convincingly as follows. Let A and B, Fig. 274, be two parts of a journal bearing or Michell thrust block separated by a film of oil. Suppose that A is stationary and that B moves with a velocity u_1 . The oil particles in contact with A will be at rest, while those in contact with B will be

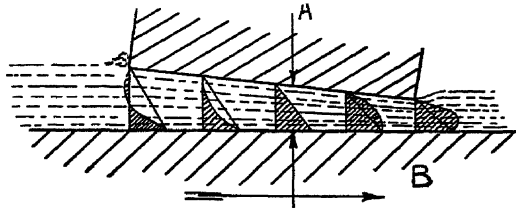


FIG 273

The actual inclination of A to B is about 1 in 3,000

moving with a velocity u_1 . The linear velocity of fluid particles in the film will vary in a definite manner across the depth of the film.

It is assumed that the depth of the bearing perpendicular to the paper is infinite, and that fluid particles move only in planes parallel

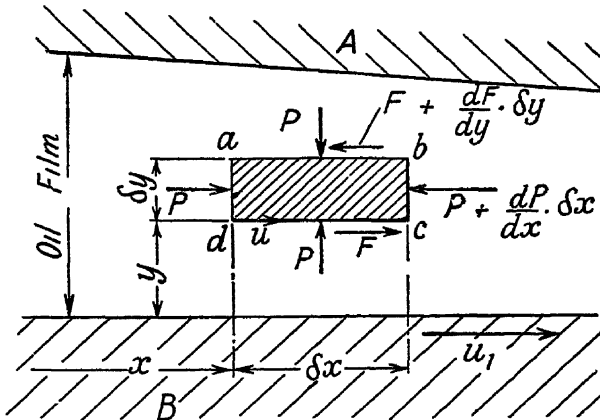


FIG 274

to the plane of the paper. Consider a small prismatic element of fluid $abcd$ of length δx , height δy , and unit depth. It will be assumed that for any value of x , the pressure intensity at any point in the film is P , i.e. P is independent of y . When x increases to $x + \delta x$, P increases to $P + \frac{dP}{dx} \cdot \delta x$. Let the velocity of the fluid

particles at a height y above B be u , and suppose that this velocity does not vary in the z direction. If the velocity gradient in the y

direction is $\frac{du}{dy}$, then the shearing stress in the oil film is

$$F = -Z \left(\frac{du}{dy} \right), \text{ because } \frac{du}{dy} \text{ is negative.}$$

Now, if $\frac{du}{dy}$ is constant, i.e. if the change of velocity is uniform across the depth of the film, then F is constant. If $\frac{du}{dy}$ varies, then F will also vary, and

$$\frac{dF}{dy} = -Z \left(\frac{d^2u}{dy^2} \right)$$

Considering the forces acting on the fluid element in the direction of motion (others balance), we have—

$$\frac{dP}{dx} \cdot \delta x \cdot \delta y = - \frac{dF}{dy} \cdot \delta y \cdot \delta x$$

$$\therefore \frac{dP}{dx} = - \frac{dF}{dy} = Z \left(\frac{d^2u}{dy^2} \right)$$

Consider the journal bearing used in Tower's experiments. Suppose that the two surfaces are developed so that one of them, e.g.

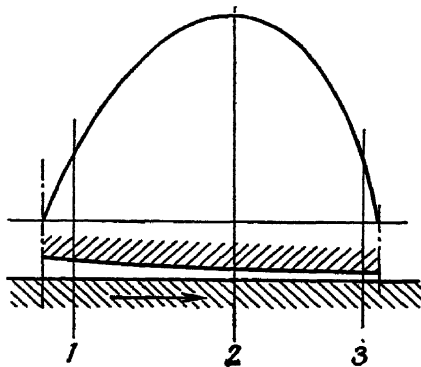


FIG. 275

the moving surface, is plane (Fig. 275). Then at the "on" side, section 1, $\frac{dP}{dx}$ is positive; at section 2, where P is a maximum, $\frac{dP}{dx} = 0$; while at the "off" side, section 3, $\frac{dP}{dx}$ is negative. In order to satisfy these requirements, the variation of velocity across the oil film at the points considered must be such that $\frac{d^2u}{dy^2}$ is positive, zero, and negative at sections 1, 2, and 3 respectively. The

necessary variations are shown in Fig. 276. It will be seen that these are the same as shown in Fig. 273

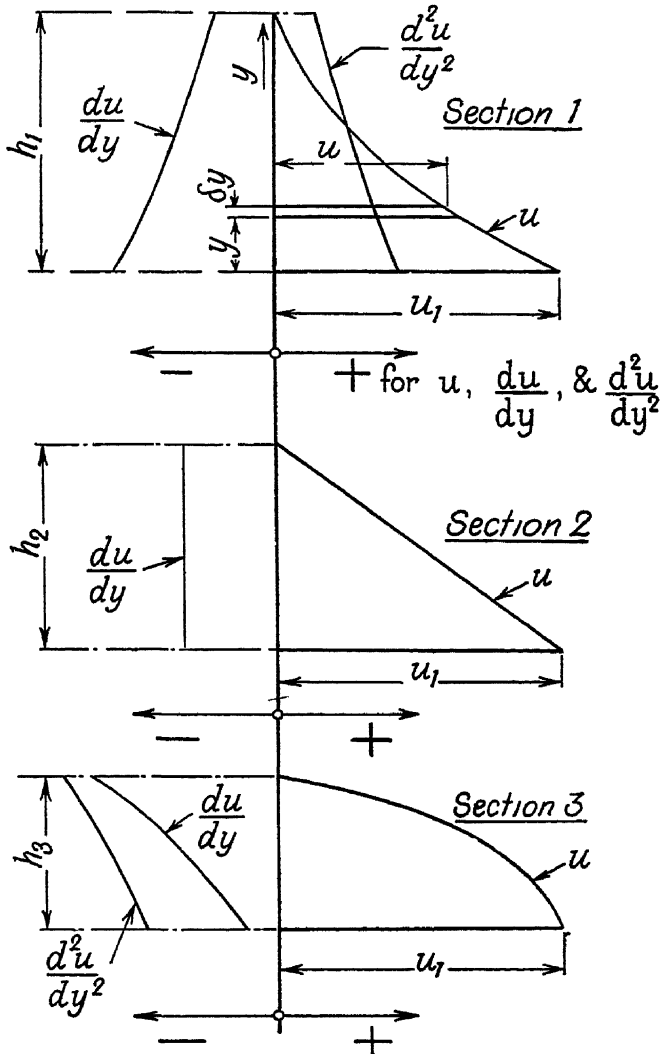


FIG. 276.—Variation of Velocity in Oil Film at Different Sections of Fig. 275

Consider now the quantity of fluid flowing past any section per unit depth perpendicular to the paper. This is obviously

$$Q = \int_0^h u \cdot dy = h \cdot u_m$$

where h is the thickness of the oil film measured perpendicular to the moving element, and u_m is the mean velocity of the fluid film. Since there is no side leakage, the volume of oil passing all three sections must be the same. Then since $u_{m1} < u_{m2} < u_{m3}$, $h_1 > h_2 > h_3$. That is to say, the oil film must converge in the direction of motion of the moving element.

If, as in actual bearings, there is side leakage, then the quantity of oil moving forward is continually being diminished, and an even greater convergence of the oil film becomes necessary.

209. The Journal Bearing. The essential parts of a journal bearing are shown in Figs. 277 and 278, the clearances being greatly

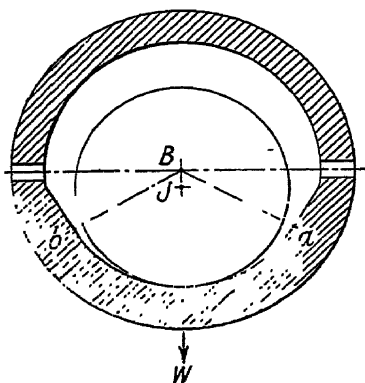


FIG. 277

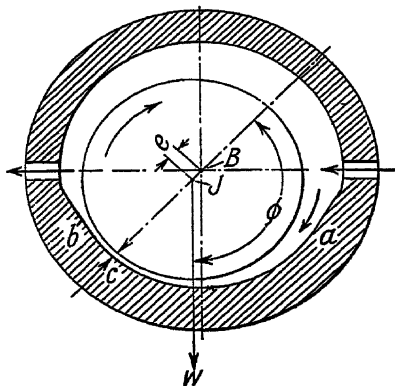


FIG. 278

exaggerated for the sake of clearness. It was formerly the practice to "bed" the journal into the "brass" by scraping the white-metal lining until the radius of the lower part of the brass was equal to that of the journal. In such a case, any possible movement of the journal which takes place when the journal begins to rotate does not lead to the formation of a satisfactory wedge-shaped film of oil, and it is difficult, if not impossible, for true film lubrication of such a bearing to take place.

In most cases, the lower part of the bearing is now bored out to a radius which is slightly larger than that of the journal itself. In some cases the clearance in the upper part of the bearing is made still larger, except at the ends of the bearing where a smaller clearance is employed in order to prevent excessive side leakage of oil. The arc ab represents the true bearing surface. Above a and b , the white-metal is scraped away to provide a large clearance. J and B represent the centres of the journal and bearing respectively.

Fig. 277 represents the bearing with the journal at rest. The journal is lying in the bottom of the brass, contact being along a comparatively narrow strip of bearing surface with a high intensity of pressure. Assuming clockwise rotation of the journal, it would

appear that at the first instant of movement the journal would tend to roll slightly to the right and upwards. But slipping would soon take place and there would be appreciable relative motion between the journal and the brass. During this stage the lubrication would be of the boundary type, although if the journal were kept rotating slowly it is quite likely that metallic contact and wear would ultimately take place. If, however, the speed be gradually raised and if oil be supplied at the "on" side, then oil will

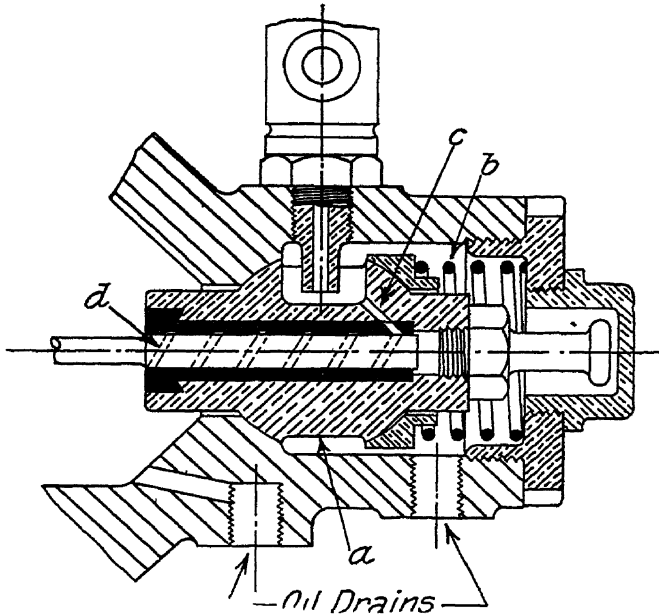


FIG. 279 —Bearing for De Laval Turbine

be dragged into the clearance space to a greater and greater extent. If the conditions are favourable, a continuous film of oil will form from *a* to *b*. It will be seen from Fig. 278 that for the formation of the necessary converging film of oil, the journal must move upwards and to the left. The final position of the journal is such that the resultant force due to the oil pressures generated in the oil film just balances the load on the journal. The distance *JB* is termed the "eccentricity" of the bearing, and the angle ϕ between the line of centres and the load line is termed the "attitude."

It will be clear that the oil film has its least thickness at *c*, Fig. 278, where the line of centres crosses the film. This minimum thickness equals $r - e$, where r is the radial clearance in the lower half and e is the eccentricity *JB* in the running position. From *c* to *b* the film is divergent, and in this part the pressure might fall below atmospheric so that air is drawn in. In such a case the film might

break down and metallic contact ensue. For this reason, the arc ab should not subtend too great an angle at the bearing centre. An angle of 110 to 120° is commonly used in turbine work. Professor Goodman, experimenting in 1885, showed that by reducing the arc of contact from 180° to 30° , other things remaining equal, the frictional resistance was progressively reduced.

210. Construction of Journal Bearings. In very small turbines drip feed lubrication may be employed. Fig. 279 shows a bearing for a small de Laval turbine. The bearing shown supports the

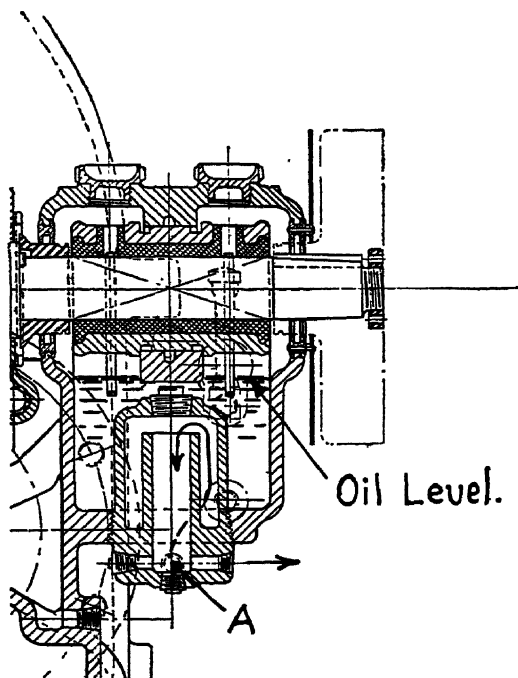


FIG. 280.—Water-cooled Ring-oiled Bearing on Terry Steam Turbine

turbine spindle at the end remote from the gearing, and consists of a bronze bush a , in one piece, and lined with white-metal. The bush is ball seated and, having little more than its own weight to support, is held in position by the compression of a light spring b . The oil dropping in from the sight feed enters the bearing at one end by the hole c . To distribute the oil satisfactorily, a shallow helical groove d is cut in the bush, the oil being carried to the other end of the bearing by the drag of the shaft.

A ring-oiled bearing for a Terry steam turbine is shown in Fig. 280. The bearing comprises a bush lined with white-metal and is provided with two rings dipping into an oil bath. Rotation of the

shaft causes the rings to rotate and oil to be lifted into the bearing. A simple method of cooling the oil is shown. Cooling water is supplied at A, rises in the annular space between the core and the outer chamber, and flows downwards through the central core to the outlet pipe

Considering the larger forced-lubricated bearings, it may be stated that practice differs chiefly in respect of the following—

- (a) Methods of boring out the white-metal.
- (b) Methods of supporting the bearing.
- (c) Methods of introducing the oil into the bearing.

Three methods of boring out the white-metal lining are indicated in Fig. 281. In (a) the brass is simply bored out truly circular with a diametral clearance of about 0.002 in. per inch of journal

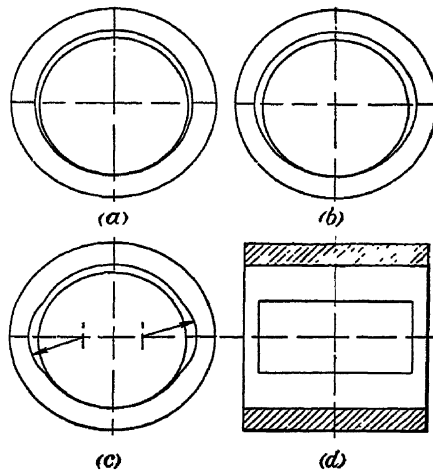


FIG 281

diameter. With small bearings, the clearance would be larger—from 0.003 to 0.005 in. In (b) the brass is bored out circular, but with temporary liners between the two halves. When the liners are removed and the bearing is assembled the horizontal clearance is greater than the vertical. In (c) the bearing is first bored out cylindrical with the clearances stated above, and then the sides are eased as shown, except at the ends of the bearing (see (d)).

Methods of supporting the brass and of introducing the oil will be considered together. The rigid type of support is well exemplified by the marine turbine bearing shown in Fig. 282. The bearing consists of bronze shells *a* lined with white-metal and bedded into the bearing block and keep. Lubricating oil is supplied through a hole in the side of the bearing block to the annular passage *b*, from whence it passes through the rectangular opening *c* into the space

between the journal and the brass. The curved dotted line *d* indicates the boundary of that part of the white-metal on either side of the horizontal joint which is eased off to facilitate entry of the oil. The dimension *E* is made about $\frac{D}{6}$.

Instead of fitting a pressure gauge (which might soon get out of order) to indicate the presence, or otherwise, of a plentiful supply

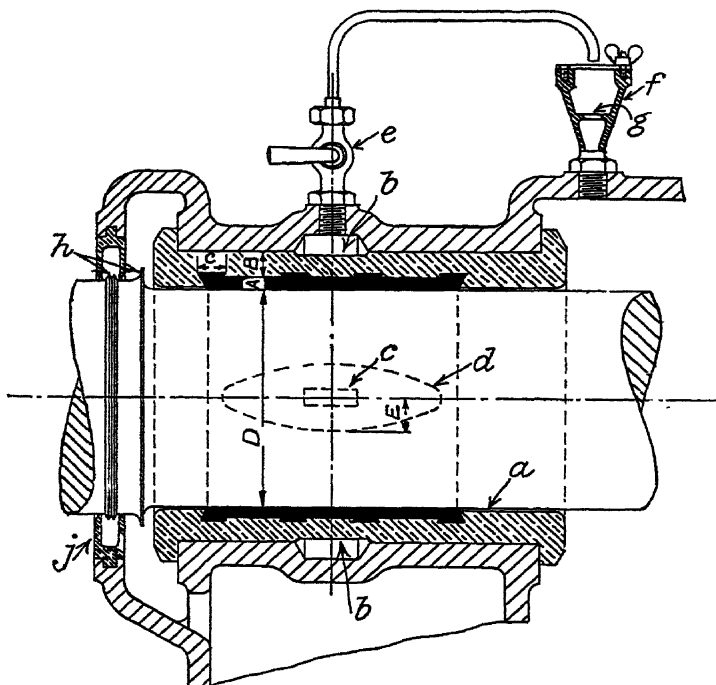


FIG. 282.—Bearing for High-speed Marine Turbine

of oil, a test cock *e*, which may be opened occasionally, is fitted. This cock is in communication with the oil supply passage *b*, and discharges into a funnel *f* fitted into the side of the bearing keep. The oil thus returned to the oil well is strained through a strip of wire gauze *g*. The oil leaking from the side of the bearing is prevented from creeping along the shaft into the turbine glands by the centrifugal throwers *h*, while the light baffle *j* serves the dual purpose of preventing oil and oil vapour from leaking out of the bearing housing and of minimizing the heating of the bearing by leakage of steam from the gland.

It will be noticed that the bronze portion of the bearing extends somewhat on either side of the white-metal, forming what is termed

a "safety strip." Should the white-metal flow, owing to overheating of the bearing, the rotor drops through a small distance on to the safety strip and may be so supported until steam is shut off. This would prevent a dangerous stripping of the blades, and would for a short time, provided the supply of oil is maintained,

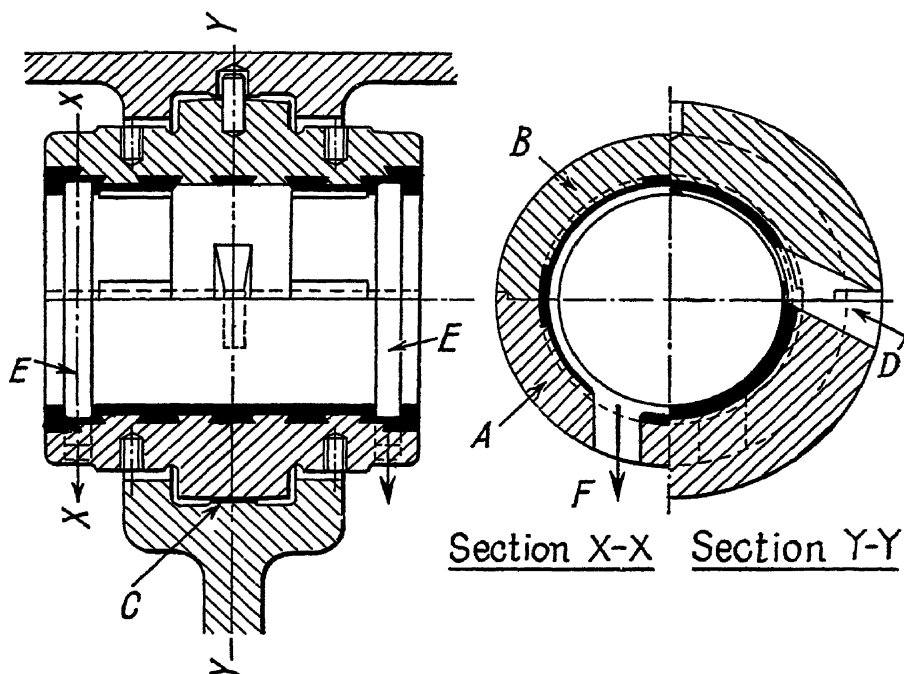


FIG 283

prevent serious damage to the shaft. The safety strip is usually about 0.015 in. below the white-metal all round.

The minimum thickness of the white metal = $A = \frac{D}{50} + \frac{3}{16}$ in. ;

$B = \frac{D}{10}$ for journals 4 in. dia. and over ; $C = \frac{D}{13} + \frac{1}{2}$ in.

Fig. 283 shows a turbine bearing by Messrs. Brown Boveri & Co., Ltd. The bearing consists of two halves A and B spigoted together and supported at C over a short length, so that the bearing can tilt slightly and thus yield to change in slope of the shaft. Oil enters at D at the "on" side. Oil leaking from the sides of the bearing flows into the grooves E and issues from the openings F. This device, widely used on turbine bearings, prevents oil from

creeping along the shaft in quantity and considerably lightens the duty of the oil throwers.

A bearing construction of considerable interest is shown in Fig.

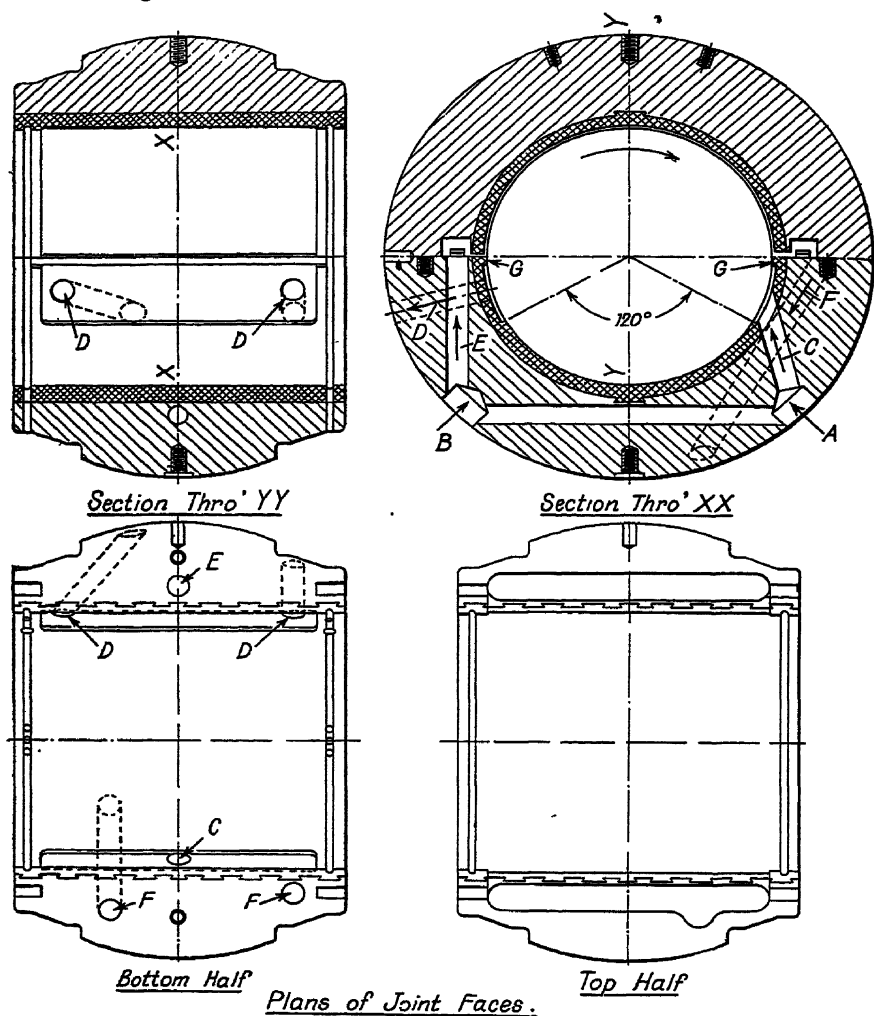


FIG. 284.—Spherically-seated Journal Bearing

284. It is designed to support a journal 20 in. diameter on a British Thomson-Houston steam turbine. The bearing is supported in a spherical seating in the pedestal. There are two definite supplies of oil, one for lubrication purposes and the other for cooling. The oil is supplied through ports in the seating and enters the lower half of the bearing at A and B. The lubricating oil proper enters

the bearing through the hole C and, after passing round the lower part, escapes through holes D. The cooling oil passes in through the hole E and, after passing through a narrow slit, enters the top half. It is dragged round by the journal surface and escapes through a long slit and finally by way of the holes F. At each side of the bearing the white-metal is carried out at G to within 0.003 in. of the shaft to form a wiper or scraper. This wiper prevents either the lubricating oil or the cooling oil from being circulated round

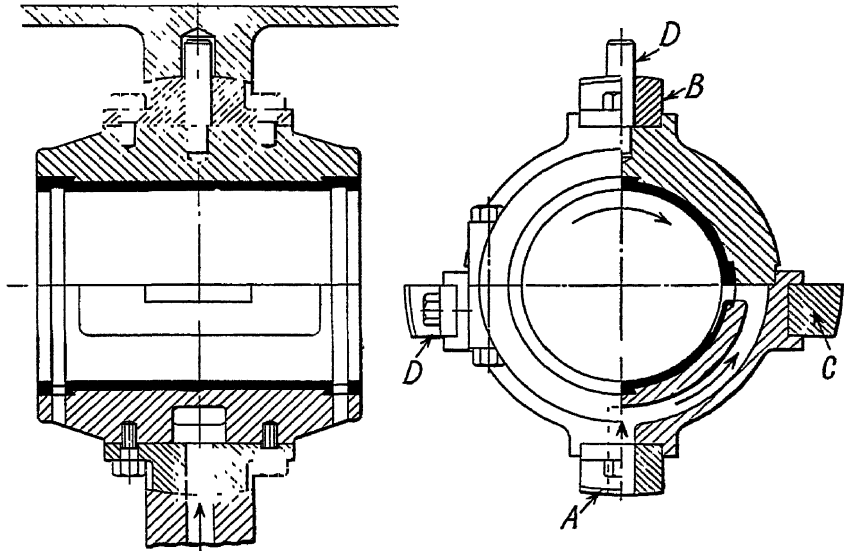


FIG 285.—Spherically-seated Bearing with Detachable Pads

the bearing, and has been found to effect a considerable reduction in power loss.

A form of bearing which is widely used in steam turbines is shown in Fig 285. The bearing is again supported in a spherical seating which allows freedom of alignment, but the steel pads A, B, C, and D are detachable. Between the pads and the bearing shell are a few thin liners which differ in thickness by small amounts. By changing these liners, the centre of the bearing may be adjusted for vertical or horizontal position. Lubricating oil is supplied through a hole in the pedestal and passes through a cored passage in the lower half of the bearing. A stop-pin D prevents the bearing from rotating.

211. Materials. The material most commonly used for the bearing shell is cast iron. Where greater strength or rigidity is desired, cast steel is employed, while in naval work bronze is mostly used. In all cases, the bearings are lined with white-metal. Longitudinal and circumferential dovetail grooves secure the lining in

the shell. In addition a bond is obtained by tinning the inside of the bearing shells before the white-metal is poured in.

Typical analyses of white-metal, given in a paper by Drs. H. L. Guy and D. M. Smith (3), are as follow—

	For Ordinary Conditions	For High Temp. Conditions
Tin . . .	78	86
Antimony . . .	13	8.5
Copper . . .	6	5.5
Lead . . .	3	

212. Design of Journal Bearings. The basic data for design are the total load W and the speed of rotation. The quantities which have to be determined include the radius of the journal, the axial width of the bearing, the angle which the effective bearing surface subtends at the centre of the bearing, the radial clearance, and the quantity of oil to be supplied per unit time.

The design of turbine bearings has been largely based on experience. Before film lubrication was understood and long after it had been explained by Osborne Reynolds, it was thought that, as in dry friction, the coefficient of friction would be constant and, in consequence, the heating effect in a bearing would be proportional to the product pu , where p is the pressure in lb. per sq. in. of projected area and u is the velocity of the journal surface. In accordance with this idea, the value of this product was kept within certain limits. This led, in many cases, to turbine bearings in which the ratio of width to radius was sometimes greater than 6. It is now realized that the pu product has little or no significance and, as a result, shorter bearings are used, the ratio of width to radius being from 2 to 3, and occasionally as low as 1.5. The pressure per sq. in. of projected area varies from 30 to 200 lb., the normal values ranging from 80 to 120. Owing to the increase in output of high-speed turbines, the journal surface speed has steadily increased and now often reaches 150 to 160 ft. per sec. As already stated, the effective bearing angle is normally 110° to 120° . The radial clearance ranges from 0.001 to 0.0015 in. per inch diameter.

There are many factors which tend to make the design and operation of turbine bearings a matter of trial and error. In the first place, the variation in the viscosity of the oil due to the rise in temperature which takes place during its passage through the bearing makes any theoretical treatment of the subject difficult. The radial expansion of the journal due to temperature rise is often

commensurate with the radial clearance allowed and should be specially considered in the case of turbines operating with high temperature steam. In this connection a water seal provides a valuable cooling agent between the gland and the bearing. Then finally the distortion of the bearing under heavy loads may produce appreciable changes in curvature and clearance.

A method of design, based on hydrodynamic principles, has, nevertheless, been put forward by Professor H. W. Swift (4). This method is based partly on a theoretical paper (5) by himself, and partly on a paper by Dr. Albert Kingsbury (6). The following brief summary of the method is set out for 120° bearings only, since this is the type approaching nearest those used in steam turbines. For the more complete application of the method the reader is referred to the original paper.

Let W = total load on bearing, lb.

R = radius of journal, in.

B = axial width of bearing, in.

L = circumferential length of effective bearing arc, in.

r = radial clearance, in.

ω = angular velocity of journal

P = load per unit width = $\frac{W}{B}$.

Z = viscosity of oil, lb. wt. in. sec. units.

u = surface speed of journal, in. per sec.

ε = eccentricity = $\frac{e}{r}$ (see Fig. 278).

Then it may be shown for an infinitely wide bearing without side leakage that $\left(\frac{P}{Zu}\right)\left(\frac{r^2}{R^2}\right)$ is a function of the eccentricity ε for a given bearing angle. $\left(\frac{P}{Zu}\right)\left(\frac{r^2}{R^2}\right)$ is a dimensionless quantity which is termed the load criterion. Writing Δ for this, the calculated relationship between Δ and ε is shown in Fig. 286. It will be observed from this that greater values of the load criterion Δ are associated with greater values of ε and, therefore, with thinner oil films.

It may also be shown mathematically that $\frac{\mu R}{r}$ is a function of ε ; the relationship for a bearing without side leakage is shown in Fig. 286, the curve being a rough interpolation between curves for bearing angles of 90° and 180° .

The design of the bearing should be such that—

(1) The thickness of the oil film at the point of closest approach should be as great as possible. This leads to maximum reliability.

(2) The coefficient of friction μ should be as small as possible.

The minimum thickness of the oil film is $t = r(1 - \varepsilon)$. For any value of ε , the corresponding value of Δ may be obtained from

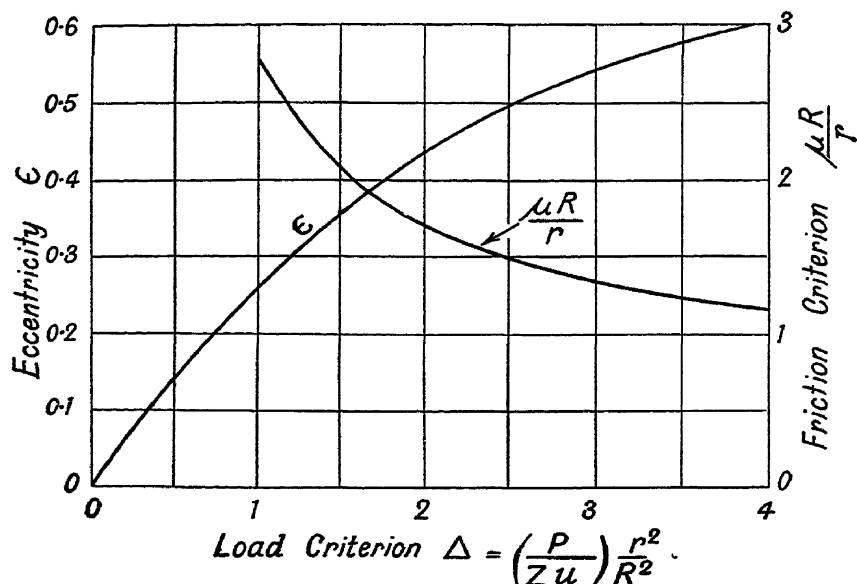


FIG. 286.—120° Journal Bearings of Infinite Width

Fig. 286. From this we may calculate $\frac{r}{R} \sqrt{\frac{P}{Zu}}$ and hence $\frac{t}{R} \sqrt{\frac{P}{Zu}}$, termed the film thickness characteristic of the bearing. Values of this are plotted in Fig. 287.

Similarly, since for any given value of ε we have corresponding values of $\frac{r}{R} \sqrt{\frac{P}{Zu}}$ and $\frac{\mu R}{r}$, it follows we have also values of $\mu \sqrt{\frac{P}{Zu}}$, the friction characteristic of the bearing, Fig. 287. Inspection of Fig. 287 shows that the thickness characteristic is a maximum and the friction characteristic a minimum when ε is about 0.5 for the infinitely wide 120° bearing, and

$$\left[\frac{t}{R} \sqrt{\frac{P}{Zu}} \right]_{B = \infty} = 0.794 \quad . \quad . \quad . \quad (202)$$

$$\left[\mu \sqrt{\frac{P}{Zu}} \right]_{B = \infty} = 2.34 \quad . \quad . \quad . \quad (203)$$

The effect of side leakage of oil in a bearing of finite width is to reduce the load-carrying capacity and to increase the coefficient of

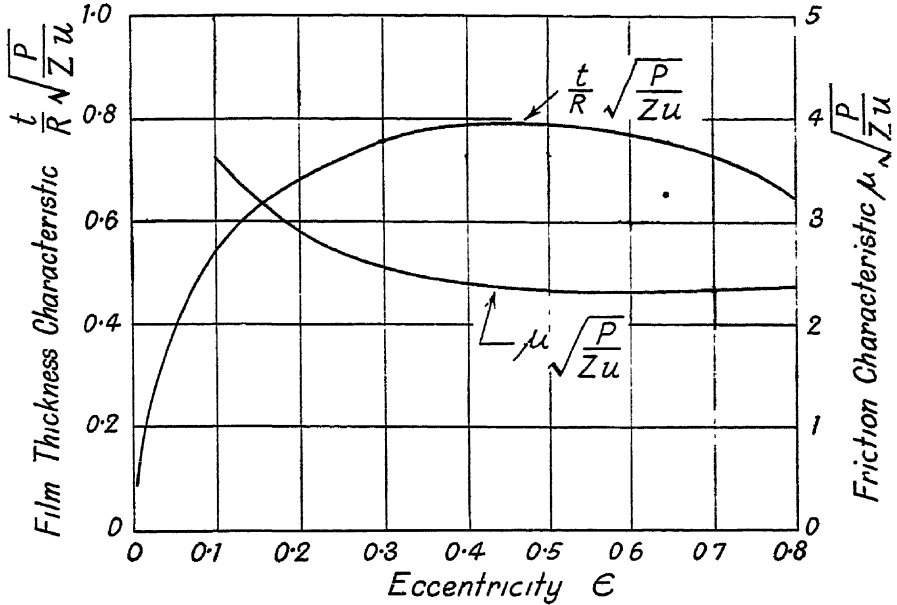


FIG 287 —120° Journal Bearings of Infinite Width

friction. Kingsbury, employing an electrical analogy, has obtained the relationship between p_0 , p , μ_0 and μ , where—

p_0 = pressure intensity in bearing of infinite width

p = pressure intensity in bearing of finite width

μ_0 = coefficient of friction in bearing of infinite width

μ = coefficient of friction in bearing of finite width.

These ratios are plotted in Fig. 288 for various values of L/B .

$$\text{Now} \quad \frac{t}{R} \sqrt{\frac{P}{Zu}} = \frac{t}{R^2} \sqrt{\frac{R}{B}} \sqrt{\frac{W}{\omega Z}}$$

$$\therefore \quad \frac{t}{R^2} \sqrt{\frac{W}{\omega Z}} = \frac{t}{R} \sqrt{\frac{B}{R}} \sqrt{\frac{P}{Zu}}$$

$$= \sqrt{\frac{p}{p_0} \cdot \frac{B}{R}} \left[\frac{t}{R} \sqrt{\frac{P}{Zu}} \right]_{B=\infty}$$

$$= 0.794 \sqrt{\frac{p}{p_0} \cdot \frac{B}{R}} \quad \quad \quad (204)$$

$$\text{Also} \quad \mu \sqrt{\frac{P}{Zu}} = \frac{\mu}{R} \sqrt{\frac{R}{B}} \sqrt{\frac{W}{\omega Z}}$$

$$\begin{aligned} \therefore \quad \frac{\mu}{R} \sqrt{\frac{W}{\omega Z}} &= \sqrt{\left(\frac{\mu}{\mu_0}\right)^2 \left(\frac{p}{p_0}\right) \frac{B}{R} \left[\mu \sqrt{\frac{P}{Z\mu}} \right]_{B=\infty}} \\ &= 2.34 \sqrt{\left(\frac{\mu}{\mu_0}\right)^2 \left(\frac{p}{p_0}\right) \frac{B}{R}} \quad \quad \quad (205) \end{aligned}$$

If varying values of $\frac{B}{R}$ be taken it will be found that the friction is least when $\frac{B}{R}$ equals 2. It increases rapidly for lower values but only slowly for higher values.

Although such methods might give reliable results for partial bearings, the power losses worked out on the coefficient of friction

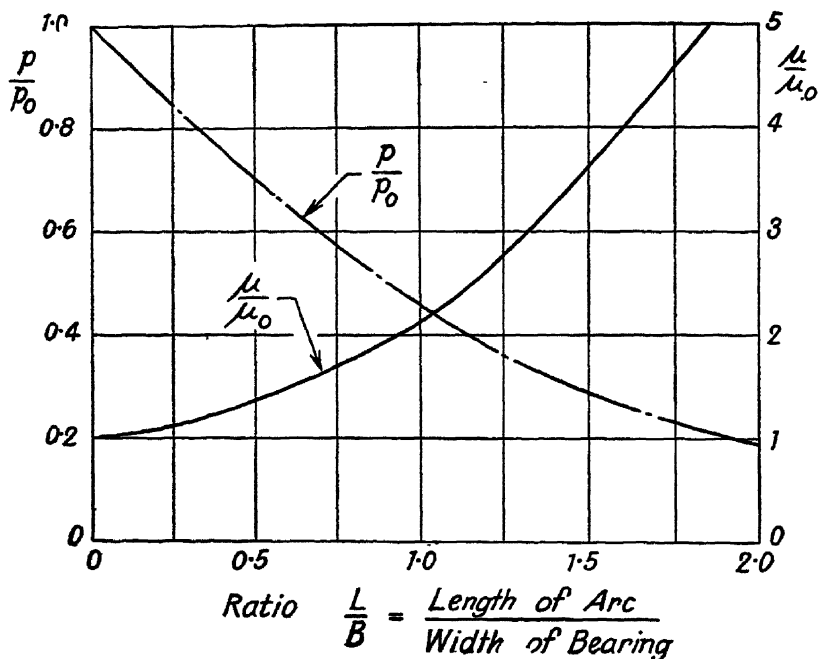


FIG. 288.—Correction Curves for Bearings of Finite Width

derived by their use are generally less than those measured experimentally on actual turbine bearings, the ratio of the actual to the theoretical value being from 1.5 to 2.0. This is probably due to the fact that the actual bearing, although sometimes provided with a fairly large clearance at the top, does embrace the journal for 360 degrees. Moreover, oil is circulating around the complete bearing. Tests on a given type of bearing have shown quite definitely that the power absorbed is increased when a greater quantity of oil is supplied to the bearing.

EXAMPLE 29. The load on a journal bearing is 12,000 lb., and the speed 3,000 r.p.m. Calculate suitable proportions for the bearing, assuming a pressure of 120 lb. per sq in. of true projected area. Calculate also the least thickness of the oil film, the radial clearance, the coefficient of friction and the theoretical power loss due to friction. Assume that the mean viscosity of the lubricating oil is 0.2 poise

We shall assume a bearing angle of 120° and a "square" bearing, i.e. $B = 2R$

$$\begin{aligned}\text{Then the projected area} &= 2BR \cos 30^\circ \\ &= 4R^2 \cos 30^\circ = 3.464R^2 \\ 3.464R^2 &= 12,000 \div 120 = 100 \text{ sq in.} \\ R &= 5.375 \text{ in}\end{aligned}$$

$$\begin{aligned}\therefore \text{Dia. of journal} &= 10\frac{3}{4} \text{ in} \\ \text{Length of bearing} &= 10\frac{3}{4} \text{ in.}\end{aligned}$$

Assuming these dimensions are acceptable we would proceed as follows, basing the calculations on the assumption that $\varepsilon = 0.5$.

$$\omega = 314.2 \text{ radians per sec}$$

$$Z = 0.2 \text{ poise} = 0.0000029 \frac{\text{lb. wt. sec.}}{\text{in}^2}$$

$$\text{Length of bearing arc } L = 11.26 \text{ in}$$

$$\therefore \frac{L}{B} = \frac{11.26}{10.75} = 1.05$$

$$\text{Then by Fig 288, we have } \frac{p}{p_0} = 0.43, \text{ and } \frac{\mu}{\mu_0} = 2.27.$$

$$\text{Then } \frac{t}{R^2} \sqrt{\frac{W}{\omega Z}} = 0.794 \sqrt{0.43 \times 2} = 0.736$$

$$\therefore t = 0.736R^2 \sqrt{\frac{\omega Z}{W}} = 0.00586 \text{ in.}$$

This is the least thickness of the oil film, i.e. about 6 thousandths of an inch. Since $\varepsilon = 0.5$, the radial clearance must be $2 \times 0.00586 = 0.0117$, say 0.012 in. This is 0.0011 in. per in. diameter, which is in line with current practice.

$$\frac{\mu}{R} \sqrt{\frac{W}{\omega Z}} = 2.34 \sqrt{\left(\frac{\mu}{\mu_0}\right)^2 \left(\frac{p}{p_0}\right) \frac{B}{R}} = 4.92$$

$$\therefore \mu = 4.92R \sqrt{\frac{\omega Z}{W}} = 0.00727$$

$$\therefore \text{Frictional force} = 0.00727 \times 12,000 = 87.2 \text{ lb}$$

Since the surface speed of the journal is 141 ft. per sec.—

$$\text{Theoretical power loss} = \frac{87.2 \times 141}{550} = 22.3 \text{ h.p. or } 16.7 \text{ kW}$$

213. **The Journal Centre Locus.** The path in which the journal centre J moves when the load on the bearing is constant and the speed of rotation or the viscosity of the oil are changed is termed

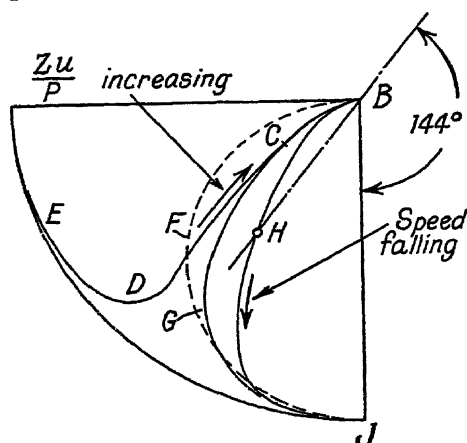


FIG. 289.—Journal Centre Loci

the journal centre locus. Let the quadrant of the circle in Fig. 289 be drawn with centre B corresponding to the centre B of the lower part of the bearing, Figs. 277 and 278, and with radius BJ equal to the radial clearance r . Then J represents the lowest and rest position of the journal. Although Reynolds published his classical equations in 1886, it was not until 1904 that Professor Sommerfeld, in a masterly study (7) of the 180° journal bearing, worked out the locus of the journal centre. BCDE shows the calculated locus for clock-

wise rotation. It may be shown that the actual position taken up by the journal centre depends on the dimensionless quantity $\frac{Zu}{P}$. If Z and P are assumed to be constant then the position of

the journal centre depends only on the surface speed u , i.e. on the speed of rotation. As the speed is increased the journal centre moves in the direction EDCB, and vice versa. This means that if the speed is falling the journal centre moves in the direction of E and ultimately the journal would seize at the *side*. In a later analysis, Professor Gumbel arrived at the conclusion that the locus was approximately the semi-circle BFJ, which gave J as the seizing point—a more rational result. Swift (5) has more recently analysed the same problem and shown that for stability the locus for the 180° bearing should be the curve BCGJ, which coincides with part of Sommerfeld's curve. For the 120° bearing the theoretical locus is the curve BHJ. With an eccentricity ϵ equal to 0.5, the running position of the journal centre is H and the attitude $\phi = 144^\circ$.

214. **Conditions during Stopping and Starting.** Consider a turbine which is being shut down. As the speed falls, the load criterion $\left(\frac{P}{Zu}\right)\left(\frac{r^2}{R^3}\right)$ increases, and as the curve of Fig. 286 shows, the

eccentricity e increases. Thus the journal centre gradually moves from H towards J, Fig. 289, the oil film becoming thinner and thinner. When the speed has fallen to a certain critical value the oil film becomes exceedingly thin and breaks down. The lubrication

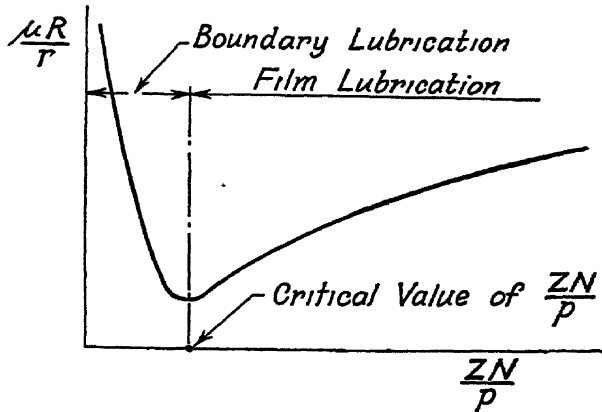


FIG. 290

changes from film lubrication to the boundary type and the coefficient of friction increases as the speed falls. The relationship between $\frac{\mu R}{r}$ and $\frac{ZN}{p}$ is shown by Fig. 290.

Messrs. S. A. and T. R. McKee (8) carried out experiments on a bearing in which $B/R = 2$, $r/R = 0.002$ approx; and $p = 192$ lb. per sq. in. Two mineral oils were used having widely different viscosities. From results obtained, the author has calculated that the critical values of $\frac{ZN}{p}$ at which the type of lubrication changes is about 0.605×10^{-6} for bronze bearings and 0.208×10^{-6} for babbitt bearings.

In a paper (9) on journal bearings, Mr. Francis Hodgkinson describes experiments made on a large steam turbine with the object of finding the lowest speed at which the oil films could be maintained. The turbine was allowed to come to rest under its own momentum. Accurate speed measurements were made and from these the retardation was carefully calculated. It was found that with oil temperatures of 115°F . and 95°F . the retardation suddenly increased when the speed had fallen to 19 r.p.m. and 16 r.p.m. respectively. From the data given in the paper, the critical values of $\frac{ZN}{p}$ work out at 0.38×10^{-6} and 0.53×10^{-6} respectively.

These values are necessarily approximate, since all four bearings would hardly reach the critical state at the same speed.

It appears, therefore, that for bearings lined with white-metal

and lubricated with mineral oils, the critical value of $\frac{ZN}{p}$ is in the neighbourhood of 0.5×10^{-6} .

In starting a turbine, the conditions are somewhat different. As already explained in Art. 209, the first tendency is for the journal to roll on to the "on" side; slipping then takes place and the lubrication is of the boundary type. In general, however, the acceleration will be relatively high and film lubrication will soon be initiated. There is no doubt, however, that the most arduous conditions as regards wear occur at starting and stopping.

215. Turning Gear. If a turbine is shut down and the rotor allowed to rest in one position for an appreciable time, then due to unequal heating the spindle bends. If the turbine is a large one, considerable vibration may occur when the turbine is started again and it may be some hours before the machine can be safely run at its full speed. For this reason, most large steam turbines are provided with motor-driven turning gear to keep the rotor turning slowly while the machine is off load. If the speed of turning is too low for film lubrication to be effective (see preceding article), a special lubricating oil pump is installed. This is a reciprocating pump, sometimes having a separate plunger to each bearing and supplying oil at a pressure of 1,000–1,500 lb. per sq. in. This provides a film of oil under each journal and, while preventing metallic contact, reduces the power required for turning. The Parsons 50,000 kW turbines at Dunston are turned at 13 r.p.m. when off load.

216. Thrust Bearings. In impulse turbines there is a certain force in the axial direction due to the change of momentum of the steam in that direction, as explained in Art. 62. This force tends to push the rotor towards the exhaust end of the turbine, but the total force is usually quite small. In many large so-called impulse turbines, however, the last few stages operate with partial reaction, and the axial forces on the rotor are considerably increased. In Parsons turbines, the axial forces on the moving blades are very large, but these are largely balanced by the steam pressures on the balance pistons. There is, however, a certain moderate residual thrust.

In all cases, a thrust bearing is provided, not only to support the axial force on the rotor but also to fix accurately the axial position of the rotor relative to the casing. The types of thrust bearing which are used in turbine work may be enumerated as follows—

(1) Plain collar type; (2) Bevelled collar type; (3) Kingsbury or Michell type; and (4) Plain collar with fixed inclined thrust pads.

217. Plain and Bevelled Collar Bearings. The plain collar type of bearing comprises a number of collars turned on the shaft working in a fixed housing which must, of course, be in halves. The housing

is lined with white-metal. Oil is supplied through a number of radial holes to the inner edges of the collars and taken off through a number of radial holes at the outer side of the recesses (Fig. 291 (a)). It has already been shown in Art 208 that the plain collar type of bearing is fundamentally unsound. Improved performance may

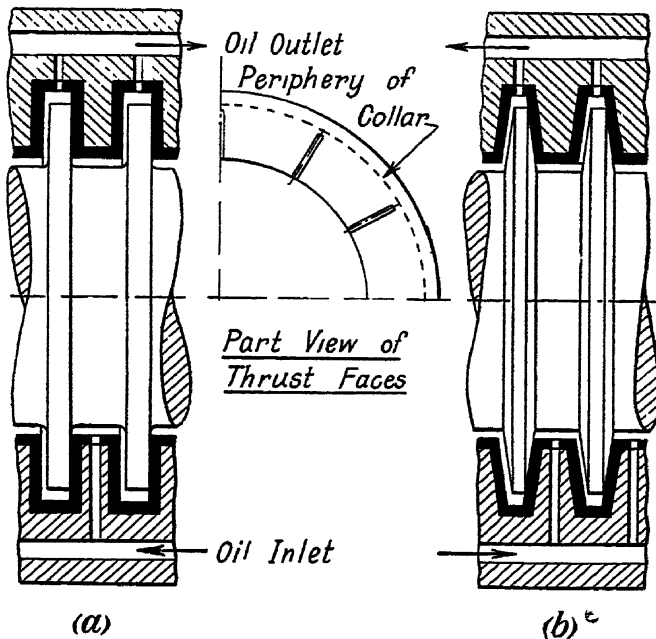


FIG 291.—Multi-collar Thrust Bearings

be obtained by dividing up the bearing surfaces into sectors and providing radial grooves to distribute the oil to the leading edges of such sectors, as shown in the figure. The sides of such grooves should be well rounded. Fig. 291 (b) illustrates the principle of the bevelled collar thrust bearing. It is said to be capable of carrying heavier loads than the plain collar bearing. A tapered film may be obtained with this type of bearing by displacing the axis of the housing with respect to that of the shaft. The allowable pressures in collar bearings is about 20 lb. per sq. in. for the plain collar and 60 lb. per sq. in. for the bevelled collar.

218. Tilting Pad Thrust Bearings. The first tilting pad thrust bearing was made by the American engineer, Albert Kingsbury, in the year 1899. It was independently developed by the Australian engineer, Mr. A. G. M. Michell, who published a mathematical study of the subject in 1905. While the priority of Kingsbury's invention is fully acknowledged, the name of Michell is usually

associated with the tilting pad bearing in this country. Accordingly, it will be described, for the sake of brevity, as the Michell bearing.

The essential feature of the Michell thrust bearing, i.e. the tilting pad, is shown diagrammatically in Fig. 292. The pad rests against a fixed abutment, being supported on a canting ridge parallel to

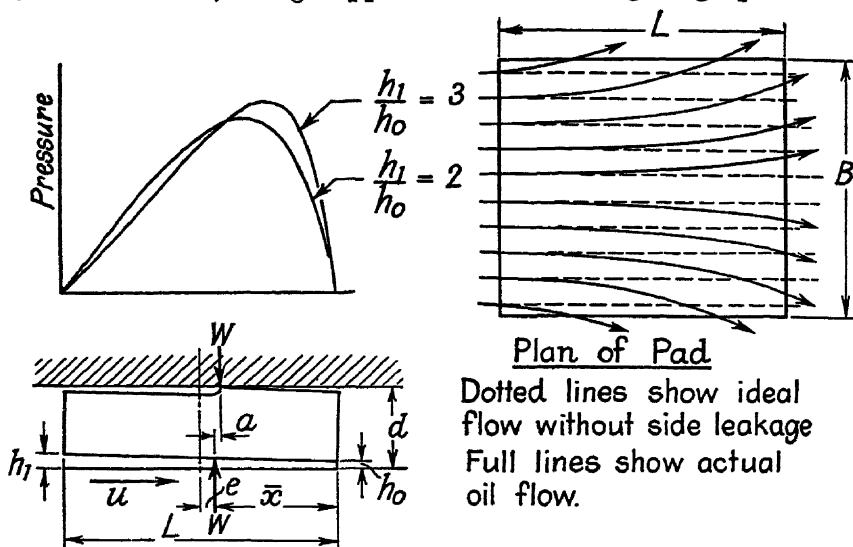


FIG. 292

the outlet edge of the pad. It is thus enabled to tilt at a slight angle to the moving surface of the thrust collar and so permit the formation of a wedge-shaped oil film. The principle has already been explained in Art. 208 and illustrated by Figs. 273 and 276. The theory is fully set out in Dr. R. O. Boswall's book (10).

Consider first the comparatively simple case of a pad of infinite width ($B = \infty$), and assuming that the viscosity Z remains constant. The moving surface is also assumed to be infinitely wide and to move with a uniform velocity u . h_1 and h_0 are the thicknesses of the oil film in inches at the inlet and outlet edges respectively.

Let L = length of pad in direction of motion, in.

u = surface velocity, in. per sec.

W_1 = load per in. width, lb.

$\bar{x}$ = distance from outlet edge to centre of pressure, in.

e = eccentricity of load, in.

Then it may be shown that $W_1 = C_1 \left(\frac{Z \cdot u \cdot L^2}{h_0^2} \right)$

and that

$$\mu_1 = C_2 \left(\frac{Z \cdot u}{W_1} \right)^{\frac{1}{2}}$$

The following values are obtained for different values of the film thickness ratio $\frac{h_1}{h_0}$ —

$\frac{h_1}{h_0}$	C_1	$\frac{\bar{x}}{L}$	$\frac{e}{L}$	C_2
1	0	0.5	0	—
2	0.1590	0.432	0.068	1.94
3	0.1476	0.392	0.108	1.82

The curves in Fig. 292 show the calculated distribution of pressure for different film thickness ratios. It will be seen that the loading is definitely eccentric, the centre of pressure being on the outlet side of the centre-line of the pad.

For equilibrium, the supporting edge must be displaced by a small amount a from the centre of pressure. Taking moments about the tilting edge—

$$W \cdot a = \mu W \cdot d$$

or

$$a = \mu d$$

Since d is normally about $0.3L$ and as μ is not usually more than 0.01 , $a = 0.003L$, an amount which may be neglected.

In actual thrust blocks, the pads are of finite width and oil leaks from the sides. The load-carrying capacity is lower than that of infinitely wide pads, while the coefficient of friction is greater. If we assume that the pads are square and that the velocity u is again uniform, then the correction for load-carrying capacity may be obtained from Kingsbury's curve, Fig. 288. The corrected ratios and constants for the *square* pad are as follow—

$\frac{h_1}{h_0}$	C_1	$\frac{\bar{x}}{L}$	$\frac{e}{L}$	C_2
2	0.0690	0.419	0.081	2.76
3	0.0678	0.374	0.126	2.38

Note.—In using the value of C_2 for finding μ , the actual value of W_1 for the square pad must be inserted in the formula.

For a full analysis of the problem and of the more practical case of the sector-shaped pads the reader is recommended to study Boswall's book.

EXAMPLE 30. The thrust block of a steam turbine contains 16 pads, each equivalent to a square of 1.75 in. side. The mean diameter of the pads is 12.5 in. and the speed 3,000 r.p.m. Assuming

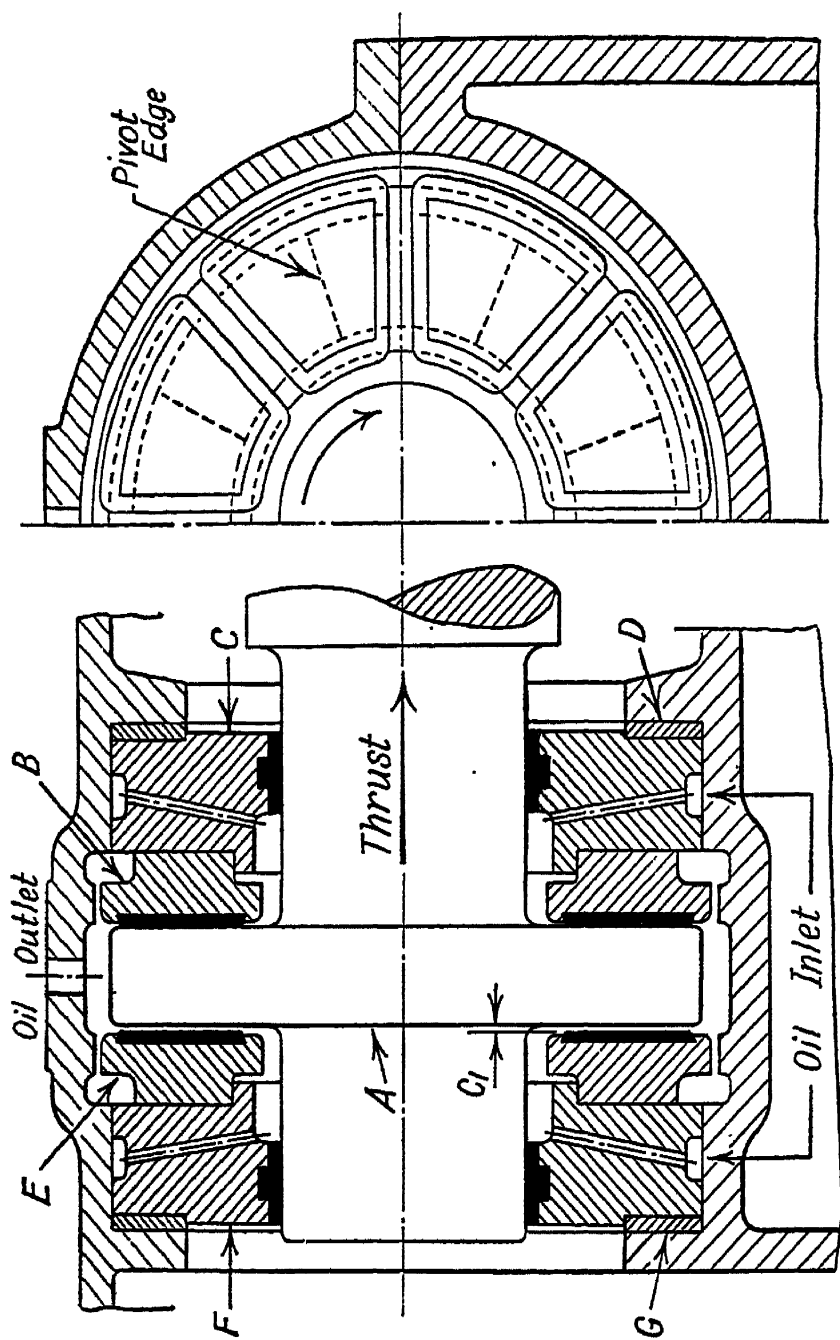


FIG. 293.—Michell Thrust Block

that the mean viscosity of the lubricating oil is 0.2 poise, that the minimum thickness of the oil film is 0.0015 in., and that the thickness ratio is 2, calculate the total load which the thrust block can support. What is the theoretical power lost in overcoming friction, and what is the required eccentricity of the pads?

Area of each pad = 3.06 sq. in.

$u = 1,962$ in. per sec.

$$Z = 0.2 \text{ poise} = 0.0000029 \frac{\text{lb wt. sec.}}{\text{in.}^2}$$

$$h_0 = 0.0015 \text{ in.} \quad L = 1.75 \text{ in.} \quad C_1 = 0.069$$

$$\text{Load per in. width } W_1 = C_1 \left(\frac{ZuL^2}{h_0^2} \right) = 535 \text{ lb.}$$

$\therefore$ Load on each pad = $535 \times 1.75 = 936$ lb.

Mean pressure on pads = 305 lb. per sq. in

$$\begin{aligned} \text{Total load on thrust block} &= 16 \times 936 \\ &= 15,000 \text{ lb.} \end{aligned}$$

$$\mu = C_2 \left(\frac{Zu}{W_1} \right)^{\frac{1}{2}}$$

$$\frac{Zu}{W_1} = \frac{0.0000029 \times 1962}{535} = 0.00001065$$

$$\left(\frac{Zu}{W_1} \right)^{\frac{1}{2}} = 0.00326. \quad C_2 = 2.76$$

$$\therefore \mu = 0.00326 \times 2.76 = 0.009$$

$$\text{Total friction force} = 0.009 \times 15,000 = 135 \text{ lb.}$$

$$\therefore \text{Theoretical power loss} = \frac{135 \times 1962}{12 \times 550} = 40.2 \text{ h.p. or } 30 \text{ kW}$$

The actual power loss would be appreciably in excess of this, because of the pumping action of the collar and other hydraulic losses.

$$\text{Eccentricity } e = 0.081 \times 1.75 = 0.142 \text{ in}$$

219. Construction of Michell Thrust Bearings. Fig. 293 shows a section through a Michell bearing for a steam turbine. A single collar A is turned on the shaft and abuts against a number of thrust pads B. These are supported by a ring C. A similar set of pads E on the non-working side of the block abut against a ring F. The rings C and F are divided by a horizontal joint. The pads E are provided to take any accidental thrust in the direction opposite to that of the normal thrust.

The axial position of the rotor is adjusted during erection by machining the liner D to a definite thickness. A small clearance c_1 ,

known as the "oil clearance," must be provided between the collar and the surging pads E when the collar is hard up against the working pads in order to allow ample space for the formation of satisfactory oil films between the collar and pads. This clearance varies from about 0.010 to 0.015 in. according to the size of the thrust block, and is fixed by giving the liner G a suitable thickness. Lubricating oil is supplied to the centre of the block under a slight pressure, and after passing through the bearing it escapes through an

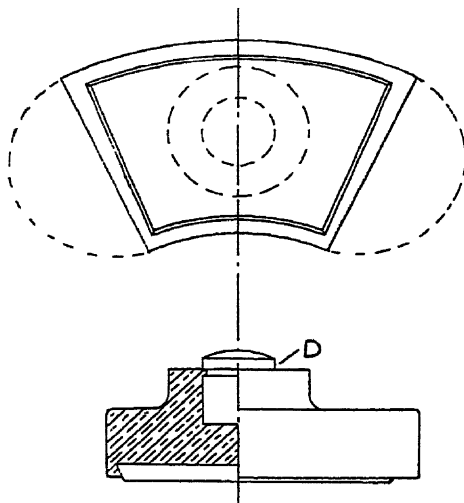


FIG. 294.—Michell Bearing Pad

outlet at the top. This ensures that the bearing is flooded with oil at all times. The rate of oil flow is controlled by a restriction orifice in the outlet pipe.

In marine turbines, where the rotor must be able to turn in both directions, it is usual to support the thrust pads on a hardened steel pin D, Fig. 294, which abuts against a hardened steel plate in the supporting ring. According to theory, this would result in an oil film of uniform thickness ($h_1 = h_0$) and thus incapable of supporting any load. In practice such pads operate quite satisfactorily provided the inlet edge is slightly rounded off. A contributory factor to this satisfactory operation is probably the drop in viscosity caused by the rise in temperature as the oil flows under the pad. This causes a reduction in oil pressure towards the "off" side with a consequent shift of the centre of pressure towards the centre of the pad. Some of the earlier thrust pads were made kidney-shaped, as shown by the dotted lines, Fig. 294. Owing to the effects of side leakage, however, pads of such shape are apt to run dry at the

"off" sides. Pads approximately square give a much better performance

In land turbines, construction takes a variety of forms. Thus the thrust bearing may be arranged (a) quite separate from the journal bearing, (b) in the same housing as the journal bearing, but to one side of it, and (c) in the centre of the journal bearing. In all cases the lubricating oil is supplied to the inner side of the pads. Fig. 295 shows part sections of a separate thrust bearing. The

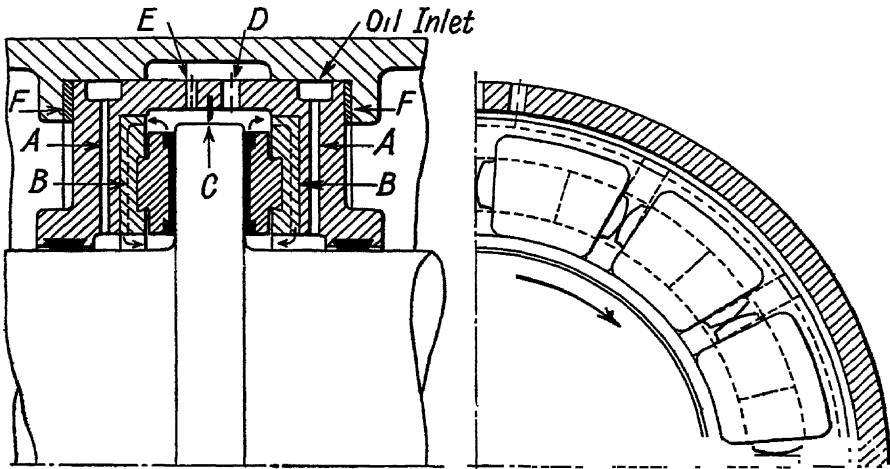


FIG 295

pads are located in a dovetail groove in a supporting ring, and are supported eccentrically on a canting ridge. They are made in a complete ring which is white-metalled and machined before being cut up into separate pads. The pads are then scraped up individually to fit a gauge. Oil is led to the centre of the housing by a number of radial holes A. The rotation of the collar produces a certain pumping action. Channels B, cut in the seating ring between the pads, enable the oil to flow back to the centre and so re-circulate through the pads.

In some cases, a baffle plate C is placed around the periphery of the thrust collar so that restriction orifices D and E of different areas may be used with the object of allowing a greater circulation of oil on the loaded side. The axial position of the rotor may be fixed by means of liners F.

In many turbines, and especially in reaction turbines with end-tightened blading, provision is made for adjusting the axial position of the rotor while the turbine is running. The two halves of the thrust block are bolted and keyed together and move as one. The axial movement is effected by screws operated through worm gear and is limited by liners.

Some manufacturers make special provision for equalizing the loads on the pads. Fig. 296 (a) shows the method employed by the American Westinghouse Co., Ltd., and is self-explanatory. Messrs. Brown Boveri & Co., Ltd., use the construction shown in Fig. 296 (b). Hardened steel discs are recessed in the ends of the pads and rest against steel balls. The latter are borne by the fixed abutment and are maintained in position by a light metal cage.

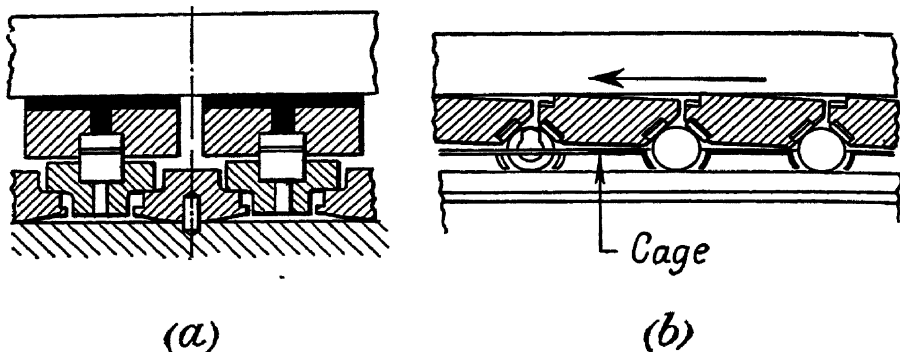


FIG. 296.—Methods of Equalizing Loads on Thrust Pads

By suitably locating the discs in the pads, the necessary eccentric loading may be achieved.

220. Plain Collar Thrust Bearing with Fixed Inclined Thrust Surfaces. In this type of bearing one or more thrust collars are employed. The bearing surfaces are divided up into a number of sectors, as shown in Fig. 297, each sector being tapered so that the oil film diminishes in thickness in the direction of motion of the collar. It is found that a better performance is obtained if the bearing surface is also tapered so that the oil film diminishes in thickness from the centre outwards at the inlet edge. The actual bearing surfaces are of white metal and oil is supplied to them by the radial grooves shown. When the taper is accurately and uniformly machined, such bearings are capable of supporting mean pressures up to a maximum of 450 lb. per sq. in. With such high pressures the total thrust may generally be taken on a single collar.

221. Forced Lubrication Systems. Fig. 298 shows the essential features of a forced lubrication system as used on land turbines. The pump, which is normally of the positive action gear type (see Fig. 299) usually delivers the whole of the oil required for lubrication and governing at the pressure required for the governor gear oil relays, i.e. about 50 to 80 lb. per sq. in. As the oil for operating the relays is only required intermittently the surplus is allowed to pass through a relief valve set to lift at the relay pressure. The oil used in the governor relays is not usually cooled. Oil for lubrication is passed through a reducing valve which is so adjusted that the oil

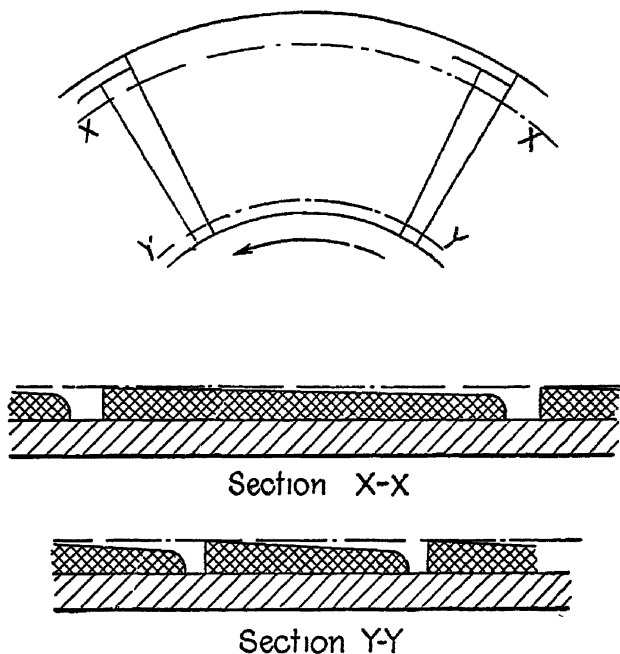


FIG 297 —Thrust Bearing with Fixed Inclined Surfaces

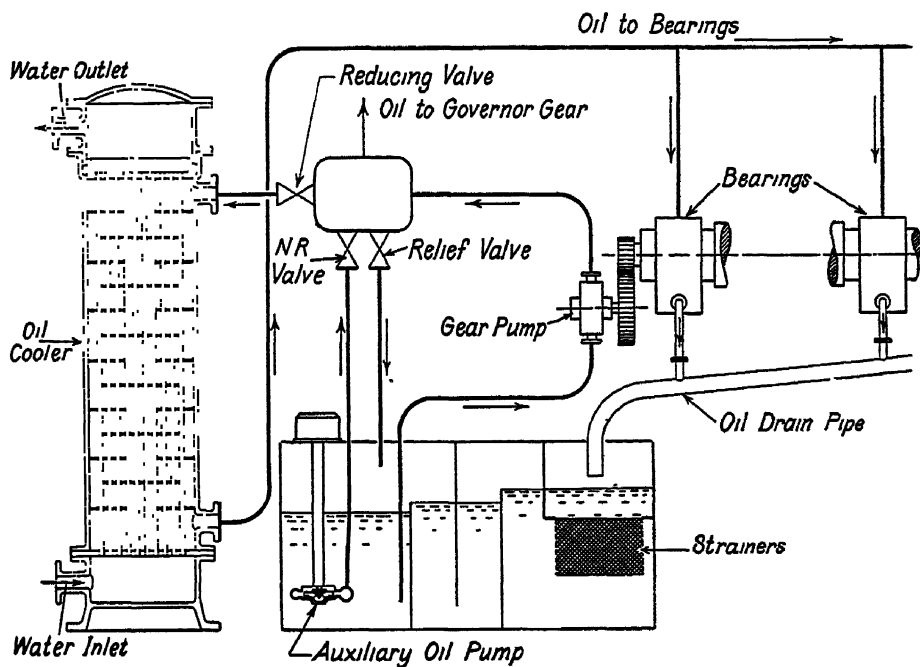


FIG. 298.—Diagrammatic Arrangement of Forced Lubrication System

supply pressure at the bearings is about 5 to 10 lb. per sq. in. It is passed through the oil cooler at the reduced pressure. A bus pipe supplies oil to all the bearings. Similarly, a large bus pipe

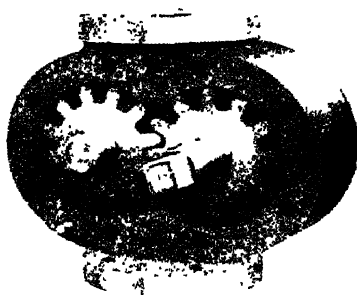


FIG. 299.—Gear Pump

collects the oil from all the bearings and leads it to the oil drain tank. This pipe must be large enough to carry the oil with the pipe only partly filled so that air bubbles have a chance to break. In the arrangement shown in Fig. 298 the oil strainers are placed

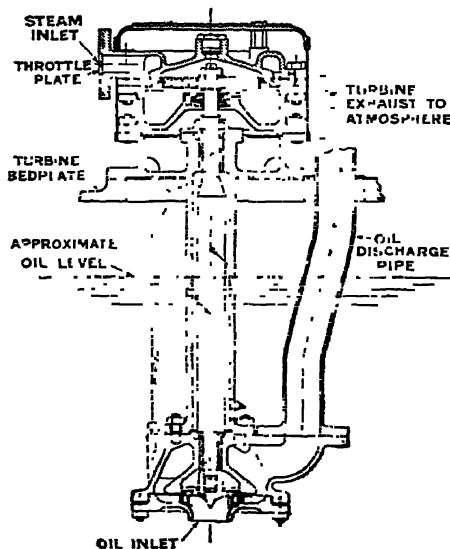


FIG. 300.—Turbine-driven Auxiliary Oil Pump

on the oil inlet side of the tank. In other cases the strainers are arranged at the pump suction.

The auxiliary oil pump comes into operation at starting and stopping when the oil pressure falls below a certain value. It is also an emergency stand-by pump and is capable of supplying the

same quantity as the main oil pump. The type most commonly used is the turbine-driven centrifugal pump, Fig. 300. In the particular example shown, there is a single impulse wheel with blades formed by semicircular channels milled in the periphery. In other

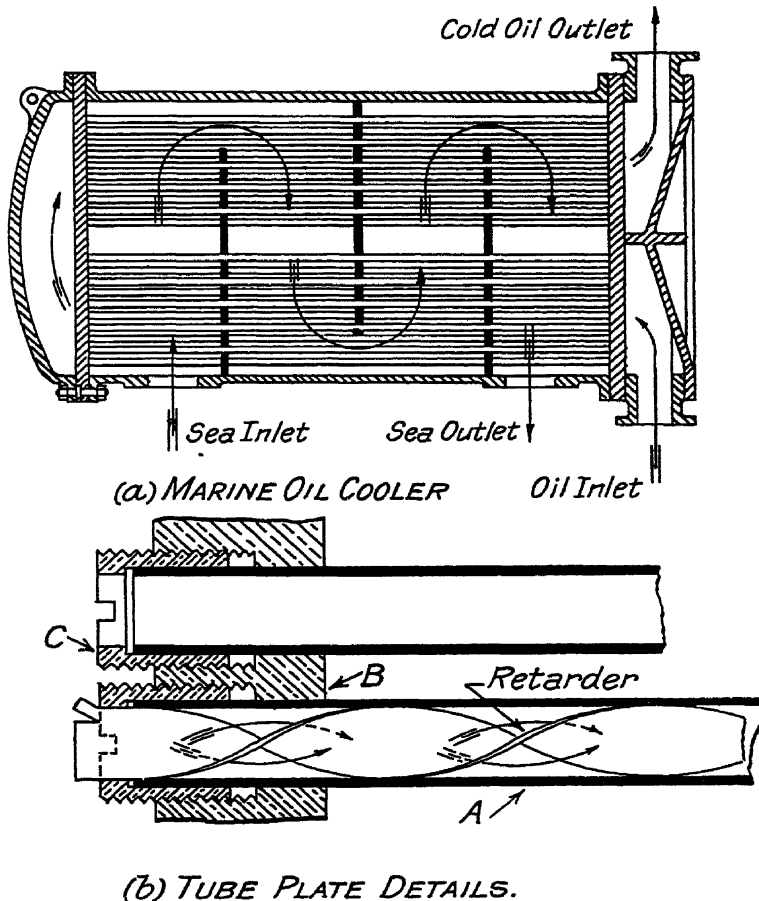


FIG 301

cases a two-row Curtis wheel is employed. The turbine is usually placed on the cover of the oil drain tank with the impeller immersed in oil. The steam supply to the turbine is controlled automatically by the oil pressure in the system.

In the oil cooler shown in Fig. 298, the cooling water passes through the tubes, the oil flow around the tubes being made as sinuous as possible by means of several baffle plates. In marine practice, it is usual to arrange the oil flow through the tubes, the sea water flowing across them (Fig. 301).

The oil tanks should be large enough to allow a considerable time to elapse between the oil draining back into the tank and re-entering the pump. This assists in breaking up the froth and is said to retard the deterioration of the oil. In modern turbine plants the tank normally contains 6 to 10 minutes' supply.

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SPEED AND OUTPUT OF STEAM TURBINES

222. Relationship between Cumulative Heat Drop, Mean Diameter, Speed of Rotation, and Number of Stages in Impulse Turbines. It has been shown in Art. 65, Chapter VII, that the efficiency of a multi-stage impulse turbine equipped with single-row wheels depends on ρ , the ratio of blade speed to steam jet speed.

Consider first of all a turbine in which the cumulative heat drop is H_c , and assume for the sake of simplicity that the mean diameter D (and hence the mean blade velocity u) is uniform throughout the turbine. Assume, further, that the nozzle angle is the same for all stages. The increase in this angle which usually occurs in the last few stages will affect the result only slightly and can be ignored.

By eq. 74, Art. 65, the required adiabatic heat drop per stage is ΔH , where

$$\Delta H = V_1^2 \{1 - \phi[k^2 + \rho^2(1 - k^2) - 2(\rho \cos \alpha - \rho^2)(k^2 + kC)]\} \div 2gJ \eta_n$$

i.e.
$$\Delta H = \frac{V_1^2 \cdot F_1}{2gJ}$$

where F_1 is a function of ρ , k , ϕ , C , α , and η_n . If k , ϕ , and C be assumed to have constant values and if values of η_n appropriate to selected values of α are taken, then F_1 is simply a function of ρ .

If n is the number of stages, then $\Delta H = \frac{H_c}{n}$, since the heat drop per stage is uniform.

$$\therefore \frac{V_1^2 \cdot F_1}{2gJ} = \frac{H_c}{n}$$

and
$$V_1^2 = \frac{2gJ \cdot H_c}{n \cdot F_1}$$

Now
$$u = \frac{\pi D}{12} \times \frac{N}{60} = \frac{DN}{229}$$

where D = mean dia. of blades, in.

N = r.p.m.

$$\begin{aligned} \therefore \rho = \frac{u}{V_1} &= \frac{DN}{229} \sqrt{\frac{n \cdot F_1}{2gJ \cdot H_c}} \\ &= \frac{1}{229 \times 223.8} \sqrt{\frac{n \cdot D^2 N^2}{H_c}} \cdot \sqrt{F_1} \end{aligned}$$

Since $u = \frac{DN}{229}$, $K = 0.0525 \Sigma u^2$

If u_1 is the mean blade velocity in metres per sec., then $K = 0.5653 \Sigma u_1^2$.

Values of $\sqrt{F_1}$ are shown in Fig. 302. These are based on the same data as Fig. 71, namely: $k = 0.86$; $C = 1$; $\phi = 0.90$,

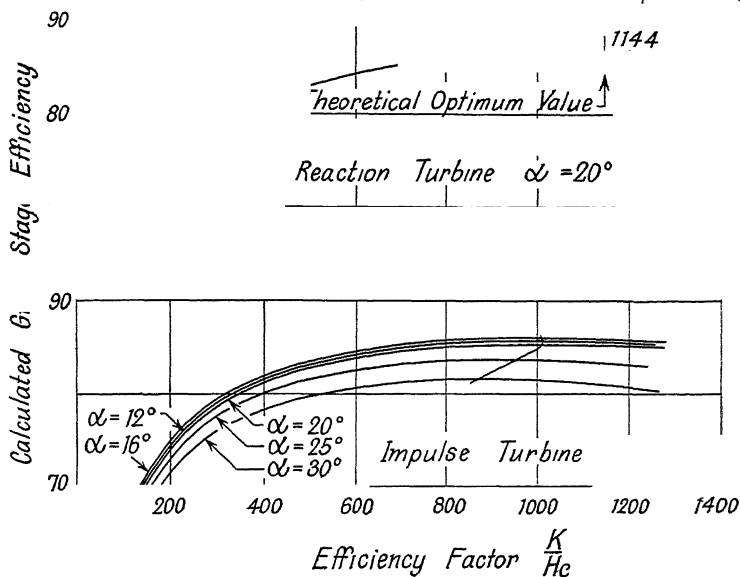


FIG 303

and the values of η_n given in Table III, page 131. The small circles show the values at the most efficient values of ρ . In Fig. 303, the values of the gross stage efficiency given in Table III are plotted against values of the efficiency factor. It must be emphasized that these are merely calculated values based on assumed values of the coefficients. Nevertheless, they show in a qualitative manner the influence of the efficiency factor on the expected efficiency of a multi-stage impulse turbine. Taking 20° as a representative nozzle angle it will be seen that the most economical value of the efficiency factor is about 1,000. In practice, no advantage would be gained by exceeding this number and for obvious reasons a smaller value would generally be employed.

223. Relationship between Cumulative Heat Drop, Mean Diameter, Speed of Rotation, and Number of Stages in Reaction Turbines. Suppose there are n stages in the turbine, each stage comprising

Since
$$u = \frac{DN}{229}, K = 0.0525 \Sigma u^2$$

If u_1 is the mean blade velocity in metres per sec., then $K = 0.5653 \Sigma u_1^2$.

Values of $\sqrt{F_1}$ are shown in Fig. 302. These are based on the same data as Fig. 71, namely: $k = 0.86$; $C = 1$; $\phi = 0.90$,

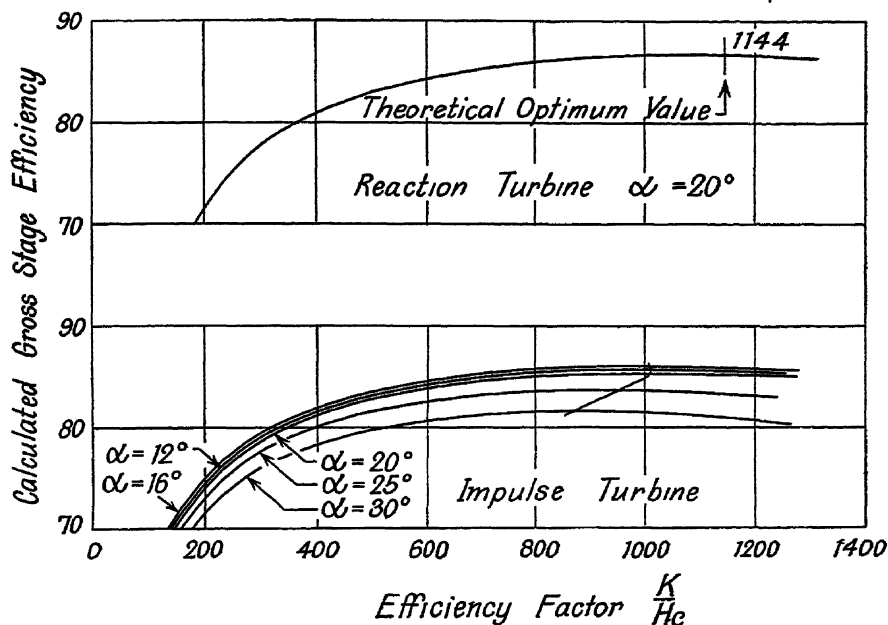


FIG 303

and the values of η_n given in Table III, page 131. The small circles show the values at the most efficient values of ρ . In Fig. 303, the values of the gross stage efficiency given in Table III are plotted against values of the efficiency factor. It must be emphasized that these are merely calculated values based on assumed values of the coefficients. Nevertheless, they show in a qualitative manner the influence of the efficiency factor on the expected efficiency of a multi-stage impulse turbine. Taking 20° as a representative nozzle angle it will be seen that the most economical value of the efficiency factor is about 1,000. In practice, no advantage would be gained by exceeding this number and for obvious reasons a smaller value would generally be employed.

223. Relationship between Cumulative Heat Drop, Mean Diameter, Speed of Rotation, and Number of Stages in Reaction Turbines. Suppose there are n stages in the turbine, each stage comprising

one fixed and one moving row of blades. Then by eq. 95, Art. 76, the heat drop per stage is—

$$\Delta H = \frac{V_1^2}{2gJ} \cdot F_2$$

where $F_2 = 2\{1 - \phi(1 + \rho^2 - 2\rho \cos \alpha)\} \div \eta_n$

$$\therefore V_1^2 = \frac{2gJ \cdot H_c}{n \cdot F_2}$$

and $\rho = \frac{u}{V_1} = \frac{1}{51.2} \sqrt{F_2} \sqrt{\frac{K}{H_c}}$

or $\frac{K}{H_c} = \frac{2620\rho^2}{F_2}$

Values of $\sqrt{F_2}$ are shown in Fig. 302, and values of the gross stage efficiency plotted against those of the efficiency factor in Fig. 303. The most economical value of the latter is 1,144 for the selected data, which are the same as those in Art. 76.

It will be seen from Fig. 303 that—

(1) the efficiency curves are very flat for both types of turbine; an appreciable reduction in the efficiency factor below the optimum value produces a relatively small change in efficiency ratio;

(2) there is not a very great difference between the optimum values of $\frac{K}{H_c}$ for both types of turbine.

An important conclusion of the above analysis is that, *irrespective of the power of a steam turbine*, for a given heat drop the K value must be the same if the same theoretical efficiency ratio is to be realized. Suppose, for example, two impulse turbines, one of 2,000 kW and another of 10,000 kW, are to be constructed to operate with steam at 250 lb. per sq. in. and 700° F. and, say, an exhaust pressure of 1 in. Hg. absolute. Suppose that the speed of the two turbines is to be the same, say 3,000 r.p.m. Then the sum Σu^2 must be the same for both turbines and if the same number of stages is employed the mean diameter of the two turbines must be the same. It is true that the nozzle and blade areas of the 10,000 kW turbine will be about five times as large as those of the 2,000 kW turbine, but as regards the leading dimensions, i.e. diameters of wheels, diaphragms, and of the cylinder, length of cylinder, size of spindle, etc., the 2,000 kW turbine will be almost identical with the 10,000 kW turbine. The difference in the total cost will not be appreciable, and hence the cost per kW capacity will be much less for the greater output. This limitation may be overcome in certain cases by the use of reduction gearing between the turbine and the driven machine, thus enabling a smaller turbine running at a higher speed to be employed,

Since, for a given set of conditions, the value nD^2N^2 must be a constant, a smaller value of the quantity nD^2 is obtained by adopting a higher speed of rotation. In other words, by increasing the speed N to the highest possible value, a smaller and consequently cheaper turbine will result. As shown in Art. 225, this procedure is limited by considerations of output.

The efficiency factor is not the sole criterion of efficiency. The actual efficiency ratio of a turbine will also depend on—

(1) the excellence, or otherwise, of the design in so far as losses due to partial admission, windage, eddies, etc., are kept within small limits;

(2) the finish of the nozzle and blade surfaces; and

(3) the characteristics of the cycle in so far as losses due to moisture in the L.P. stages are concerned.

224. Comparison of Pressure-compounded and Velocity-compounded Turbines. Consider a turbine consisting of a number of pressure-compounded single-row wheels and another containing a certain number of velocity-compounded wheels.

Let n_p, N_p, D_p , etc., refer to pressure-compounded turbine.

n_v, N_v, D_v , etc., refer to velocity-compounded turbine

μ = number of rows of moving blades in each stage of the velocity-compounded turbine

It has been shown in Art 71 that for the simple case of frictionless flow and no carry-over, the condition for maximum efficiency is that—

$$\rho_v = \frac{\rho_p}{\mu}$$

Hence
$$\sqrt{\frac{K_v}{H_c}} = \frac{1}{\mu} \sqrt{\frac{K_p}{H_c}}$$

Hence, for the same cumulative heat drop—

$$K_p = \mu^2 \cdot K_v$$

i.e.
$$n_p(D_p \cdot N_p)^2 = \mu^2 \cdot n_v(D_v \cdot N_v)^2 \quad . \quad . \quad . \quad (208)$$

Thus for the same value of DN (i.e. for the same blade velocity) and for equal relative efficiency—

$$n_p = \mu^2 \cdot n_v$$

that is to say—

One two-row velocity wheel will replace 4 single wheels.

One three-row velocity wheel will replace 9 single wheels.

One four-row velocity wheel will replace 16 single wheels.

In calculating the value of the efficiency factor $\frac{K}{H_c}$ for velocity-compounded or combination turbines, it is preferable, for the

purposes of direct comparison, to modify the value of n and express it as an equivalent number of pressure-compounded stages.

EXAMPLE 31. Compare the relative rotative speeds of a de Laval steam turbine 8 in. mean diameter and a three-row velocity-compounded two-stage turbine having a wheel 40 in. mean diameter, both turbines having the same heat drop.

$$n_p(D_p \cdot N_p)^2 = \mu^2 \cdot n_v(D_v \cdot N_v)^2$$

In this example, $n_p = 1$, $n_v = 2$, $\mu = 3$ rows.

$$\begin{aligned} \text{Hence} \quad \frac{N_p}{N_v} &= \mu \cdot \frac{D_v}{D_p} \cdot \sqrt{\frac{n_v}{n_p}} \\ &= 3 \cdot \frac{40}{8} \sqrt{\frac{2}{1}} = 21.2 \end{aligned}$$

225. Relationship between Output, Speed, and Blade Velocity in Impulse Turbines. The power output of a steam turbine depends upon (1) the mass of steam which passes through the turbine per unit time, (2) the adiabatic heat drop, and (3) the efficiency ratio.

Let HP = power output at turbine coupling, h.p.

kW = power output at turbine coupling, kW.

H_a = adiabatic heat drop, B.Th.U. per lb.

η_r = efficiency ratio of turbine.

w = weight of steam passing through turbine per second.

x_e = dryness fraction of steam at exhaust.

v_e = specific volume of dry and saturated steam at the exhaust pressure.

V_1 = steam velocity at exit from nozzles in last stage.

u = mean blade velocity in last stage.

α = nozzle angle in last stage.

l = nozzle height in last stage.

$$\text{Then} \quad w = \frac{2546 \text{ HP}}{3600 \eta_r \cdot H_a} = \frac{3413 \text{ kW}}{3600 \eta_r \cdot H_a}$$

$$\begin{aligned} \therefore \text{Volume of steam passing through last-stage nozzles} \\ = w \cdot x_e \cdot v_e \text{ cub. ft. per sec.} \end{aligned}$$

The area available through the nozzles may be set down with reference to Fig. 304. As already explained in Chapter XVI, the L.P. nozzles are formed by casting-in sheet metal guide vanes in the diaphragm. The effective area of the channel in each nozzle is

$$l(p_m \cdot \sin \alpha - t) = c \cdot l \cdot p_m \sin \alpha$$

where

$$c = \frac{p_m \cdot \sin \alpha - t}{p_m \cdot \sin \alpha}$$

The factor c may be termed the "thickness coefficient" for the nozzle, since it takes into account the partial obstruction of the

nozzle passages due to the finite thickness of the guide vanes. Values of c are given in Fig. 305 for a standard mean pitch of 2 in and different vane thicknesses

If, as is usual, the nozzles extend round the full circumference, the number of nozzles is obviously $\frac{\pi D}{p_m}$

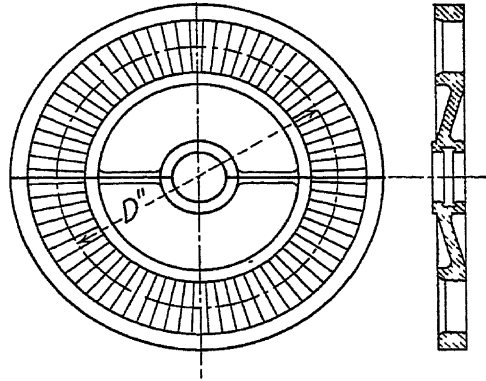


FIG 304

$$\therefore \text{Total area} = A = \frac{\pi D l}{144} \frac{c \cdot \sin \alpha}{144} \text{ sq ft}$$

Writing $m = \frac{l}{D}$

$$A = \frac{\pi m D^2 c \cdot \sin \alpha}{144}$$

$\therefore$ Volume of steam flowing per second

$$= AV_1 = \frac{\pi m D^2 c \cdot V_1 \cdot \sin \alpha}{144}$$

Since $D = \frac{229u}{N}$, and $V_1 = \frac{u}{\rho}$

$$\text{Vol. per sec.} = \frac{\pi \times (229)^2}{144} \times \frac{mcu^3 \cdot \sin \alpha}{N^2 \cdot \rho}$$

Equating this to the volume based on the steam flow and transposing, we have—

$$\left(\frac{u}{100}\right)^3 \cdot \sin \alpha = 0.618 \left(\frac{\text{HP}}{1000}\right) \left(\frac{N}{1000}\right)^2 \cdot \frac{x_e v_e \cdot \rho}{mc \eta_r H_a} \quad (209)$$

$$\left(\frac{u}{100}\right)^3 \cdot \sin \alpha = 0.829 \left(\frac{\text{kW}}{1000}\right) \left(\frac{N}{1000}\right)^2 \cdot \frac{x_e v_e \rho}{mc \eta_r H_a} \quad (210)$$

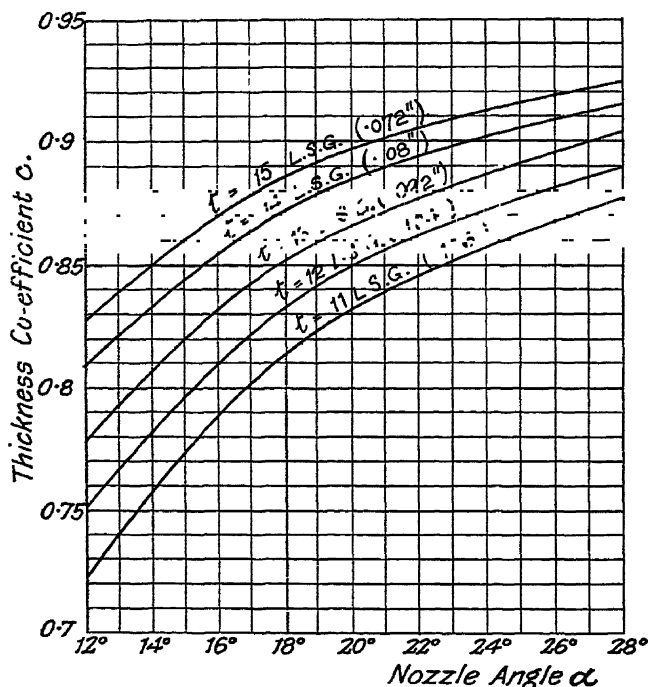


FIG. 305.—Thickness Coefficients for Turbine Nozzles

Alternatively—

$$\text{HP} = 1618 \frac{u^3}{N^2} \cdot \frac{mc \cdot \sin \alpha \cdot \eta_r H_a}{x_e v_e \rho} \quad (211)$$

$$\text{kW} = 1206 \frac{u^3}{N^2} \cdot \frac{mc \cdot \sin \alpha \cdot \eta_r H_a}{x_e v_e \rho} \quad (212)$$

These expressions are correct only for a non-extraction turbine with a single exhaust. For two similar exhausts, the constants in eqs. 209 and 210 are halved, while those in eqs. 211 and 212 are doubled, and so on. Moreover, the expressions are only applicable to pure impulse turbines.

Considering in turn the factors involved—

(1) x_e is found direct from the Mollier chart.

(2) v_e depends on the exhaust pressure, i.e. the vacuum. The higher the vacuum, the greater v_e and the smaller the limiting output of the turbine, other things being equal.

(3) ρ depends on the efficiency required but is arbitrary. If u is high in the last stage and ρ is given the same value as in preceding stages it is possible that the jet velocity will exceed the critical, in which case unless convergent-divergent nozzles are employed a certain amount of jet deflection will take place (see Art. 51). In such cases partial reaction might be employed and a more detailed method of analysis must be used.

(4) The value of m is arbitrary, but in a turbine in which the output is relatively high for the speed of rotation the value of m must be made as great as possible, otherwise a large nozzle angle will have to be used. Formerly, the value of m was limited to about 0.2, but values of m up to 0.32 have been used in extreme cases. Owing to the effects of inertia on the steam during its passage through the blades, it is doubtful if such long blades could operate on the pure impulse principle, and the maximum value of m for which the above formulae could be safely employed is probably about 0.2, or even less at high peripheral speeds.

(5) The value of c depends on the nozzle angle α and the thickness of the guide blades. Since α may be the unknown, the calculation may have to be made by trial and error.

(6) η_r and H_a are known or assumed.

If N is known, then the product $u^3 \sin \alpha$ is derived, and in general u will be fixed by the size of the nozzle angle. If, however, N is arbitrary, as, for example, in the case of small geared sets, α and u may be chosen arbitrarily and the required value of N calculated. If this proves unsuitable, then either u or α , or both, may be modified. It will be clear that, given the output and steam conditions, the number of solutions is very large indeed.

EXAMPLE 32. An impulse turbine is required to develop 3,000 kW at 3,000 r.p.m., with the following steam conditions—

Initial pressure	.	.	300 lb. per sq. in. abs.
Initial temperature	.	.	700° F.
Vacuum	.	.	29 in. Hg.

Calculate the required mean blade velocity in the last stage and the mean diameter of the blade ring in that stage for the following nozzle angles: 20, 22, 24, 26, 28, and 30 degrees. Make the following assumptions—

Efficiency ratio = 0.75; m = 0.18; ρ = 0.55; mean pitch of nozzles = 2 in.; guide vane thickness = 0.104 in.

From the ϕ -I chart we find the adiabatic heat drop to be 467.6, and the dryness fraction of the exhaust steam, after allowing

for losses, to be 0.932. On substituting in eq. 210, we obtain the following expression—

$$\left(\frac{u}{100}\right)^3 = \frac{118.7}{c \cdot \sin \alpha}$$

The remainder of the calculation is best arranged in tabular form as follows—

α	$\sin \alpha$	c	u ft. per sec.	D in.
20	0.3420	0.850	742	56.6
22	0.3746	0.862	717	54.8
24	0.4067	0.872	695	53.0
26	0.4384	0.881	675	51.5
28	0.4695	0.890	657	50.1
30	0.5000	0.896	642	49.0

EXAMPLE 33. Calculate suitable speeds of rotation and corresponding mean diameters (of last stage) for a geared turbine of 800 h.p. under the following conditions—

Initial steam pressure	200 lb. per sq. in. abs.
Initial steam temperature	600° F.
Vacuum	28 in. Hg.
Efficiency ratio	0.72
Mean blade velocity	500 ft. per sec.

Make calculations for the following nozzle angles in the last stage: 20, 22, 24, 26, 28, and 30 degrees, and assume: Admission = Complete; $m = 0.15$; $\rho = 0.5$; $p_m = 2$ in.; $t = 0.104$ in.

From the Mollier diagram we find that the adiabatic heat drop is 386.3 and the actual exhaust dryness fraction is 0.947. Substituting values in eq. 211, it will be found that—

$$\left(\frac{N}{1000}\right)^2 = 65.8 c \cdot \sin \alpha$$

$$\text{Since } u = \frac{DN}{229}, D = \frac{229 \times 500}{N} = \frac{114,500}{N}$$

The following results are then obtained—

α	$\sin \alpha$	c	N r.p.m.	D in.
20	0.3420	0.850	4,370	26.2
22	0.3746	0.862	4,605	24.9
24	0.4067	0.872	4,830	23.7
26	0.4384	0.881	5,095	22.5
28	0.4695	0.890	5,245	21.8
30	0.5000	0.896	5,425	21.1

This example shows that by the use of reduction gearing, turbines

of relatively small output may be constructed in the form of high-speed machines of small dimensions.

226. Relationship between Output, Speed, and Blade Velocity in Reaction Turbines. In Art. 77 it is shown that for an impulse-reaction turbine—

$$D^2 l = \frac{10,510 w_1 v_s \cdot \rho}{N \cdot \sin \alpha}$$

In the symbols of the previous article, v_s is equal to $x_e v_e$. w_1 is the "working steam" in lb. per sec. In the last stage of the turbine,

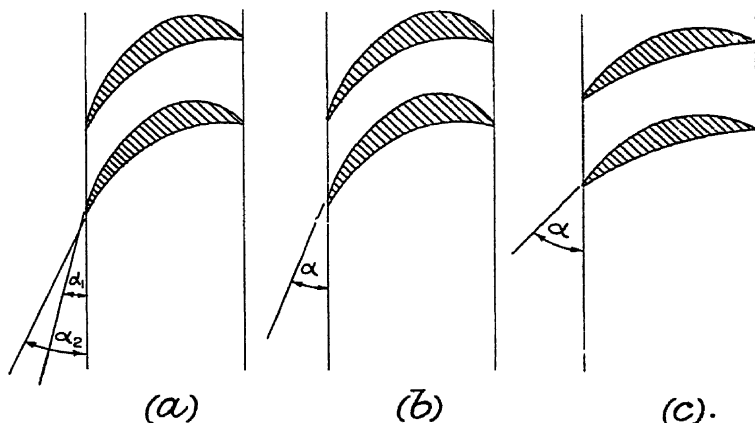


FIG 306

- (a) Section through "normal" reaction turbine blade
Mean outlet angle 18°-20°
(b) "Semi-wing" blade. Mean outlet angle 30°
(c) "Full-wing" blade. Mean outlet angle 45°

the tip clearance loss will generally be so small that w_1 is practically equal to w , the quantity of steam supplied to the turbine per second.

Writing $l = mD$, $D = \frac{229u}{N}$, and substituting $\frac{2546\text{HP}}{3600\eta_r H_a}$ for w_1 —

$$\left(\frac{u}{100}\right)^3 \cdot \sin \alpha = 0.618 \left(\frac{\text{HP}}{1000}\right) \left(\frac{N}{1000}\right)^2 \frac{x_e v_e \rho}{m \cdot \eta_r \cdot H_a} \quad (213)$$

Similarly—

$$\left(\frac{u}{100}\right)^3 \cdot \sin \alpha = 0.829 \left(\frac{\text{kW}}{1000}\right) \left(\frac{N}{1000}\right)^2 \frac{x_e v_e \rho}{m \cdot \eta_r \cdot H_a} \quad (214)$$

As the value of ρ must be greater for the reaction turbine than for the impulse turbine in order to realize the same relative efficiency, the product $u^3 \cdot \sin \alpha$ must be greater for the same output. In other words, the limiting output of the reaction turbine is smaller than that of an impulse turbine operating under otherwise identical conditions. This limit to the output of reaction turbines is obviated

in a manner similar to that employed to increase the output of impulse turbines, namely, by gradually increasing the blade (equivalent nozzle) angles in the final stages. The last few groups of blading are made of the same height, but the blade outlet angle is progressively increased by the use of quarter-wing, semi-wing, three-quarter-wing, and full-wing blades (Fig. 306). It is difficult, if not impossible, to state the outlet angle of a reaction blade because the tangents to the back and front curved surfaces give two different values α_1 and α_2 , as shown in Fig. 306 (a). The values given under Fig. 306 are mean values, but in any case they are varied to suit requirements. Instead of stating the outlet angle of the blade, the ratio of the effective area through the blade outlet to the full area of the annulus is sometimes given. This is termed the "opening." Interesting particulars are given in Mr. R. H. Parsons' book, *The Development of the Parsons Steam Turbine*, of an 8,500 kW steam turbine installed in Sheffield power station. The last four pairs of blades were made 9 in. high on a rotor 60 in. diameter.

	Opening	$\sin^{-1} \theta$
1st pair, fixed and moving	0.35	20.5 deg.
2nd pair, fixed and moving	0.50	30.0 "
3rd pair, fixed and moving	0.61	37.6 "
4th pair, fixed	0.71	45.3 "
4th pair, moving	0.83	56.1 "

As in impulse turbines, the employment of long blades (large values of m) involves variations in steam pressure within the blade due to inertia, with the result that the degree of reaction is variable along the length of the blade. The expressions given in this article are, therefore, only applicable to moderate values of m .

The effect of vacuum on the limiting output is well illustrated by the following example.

EXAMPLE 34. Calculate the limiting outputs of reaction turbines operating with live steam at 400 lb. per sq. in. abs. and at 720° F. for the following exhaust pressures: 1.5, 1.0, 0.8, 0.6, and 0.5 lb. per sq. in. abs. The following data are given: α for final row of blades = 45°; u = 600 ft. per sec.; N = 3,000; ρ = 0.75; m = 0.2; and η_r = 0.80.

Substituting values in eq. 214, and transposing, we have—

$$\text{Limiting kW} = \frac{4370H_a}{\alpha_e v_e}$$

The adiabatic heat drops and actual exhaust dryness fractions are obtained from the Mollier diagram and inserted in the table on page 493.

227. Turbine Output and Stress in Last Row of Blades. It has been shown in Art. 140 that the centrifugal stress in turbine blades of uniform cross-section is given by the expression—

$$f_c = 0.231m_1u^2 \text{ for brass blades}$$

$$f_c = 0.215m_1u^2 \text{ for steel blades}$$

p_e	H_a	x_e	v_e	Limiting Output kW
1.5	431	0.924	227.4	9,000
1.0	453	0.914	333.1	6,500
0.8	464	0.910	411.1	5,400
0.6	477	0.902	539.1	4,300
0.5	486	0.899	640.5	3,700

where m_1 is the ratio of the blade length to the mean diameter of the blade ring. Eq. 212 may be written

$$\text{kW} = \frac{1206u}{N^2} \cdot \frac{c \cdot \sin \alpha \cdot \eta_r \cdot H_a}{x_e v_e \cdot \rho} m u^2$$

Considering steel blades only (as brass is unsuitable for high stresses), we have

$$m_1 u^2 = 4.65 f_c$$

Assuming, for simplicity, that the length of the blade is equal to the height of the nozzle (in many turbines of large output there is little difference), then $m_1 = m$.

$$\therefore \text{kW} = \frac{5610u}{N^2} f_c \left(\frac{c \sin \alpha \eta_r H_a}{x_e v_e \rho} \right) \quad (215)$$

Thus for fixed values of the quantities within the bracket, the limiting output of the turbine varies—

- (1) directly as the maximum centrifugal stress in the blades,
- (2) directly as the mean blade velocity,
- (3) inversely as the square of the speed of rotation

$$\text{Alternatively } f_c = \frac{N^2 \text{ kW}}{5610u} \left(\frac{x_e v_e \cdot \rho}{c \cdot \sin \alpha \cdot \eta_r H_a} \right) \quad (216)$$

The output may be considerably increased for the same value of the maximum stress if the cross-sectional area of the blade is gradually increased from tip to root (see Art. 144)

228. Carry-over Loss. Reference to the velocity diagram for an impulse wheel shows that the absolute velocity of the steam at exit from a wheel is approximately axial. In estimating the carry-over loss (see Art. 86) it is sufficiently accurate to assume that the steam flows into the exhaust belt in the axial direction. This statement also applies to the reaction turbine when the ratio of blade velocity to steam velocity is reasonably near to the optimum value.

(a) *Single-flow Turbine* The volume of steam flowing into the exhaust belt of a turbine without steam extraction in any form and having only one exhaust branch is

$$\frac{2546 \text{ HP}}{3600 \eta_r H_a} x_e v_e \text{ cub. ft. per sec.}$$

The area available for flow (see Fig. 307)

$$= \frac{\pi D l}{144} = \frac{\pi m_1 D^2}{144} \text{ sq. ft.}$$

$$\therefore \text{Velocity at exit} = \frac{32.42 \text{ HP } x_e v_e}{m_1 D^2 \eta_r H_a}$$

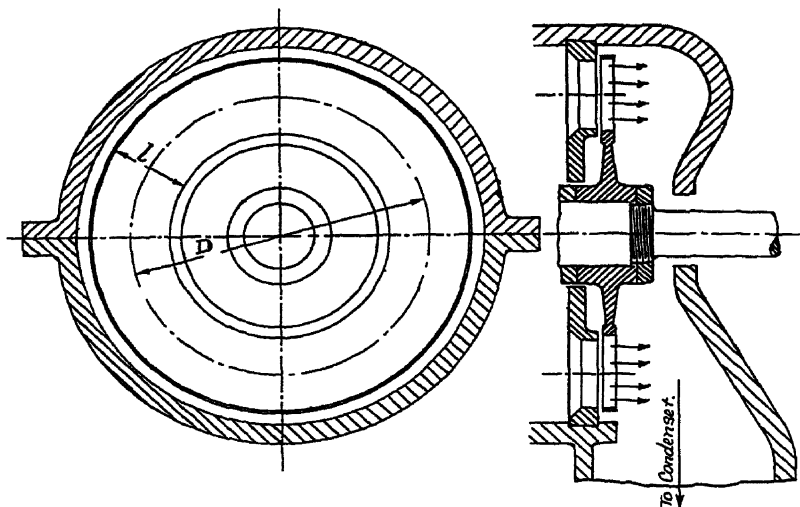


FIG. 307

The heat equivalent of the kinetic energy in B.Th.U. per lb.

$$= \left\{ \frac{32.42 \text{ HP } x_e v_e}{223.8 m_1 D^2 \eta_r H_a} \right\}^2$$

Hence we may express the residual velocity loss as a percentage λ of the adiabatic heat drop, from which

$$\begin{aligned} D \text{ inches} &= \frac{1}{\sqrt[4]{\lambda}} \sqrt{\frac{1.45 \text{ HP } x_e v_e}{m_1 \cdot \eta_r \cdot H_a^{1.5}}} \\ &= \frac{1}{\sqrt[4]{\lambda}} \sqrt{\frac{1.94 \text{ kW } x_e v_e}{m_1 \cdot \eta_r \cdot H_a^{1.5}}} \quad . \quad . \quad . \quad (217) \end{aligned}$$

(b) *Multi-exhaust Turbines.* As will be seen in Art. 230, the limiting output of steam turbines is frequently increased by the use of two or more similar exhausts, the steam flow being divided at a certain point in the expansion and the subsequent expansion

of the steam taking place in two or more L.P. blade systems working in parallel. Then since the area varies as D^2 —

if D_1 = calculated diameter of single-flow turbine

„ D_2 = calculated diameter of double-flow turbine

„ D_3 = calculated diameter of triple-flow turbine, etc.,

then $D_2 = \frac{D_1}{\sqrt{2}}$; $D_3 = \frac{D_1}{\sqrt{3}}$; $D_4 = \frac{D_1}{2}$, etc.

229. Methods of Increasing the Limiting Output of Steam Turbines. The increasing use of electrical energy for domestic and industrial purposes has led to a continuous increase in the size of generating stations. In the replacement of obsolete plant, or the design of new extensions, the engineer is faced with the problem of selecting a size for the generating units. Choice is influenced by a number of factors such as reliability, space available, necessity of providing a standby machine, first cost, and thermal efficiency. Turbines of larger size are not, in general, so reliable as smaller machines. They require more time for warming-up, and load must not be applied too quickly, otherwise dangerous heat stresses and also vibrations may be set up. On the other hand, the first cost of turbine plant per unit capacity diminishes as the size is increased, and, further, the overall thermal efficiency of large units is generally higher than that of smaller machines. The tendency has been, and still is, to install larger and still larger machines. Thirty-five years ago a turbine of 5,000 kW was regarded as a big machine; since then we have witnessed the evolution of noteworthy machines of 15,000 kW, 25,000 kW, 50,000 kW, then 160,000 kW, and finally a “unit” developing 208,000 kW.

The high-powered turbine presents an interesting and sometimes difficult problem in design, chiefly on account of the enormous volume of steam which must pass through the final stages of the turbine. The limiting output of a turbine is given by eq 212—

$$\text{kW} = \frac{1206 \, m \cdot c \cdot \sin \alpha \cdot \eta_r \cdot H_a}{x_e \cdot v_e} \cdot \frac{1}{\rho} \cdot \frac{u^3}{N^2}$$

A brief study of this relation will show at once the influence of various factors on the maximum output for which a turbine may be designed. Considering these in detail—

Nozzle Angle α . For the highest capacities, α must be made as large as possible. There is likely to be a slight sacrifice in efficiency, as indicated by Figs. 71 and 303.

Adiabatic Heat Drop H_a . Other things being equal, the specific steam rate per kWh is inversely proportional to the adiabatic heat drop. Thus it is easier, from this point of view, to design a machine for a large power if it utilizes high-pressure steam, this is

further emphasized by the reduced dryness fraction x_e of the exhaust steam when using higher pressures.

Blade Velocity u . The capacity is proportional to the cube of the blade speed, hence the importance of this factor is paramount.

Volume per lb. of Exhaust Steam $x_e \cdot v_e$. This is influenced chiefly by the vacuum reached in the condenser. The higher the vacuum the larger the exhaust specific volume and the smaller the limiting output of a given turbine frame.

Blade Speed Ratio ρ . Only two types of turbine demand consideration here, the single-row impulse type and the Parsons type. In the former, the most economical value of ρ varies from about 0.52 to 0.6, whereas for the latter it is about 0.7 for wing blades. As the efficiency curves are flat in the region of the optimum values, a lower value of ρ may be used in the final stage with considerable advantage.

Speed of Rotation N . The significance of rotational speed has already been pointed out in Art. 223, where it is shown that in order to reduce the dimensions and cost of a turbine, the rotational speed must be made as high as possible. Here, however, we see that a high rotational speed runs counter to a high output, and so we find turbines of high power designed for lower running speeds. An interesting example of this is described in Art. 232. Various methods of increasing the output will be considered in the following articles.

230. Multi-exhaust Turbines. The speed of turbines driving alternating current generators is given by the expression—

$$N = \frac{120f}{p} \text{ r.p.m.}$$

where f = frequency of alternating current per sec.

p = number of poles on rotor.

In this country the standard frequency is 50 cycles per second and the possible turbine speeds are 3,000, 1,500, 1,000, and 750 r.p.m. for 2, 4, 6, and 8 poles respectively. Eq. 212 shows that if other things are equal, the limiting outputs for these speeds are in the ratios 1, 4, 9, and 16. The 3,000 r.p.m. turbine is an attractive machine owing to the small sizes and masses of its parts, and the limitations to the output of a single-flow machine may be readily overcome by carrying out part of the expansion in a normal single-flow turbine and the subsequent part in two or more similar low-pressure turbines operating in parallel.

Various arrangements of two-exhaust turbines are shown diagrammatically in Fig. 308. In (a) there is a H.P. turbine with a double-flow L.P. turbine necessitating the use of two condensers; (b) shows a H.P. turbine having two receiver connections leading steam to the opposite ends of a double-flow L.P. turbine with a

single exhaust branch and a single condenser. In (c) the H.P. and one section of the L.P. turbine are arranged in the same cylinder, the other half of the steam being led to a second L.P. turbine identical with the other. Fig 308 (d) shows a turbine with a divided

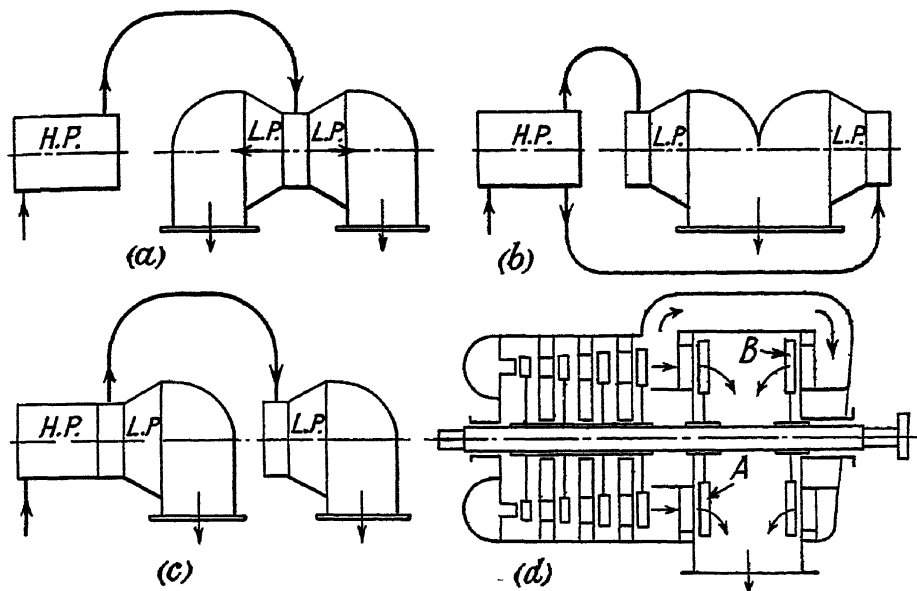


FIG 308 —Turbines with Two Exhausts

flow, in which half the steam flows direct through the wheel A into the exhaust branch, the other half passing through wheel B. In calculating the K value of multi-exhaust turbines the sum

$$\sum \left(\frac{D}{10} \right)^2 \left(\frac{N}{100} \right)^2$$

is calculated for only one of the L.P. stage groups and added to that for the H.P. stages down to the point where the flow is divided.

An interesting example of a British design of a divided-flow turbine is illustrated in Fig 309. It was constructed some years ago (about 1919) by Messrs. Fraser & Chalmers Engineering Works for the Dalmarnock Power Station of Glasgow Corporation, for the following conditions—

Maximum continuous load	.	.	.	18,750 kW
Maximum overload	.	.	.	23,500 "
Most economical load	.	.	.	15,000 "
Initial steam pressure	.	.	.	250 lb. persq. in. g.
Initial steam temperature	.	.	.	650° F.
Vacuum	.	.	.	29.1 in. Hg.
R.p.m.	.	.	.	1,500

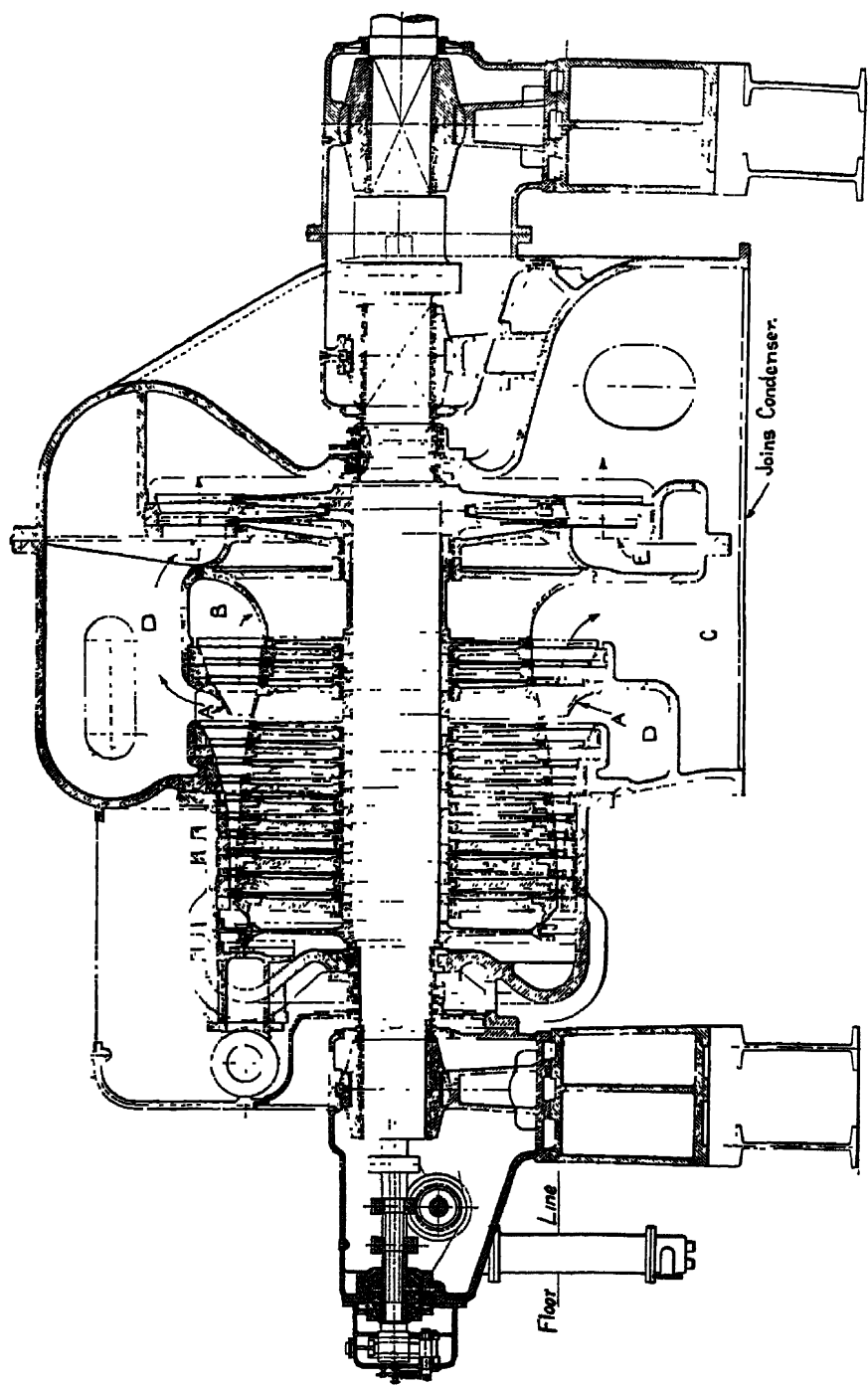


FIG. 309.—187,50 kW 1,500 r.p.m. Steam Turbine by Messrs. Fraser & Chalmers Engineering Works

It will be seen that the turbine has a two-row velocity-compounded wheel in the first stage, followed by ten single-row wheels. The exhaust steam from the tenth wheel is divided into two portions by a deflector A. One portion passes through three stages of approximately the same mean diameter as the former stages and enters the space B, which is a volute leading into the exhaust branch C. The other portion flows into the space D, which is also of volute form, and

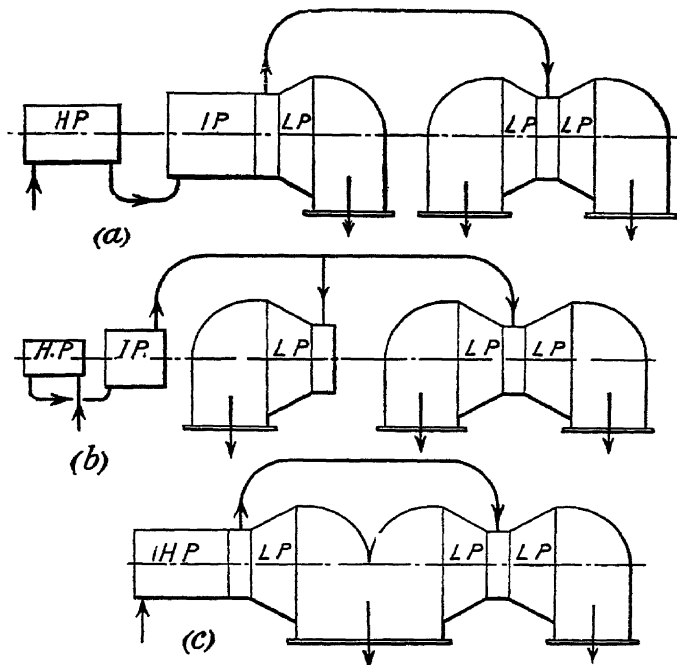


FIG. 310 —Turbines with Three Exhausts

enters the nozzles E, passing through two stages. The mean diameter of these wheels is larger than that of the earlier stages, and is such that they utilize the same heat drop as the three wheels which absorb the available energy of the first stream. By this construction the output is approximately doubled.

Various arrangements of turbines with three exhausts are shown in Fig. 310. Each diagram shows an arrangement which has actually been used in practice and is self-explanatory. An output of 50,000 kW at 3,000 r.p.m. has been obtained with the arrangement shown in Fig. 310 (b). Fig. 311 shows turbines operating with four exhausts, the largest output obtained at 3,000 r.p.m. being 60,000 kW.

Data relating to turbines of large output are given in Tables XVIIA and XVIIIB. These have been collected from various sources, but for much of the information given the author is indebted to two

TABLE XVIIA

PARTICULARS OF TURBINES OF LARGE POWER

Meanings of Abbreviations: M.C.R. = Max. continuous rating, i.e. the maximum output which the generator is capable of giving continuously. E.R. = Economic rating, i.e. the power at which the turbine operates most economically. Where this has not been stated by the manufacturer, it has been taken as 80 per cent of the M.C.R. p_s = stop valve pressure, lb. per sq. in. abs. t_s = steam temp. at stop valve, °F. Firms: A.E.G. = Allgemeine Elektrizitäts-Gesellschaft, Berlin; A.I. = Alsthom; B.T.H. = British Thomson-Houston Co., Ltd.; E.E. = English Electric Co., Ltd.; F. & C. = Messrs. Fraser & Chalmers Eng. Works; G.E. = General Electric Co., U.S.A.; M.A.N. = Maschinenfabrik Augsburg-Nürnberg A.G.; M.V. = Metropolitan-Vickers Electrical Co., Ltd.; A.C.O. = Ateliers de Construction Oerlikon; C.A.P. = Messrs. C. A. Parsons & Co., Ltd.; W.E. = Westinghouse Electric & Manufacturing Co., U.S.A.

Ref. No.	Firm	Power Station	Year	Output kW		Operating Conditions			No. of Shafts	No. of Cyls.	No. of Exhs.	kW per Exhaust (E.R.)	
				M.C.R.	E.R.	p_s	t_s	Vac. at 1 in. Hg. Abs.				Actual	Reduced to 1 in. Hg. and 3,000 r.p.m.
1	C.A.P.	Commonwealth, Fisk Street, Chicago	1912	25,000	20,000	215	588	1.0	1	2	2	10,000	625
2	C.A.P.	Lots Road, London	1915	18,000	15,000	215	600	1.0	1	2	2	7,500	833
3	W.B.	Commonwealth, N.W., Chicago	1917	30,000	25,000	—	—	1.0	1	2	2	12,500	2,000
4	G.E.	Detroit Edison, Connor's Creek	1918	45,000	35,000	—	—	1.0	1	1	1	35,000	5,600
5	G.E.	Commonwealth, N.W., Chicago	1918	35,000	25,000	—	—	1.0	1	1	1	25,000	5,250
6	G.E.	Commonwealth, Fisk Street, Chicago	1918	30,000	22,500	—	—	1.0	1	1	1	22,500	8,100
7	A.F.G.	Rheinische Westfälische Kraftwerke, Cologne.	1919	50,000	40,000	—	—	2.0	1	1	1	40,000	5,000
8	C.A.P.	Mersey Power Co.	1921	12,500	10,000	—	—	1.0	1	2	2	5,000	6,670
9	C.A.P.	Lots Road, London	1924	18,000	15,000	—	—	1.0	1	1	1	15,000	8,000
10	C.A.P.	Brimsdown B., London	1926	25,000	20,000	—	—	1.0	1	2	3	6,670	8,000
11	M.A.N.	Hamburg	1928	36,000	27,000	—	—	1.14	1	3	3	9,000	5,200
12	A.L.G.	Zschornitz	1929	85,000	70,000	—	—	1.8	1	3	2	12,000	10,400
13	Al.	Comines	1929	30,000	24,000	—	—	1.17	1	3	2	66,000	—
14	G.E.	Phil, Ohio Power Co.	1929	165,000	132,000	—	—	1.0	3	2	2	40,000	10,000
15	C.A.P.	Dunston	1930	50,000	40,000	—	—	1.0	1	2	2	—	—
16	M.V.	Victoria Falls, South Africa	1930	32,500	30,000	—	—	1.8	1	2	2.5	12,000	12,000
17	F. & C.	Battersea, London	1930	30,000	24,000	—	—	1.0	1	2	2	10,000	10,700
18	A.C.O.	St. Denis, Paris	1931	50,000	40,000	—	—	0.926	1	4	4	13,333	14,300
19	Al.	St. Denis, Paris	1931	50,000	40,000	—	—	0.926	1	3	3	32,000	9,500
20	C.A.P.	Hackney, London	1931	30,000	24,000	—	—	1.3	1	2	2	12,000	7,500
21	B.T.H.	Barking, London	1931	75,000	60,000	—	—	1.0	1	3	2	42,000	11,570
22	M.V.	Battersea A., London	1936	105,000	84,000	—	—	0.9	1	3	2	16,000	13,600
23	E.E.	Ferrybridge, Yorks	1938	40,000	32,000	—	—	1.2	1	2	2	—	—

TABLE XVIII

PARTICULARS OF FINAL STAGE OF TURBINES OF LARGE POWER

Note The stress in a parallel blade has been calculated for steel blades by means of the formula: $f_c = 0.215 \text{ } mu^2 \text{ lb per sq in}$ The real stress in many cases is appreciably less than the value given, due to the use of tapered blades

Ref No Table XVIIA	Diameter, inches		Useful Blade Length in	$m = \frac{l}{D}$	u ft. per sec		Leaving Area per Exhaust sq ft	f_c Parallel Blade tons per sq. in
	Mean	Tip			Mean	Tip		
1	105	124	19	0.181	344	406	43.6	2.05
2	89.5	104	14.5	0.162	391	455	28.3	2.38
3	98	116	18	0.184	514	608	38.4	4.65
4	140	174	34	0.243	734	912	104	12.53
5	120	148	28	0.234	787	970	73.3	13.90
6	100	124	24	0.24	787	975	52.3	14.25
7	134	154	20	0.149	585	674	58.5	4.89
8	47.5	57	9.5	0.2	622	747	9.85	7.43
9	80	100	20	0.25	698	873	34.9	11.67
10	54.27	68.54	14.27	0.263	711	899	16.85	12.73
11	66.93	80.95	14.02	0.210	875	1060	20.45	15.40
12	114.2	137.82	23.62	0.207	748	900	58.9	11.10
13	61	75.82	14.82	0.243	798	992	19.7	14.82
14	110	145	35	0.318	865	1140	84.0	22.80
16	47.125	61.25	14.125	0.300	617	800	14.5	10.97
17	60	76.5	16.5	0.275	785	1000	21.6	16.25
18	61.5	75.5	14.0	0.228	805	990	18.8	14.18
19	64.96	79.51	14.55	0.224	852	1040	20.6	15.60
20	59.25	73.5	14.25	0.241	775	960	18.4	13.87

well-known papers (1 and 2) by Mr. K. Baumann. The data are arranged in chronological order. In order to show the increase which has taken place in the output per exhaust, this has been reduced to a standard exhaust pressure of 1 in. Hg. and a standard speed of 3,000 r.p.m. in accordance with the following simple formula,

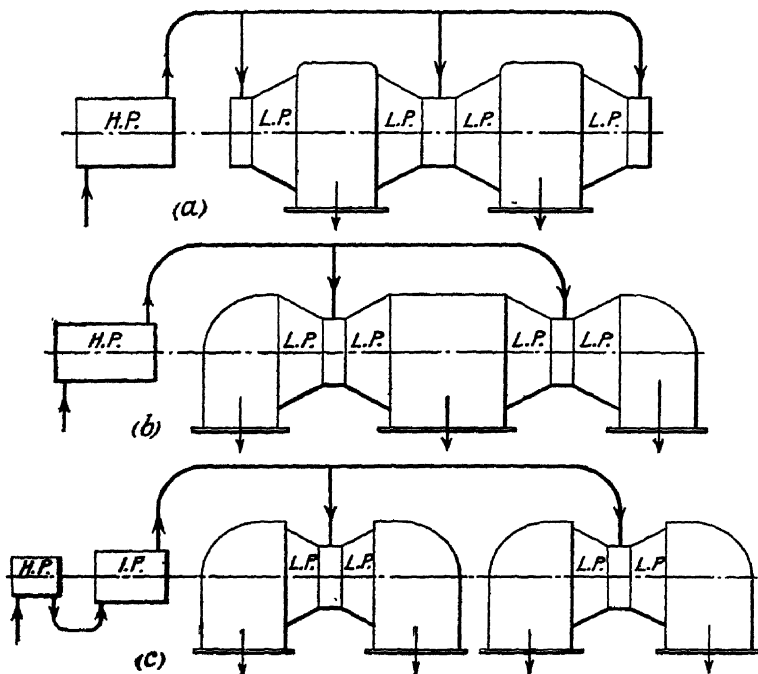


FIG. 311.—Turbines with Four Exhausts

based on eq. 212 and on the assumption that $p_e v_e$ is approximately constant for small changes of exhaust pressure—

$$kW_1 = kW \left(\frac{1}{p} \right) \left(\frac{H_{a1}}{H_a} \right) \left(\frac{N}{3000} \right)^2 \quad . \quad . \quad (218)$$

where kW_1 = reduced output per exhaust

kW = actual output per exhaust

p = exhaust pressure, in. Hg. abs.

H_{a1} = adiabatic heat drop to 1 in. Hg.

H_a = adiabatic heat drop to actual exhaust pressure

N = actual speed, r.p.m.

The largest turbine plant in the world generates 208,000 kW, and is installed in the State Line Generating Station, near Chicago. A rough diagram showing the lay-out of the plant is given in Fig. 312.

The "unit" comprises a high-pressure turbine and two double-flow low-pressure turbines. Steam is supplied to the H. P. turbine at 600 lb. per sq. in. gauge and 730° F. It is exhausted from the H. P. turbine at 71 lb. per sq. in. abs. and is then reheated by means of live steam to a temperature of 500° F. The exhaust pressure is 0.5 lb. per sq.

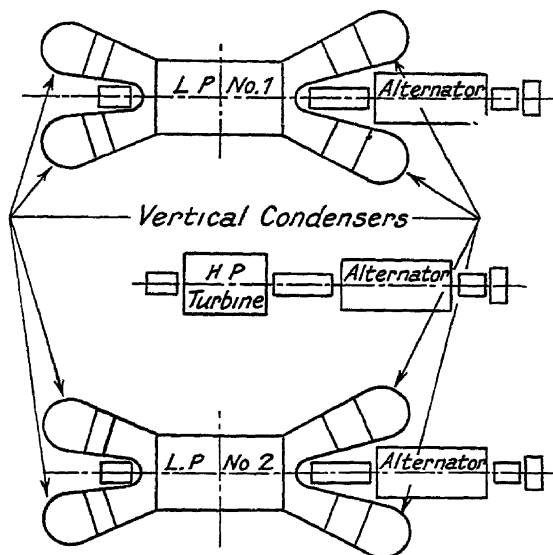


FIG. 312 —Diagrammatic Sketch Plan of 208,000 kW Triple Cross-compound Steam Turbo-alternator

in. abs. The low-pressure turbines exhaust into eight vertical surface condensers.

231. The Baumann Multi-exhaust Turbine. A very ingenious method of providing a greater area at the exhaust end of steam turbines was devised some years ago by Mr. K. Baumann of the Metropolitan-Vickers Electrical Co., Ltd. The principle of the system will be explained with reference to Fig. 313, showing a part section of the last few wheels at the exhaust end of the turbine. Steam expands in passing through the earlier stages of the turbine until it reaches the special diaphragm A, where a circular division ring B splits up the steam into two portions. The vanes C of the diaphragm A are normal in shape, forming convergent nozzles which expand the steam down to condenser pressure. The portion of the steam flowing through C passes through the outer parts of the blades (which are shaped as ordinary impulse blades) and then flows into the condenser.

The vanes E of the diaphragm A have a large outlet angle and allow the steam to flow through with only a small drop of pressure and a relatively small increase in velocity. The part F of the blade,

lying between the dividing ring G and the disc, is designed for a small amount of reaction, the steam undergoing a certain drop in pressure and an increase in velocity. The steam issuing from the blades F is now divided in the succeeding diaphragm in the same way.

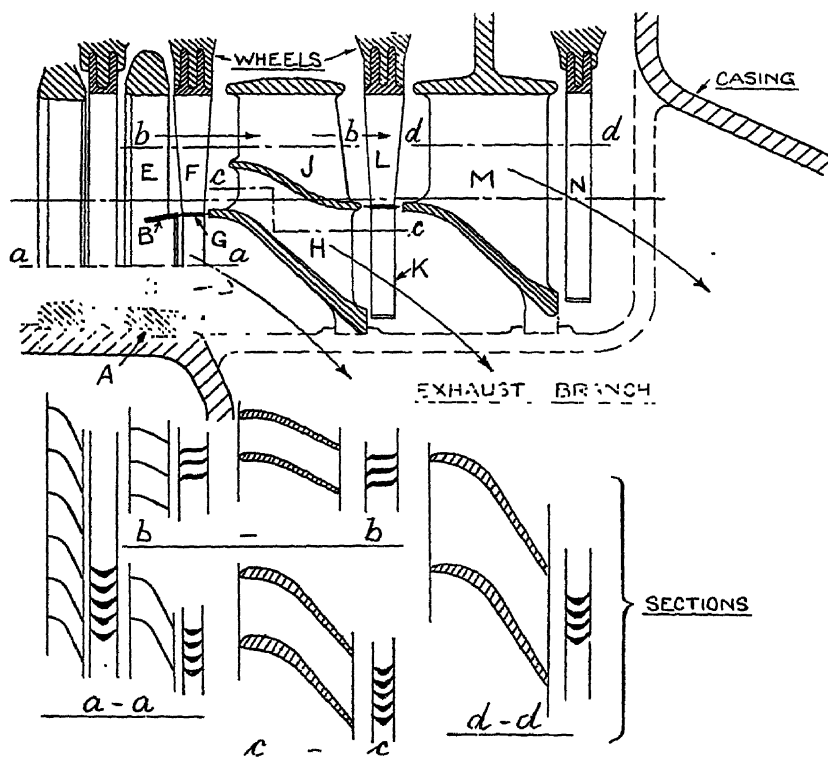


FIG. 313.—Details of the Metropolitan-Vickers "Multi-exhaust" System

One part is expanded to condenser pressure in H and does work on the blades K. The other part passes through J and L into the diaphragm M with only a small drop in pressure and, in doing so, does a small amount of work on the inner part L of the blades. On reaching the diaphragm M, this steam is expanded down to condenser pressure and does work on the impulse blades N.

The leaving area may be increased by the following amounts by this construction—

No. of Additional Rows
of Exhaust Blades

1
2
3

Percentage Increase
in Nozzle Area

170
60
120

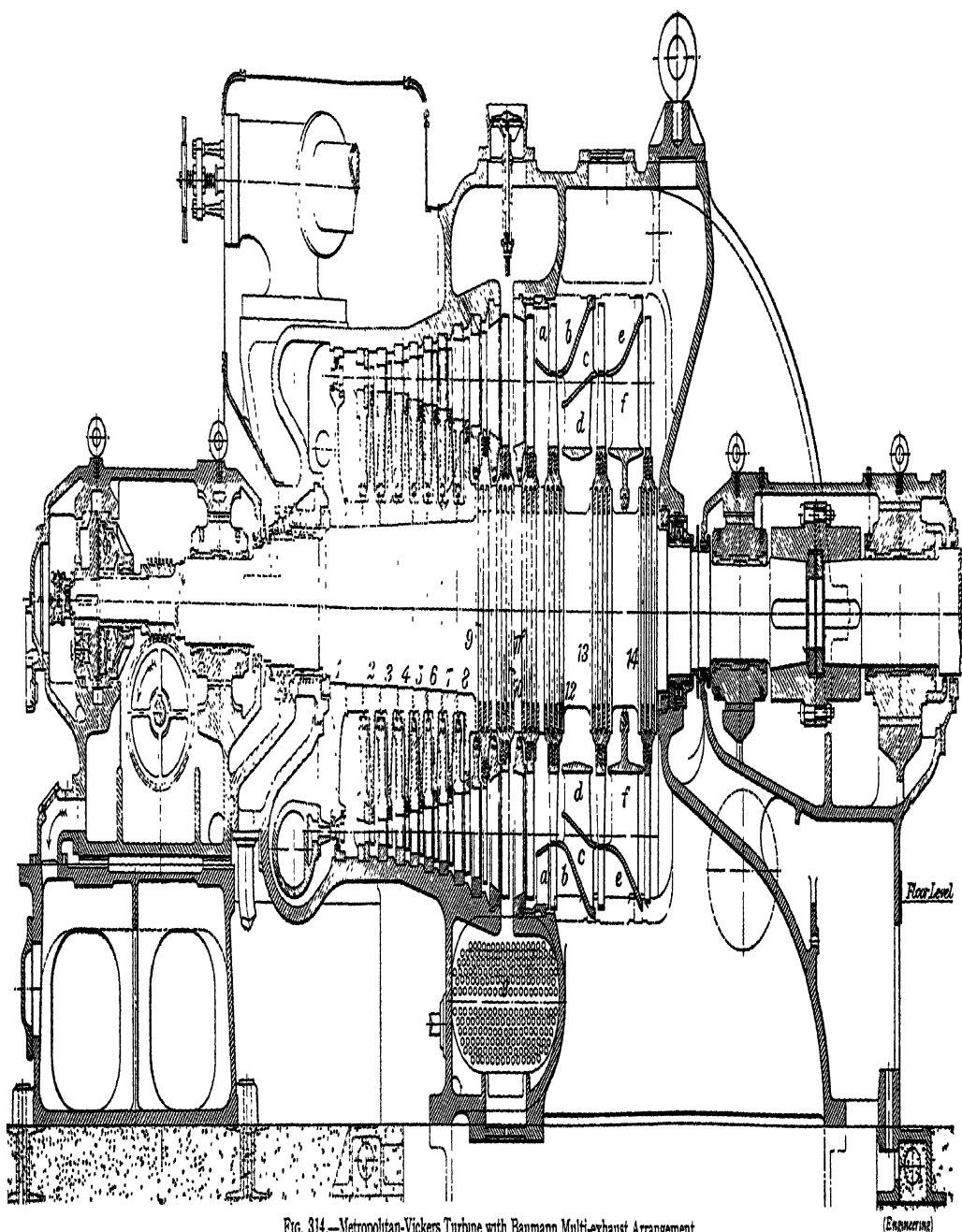


FIG. 314.—Metropolitan-Vickers Turbine with Baumann Multi-exhaust Arrangement
12,500 kW (15,600 kW overload) at 3,000 r.p.m., installed in the Lister Drive Power Station of the Liverpool Corporation

(Engineering)

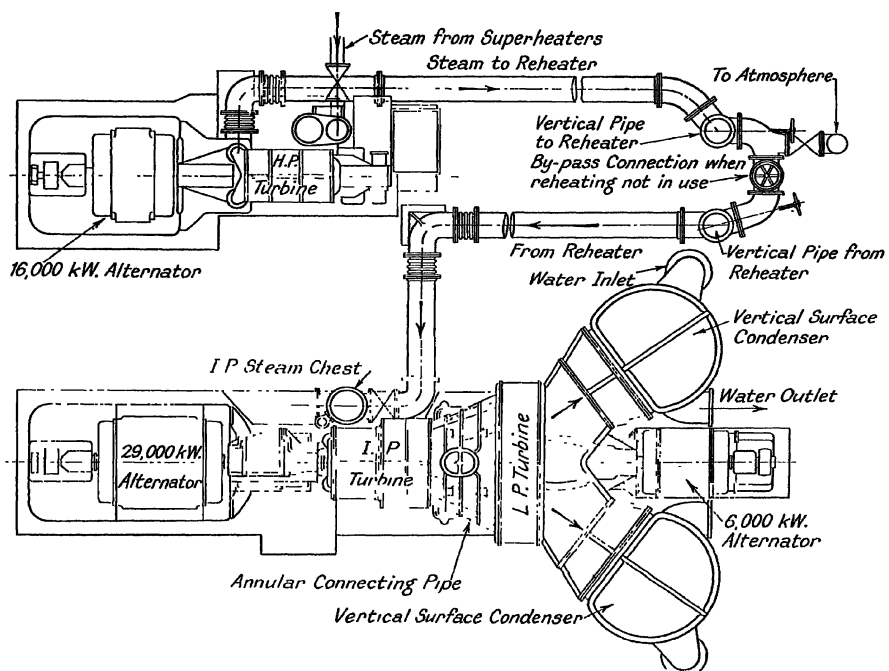


FIG. 315 —General Arrangement Plan of 51,000 kW Parsons Turbo-alternator

Owing to the special shape of the blade root, the power increase is greater than the increase in leaving area, and it is claimed that with three additional rows of blades the power may be increased four times for the same speed and factor of safety. In recent years the application of the Baumann multi-exhaust system appears to have been restricted to the use of one extra row of blades, which, when employed in a double-flow turbine, gives about three times the leaving area of a single-flow turbine of normal design.

A Metropolitan-Vickers turbine of 12,500 kW, with an overload of 15,600 kW and equipped with two extra rows of blades, is illustrated in Fig. 314. The machine is designed for a pressure of 185 lb. per sq. in. at the stop valve, a superheat of 150° F., and a vacuum of 27.5 in. The speed of rotation is 3,000, and since the mean diameter of the blades is 42 in., the mean blade velocity has the moderate value of 550 ft. per sec. The machine occupies a very small space for the power developed, its length being 17 ft and its width 13 ft.

Attention is directed to the wheel construction in the last six wheels. The shaft diameter is made fairly large to ensure a sufficient margin between the running speed and the critical speed. The "discs" are in halves and have from three to four internal prongs which fit into grooves in the shaft. They are secured by rivets in multiple shear. The turbine is furnished with a single-stage integral feed heater for regenerative feed-heating.

232. Large-powered Turbines with Elements Running at Different Speeds. It has already been pointed out in Art. 229 that the capacity of a given turbine frame varies inversely as the square of the rotational speed when other factors remain the same. The capacity of a multi-cylinder plant may therefore be greatly increased by running the low-pressure element at a lower speed.

This is the procedure adopted in the 51,000 kW Parsons turbine constructed for the Crawford Avenue Power Station of the Commonwealth Edison Co. of Chicago. A plan view of the set is shown in Fig. 315, from which it will be seen there are three turbines arranged in series. The leading particulars relating to the plant are given in the table on page 507.

The exhaust steam from the H.P. turbine leaves at a temperature of about 425° F. and is reheated in a special reheater boiler (having the greater part of its heating surface devoted to the purpose) to a temperature of 700° F. The exhaust pressure of the I.P. turbine is about 2 lb. per sq. in. abs., at which pressure the specific volume of the steam is about 170 cub. ft. per lb. The actual flow per second at this point is about 91 lb., so that the volume of steam transferred per second from the I.P. to the L.P. turbine is about 15,500 cub. ft. In order to effect this transference with the minimum drop of pressure, the two turbines are arranged co-axially and connected by a double-walled conical connecting piece, Fig. 316

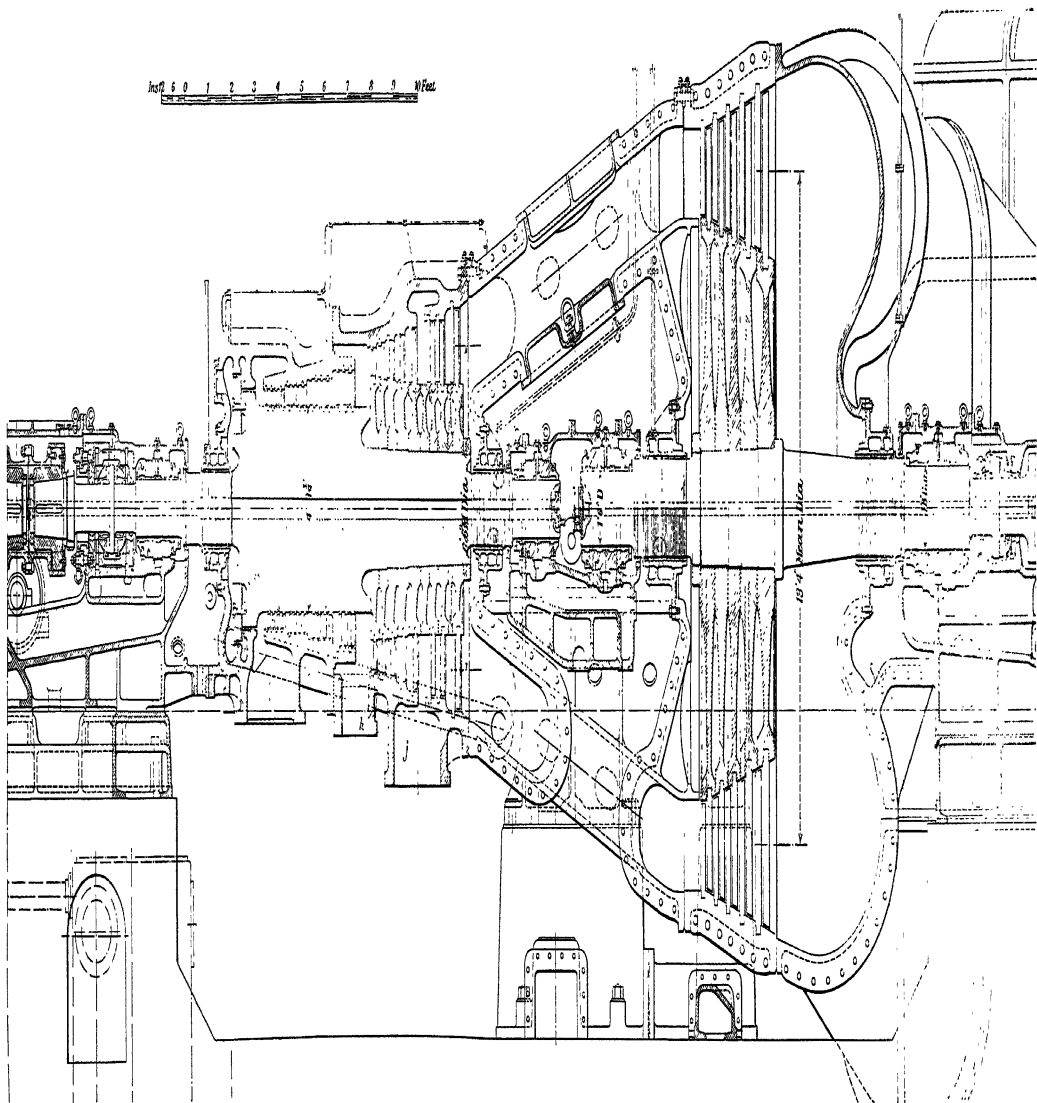


FIG. 316.—Section through I.P. and L.P. Turbines of Parsons 51,000 kW Turbo-alternator

(T.5000)

[Engineering]

	TURBINE		
	H.P.	I.P.	L.P.
Admission pressure, lb. per sq. in. (abs.)	565	105	2
Exhaust pressure, lb. per sq. in. (abs.)	115	2	0.37
Admission steam temperature ° F.	750	700	—
		(after reheating)	
Alternator output . . . kW	16,000	29,000	6,000
Speed of rotation . . . r.p.m.	1,800	1,800	720
Frequency of A.C. current . . .	60	60	60
No. of poles on alternator rotor . . .	4	4	10
Mean diameter of blading at outlet end inches . . .	40 $\frac{3}{4}$	78	160
Mean blade speed . . . ft. per sec.	317	614	503
Blade height in 1st stage . . .	2 $\frac{1}{4}$	3 $\frac{1}{4}$	21
Blade height in last stage . . .	3 $\frac{1}{4}$	18	42

The area through the connecting piece gradually increases so that a certain amount of diffuser action is obtained. The L.P. turbine exhausts into two vertical surface condensers. A view of the turbines, alternators, and condensers taken from the overhead crane is given in Fig. 317.

As an indication of the progress made in the turbine art, it may be mentioned that Messrs. C. A. Parsons & Co., Ltd. constructed, in 1939, a 50,000kW turbo-alternator for Bunnerong, Australia. This machine develops its power at 3,000 r.p.m. The turbine is supplied with steam at 600 lb. gauge, 825° F., and exhausts at 28.5 in. vacuum. There are three cylinders, in tandem, a high-pressure, intermediate-pressure, and a double-flow, low-pressure cylinder.

In order to limit the stress in the rotor blades of the last stage of the L.P. turbine, and also the edge loading of the wheel, the blades in that stage are made hollow from end to end. The internal cavity is given the same shape as the blades and the blades are tapered outwards both in width and thickness.

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2. BAUMANN, K. "Neuere grosse Dampfturbinen." Paper read at World Power Conference. *Z.V.d.I.*, 1930; 74; 805.

EXAMPLES XXIV

1. Calculate the required K value for a multi-stage impulse turbine under the following conditions: Cumulative heat drop = 465 B.Th.U. per lb., nozzle angle = 18°; $\rho = 0.5$; $C = 1$; $\eta_n = 0.92$; $\phi = 0.9$; and $k = 0.85$.

If the speed of rotation is 3,000 r.p.m. and the mean blade velocity is to be 600 ft. per sec., calculate the number of single-row stages required if the mean diameter is uniform throughout the turbine.

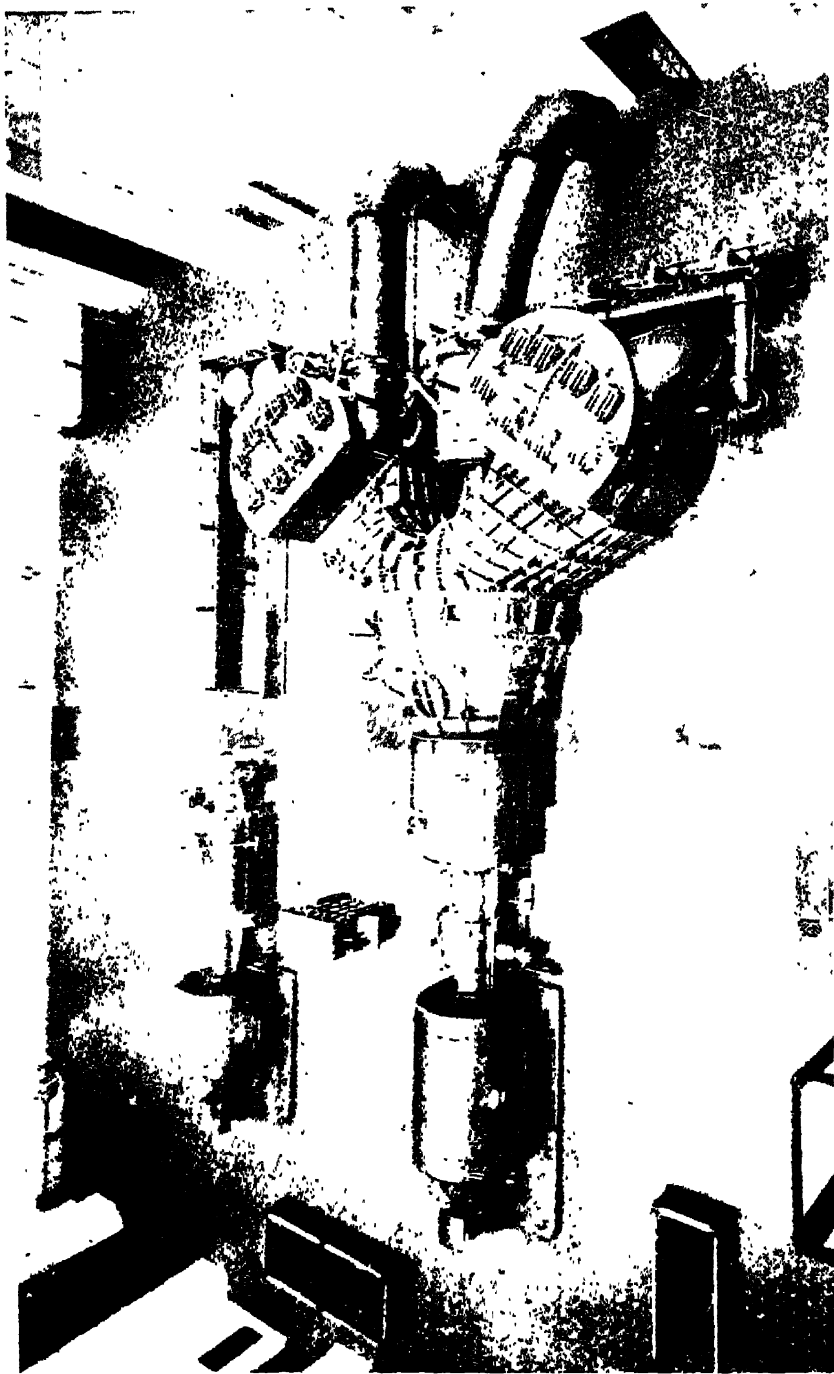


FIG. 317 —51,000 kW Parsons Reaction Turbo-Alternator installed in the Crawford Avenue Power Station of the Commonwealth Edison Co. of Chicago. View taken from the overhead crane

2. The leading dimensions of a 30,000 kW steam turbine are as follows—

Stage No.	Mean Dia. in.	Stage No.	Mean Dia. in.
1	66.5	8	81.5
2	62.0	9	86.0
3	64.0	10	90.0
4	66.5	11	95.0
5	70.0	12	102.0
6	74.0	13	109.0
7	77.5	14	118.0

The first stage is a two-row wheel, the remainder single-row wheels.

Calculate the K value and the efficiency factor, given the following data—

Initial steam pressure	200 lb. per sq. in. abs.
Initial steam temperature	640° F.
Exhaust pressure	0.7 lb. per sq. in. abs.
Reheat factor	1.05
R.p.m.	1,500

Hence calculate the ratio of blade speed to steam speed in the single-row stages, assuming that it is uniform in those stages and has one-half of this value in the two-row stage. Assume that F_1 is unity.

3. Compare the relative rotative speeds of the following steam turbines when using steam having the same initial and final pressures. The speed of the de Laval turbine may be taken as 25,000 r.p.m.

(a) De Laval turbine, 6 in. mean diameter.

(b) Velocity-compounded turbine having four two-row wheels, 42 in. mean diameter.

(c) Pressure-compounded impulse turbine having 25 wheels, 30 in. mean diameter.

Neglect the effects of carry-over energy from stage to stage.

4. A set of turbines is to be designed for a net s.h.p. of 3,130. The final specific volume of the steam is 239 cub. ft. per lb., the adiabatic heat drop is 325 B.Th.U. per lb., the efficiency ratio is 0.7, the blade velocity 400 ft. per sec., and the blade speed ratio 0.4. Assume that the vane angle in the last-stage diaphragm is 28°, a thickness coefficient of 0.89, and a nozzle height 17 per cent of the mean diameter. Calculate—

(a) The mean diameter of the last-stage nozzles.

(b) The revolutions per minute.

5. A marine steam turbine is required to develop 3,600 s.h.p. with steam at 200 lb. per sq. in. abs. and 600° F. and an exhaust pressure of 1.0 lb. per sq. in. abs. The efficiency ratio of the turbine is 73 per cent and the carry-over loss is to be limited to 1.5 per cent of the adiabatic heat drop. The length of the blades in the last stage is limited to 0.2 of the mean diameter in that stage. Calculate the required mean diameter of the final L.P. stage and the speed of the L.P. turbine if the mean blade velocity is limited to 450 ft. per sec.

6. A steam turbine is to operate with steam at 400 lb. per sq. in. abs. and at 700° F., and with an exhaust pressure of 0.5 lb. per sq. in. abs. The actual enthalpy of the exhaust steam is estimated to be 981 B.Th.U. per lb., after allowing for internal losses. Steam is bled off at 40 lb. per sq. in. abs., and from the dimensions of the feed heater it is estimated that the feed temperature at outlet from the heater will be 260° F. and that the temperature of the

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condensed heating steam (which enters the steam space of the condenser) will be 90° F. The main condensate temperature may be taken as 79·5° F.

Calculate the required height of the last-stage nozzles if the output of the turbine is to be 3,000 kW at 3,000 r.p.m. Assume that $u = 600$ ft. per sec., $\rho = 0\cdot48$, $\alpha = 28^\circ$ ($c = 0\cdot877$), and that the condition curve on the ϕ -I diagram may be taken as a straight line

7. Steam is supplied to an impulse turbine at 600 lb. per sq. in. abs. and at 800° F. It is first expanded to 100 lb. per sq. in., the internal efficiency ratio being 82 per cent. It is then reheated at this pressure to 600° F, after which it is expanded to 0·5 lb. per sq. in. abs., the internal efficiency ratio during the latter expansion also being 82 per cent. If $m = 0\cdot18$, $\alpha = 30^\circ$ ($c = 0\cdot884$), $\rho = 0\cdot48$, and $N = 3,000$ r.p.m., show that the maximum output cannot exceed

$$\text{kW} = 17\cdot7 \left(\frac{u}{100} \right)^3$$

where u is the mean peripheral velocity in the last stage. What is the limiting output of a turbine operating under such conditions when there are three exhausts and $u = 700$ ft. per sec.?

8. A number of steam turbines have to be built to operate with steam at 400 lb. per sq. in. abs. and at 700° F, the exhaust pressure being 1·0 lb. per sq. in. abs. The energy loss due to the residual velocity of flow into the exhaust belt is not to exceed 2 per cent of the adiabatic heat drop, and the length of the last row of blades is not to exceed 20 per cent of the mean diameter of the blades. If the efficiency ratio of the turbines is 78 per cent, show that the mean diameter of the last ring of blades must not be less than

$$D \text{ (inches)} = 0\cdot461 \sqrt{\text{HP}}$$

where HP represents the horse-power of any turbine in question. Assume that the internal efficiency ratio is equal to the efficiency ratio.

9. Using the data given in Question 8, show that the maximum power HP that can be developed for a given centrifugal stress f_c at the root of the blade in the last stage, at a speed N r.p.m., is—

$$\text{HP} = \frac{5,740,000 f_c}{N^2}$$

Assume that the blades have a uniform section and are of steel

Hence find the limit in power for a speed of 3,000 r.p.m. and a stress of 16,000 lb. per sq. in.

THE LJUNGSTRÖM TURBINE

233. Theory of the Ljungström Turbine. The elementary features of the Ljungström turbine have already been outlined in Chapter I. It is essentially an outward-radial-flow turbine in which there are two rotors turning in opposite directions. It operates on the impulse-reaction (so-called "reaction") principle, with half-degree

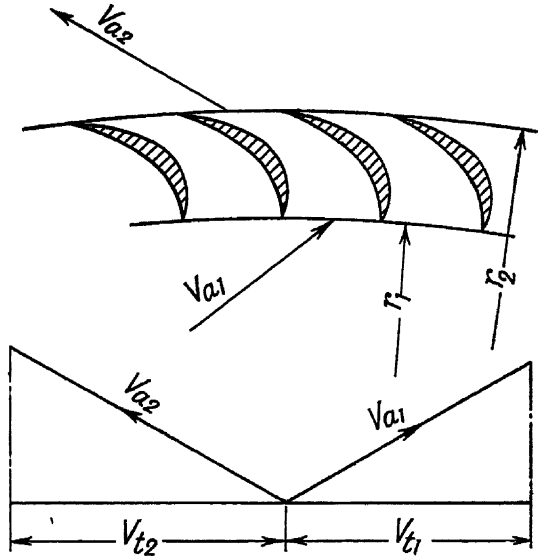


FIG. 318

reaction. In turbines of large output, the required outlet area at the low-pressure end is obtained by the use of one or two axial-flow stages. Details of this and other constructional features will be given later in this chapter.

We shall first derive an expression for the work done in a radial-flow turbine. Suppose that the peripheral velocities at the inlet and outlet of a single ring of blades (see Fig. 318) are u_1 and u_2 , and that the absolute steam velocities are V_{a1} and V_{a2} . Let V_{t1} and V_{t2} be the tangential components of these velocities, and assume that the steam flow through the blade ring is w lb. per sec. Then since the torque is equal to the change of moment of momentum per second—

$$\text{Torque } T = \frac{w}{g} (V_{t2} \cdot r_2 - V_{t1} \cdot r_1)$$

where correct account is taken of the signs of V_{t1} and V_{t2} . For instance, in Fig. 318, if V_{t2} is taken as positive, V_{t1} must be taken as negative.

Work done per sec. = ωT

$$= \frac{w}{g} (u_2 \cdot V_{t2} - u_1 V_{t1})$$

and the work done per lb. of steam

$$W = \frac{1}{g} (u_2 V_{t2} - u_1 V_{t1}) \text{ ft.-lb.}$$

$$W = \frac{1}{gJ} (u_2 V_{t2} - u_1 V_{t1}) \text{ B.Th.U.} \quad . \quad . \quad (219)$$

The upper part of Fig. 319 shows a part longitudinal section and a part transverse section through four rows of blades. It will be assumed in the following analysis that—

(1) All the blades, except those in the innermost ring, have the same cross-section and that they are all set similarly in the rotors. They have, therefore, all the same outlet angle.

(2) In all rings of blades, the ratio of the relative steam velocity at outlet to the peripheral velocity at the outlet edge of the blade ring is constant, i.e. that $\rho = \frac{u}{V}$ is a constant.

Giving all dimensions in foot units—

Let r_0 = inner radius of first ring.

r_1, r_2, r_3 , etc., be the outer radii of rings 1, 2, 3, etc., respectively.

u_1, u_2, u_3 , etc., be the peripheral velocities at the *outer* edge of the rings 1, 2, 3, etc., respectively.

Consider first of all the flow of steam through the first ring of blades. Steam will flow into that ring with a velocity V_0 (AB) which may be assumed to be radial and to be given by the expression

$$V_0 = \frac{w \cdot v_0}{2\pi r_0 \cdot l_0}$$

where l_0 = axial length of blades at r_0

v_0 = specific volume of steam at blade inlet.

This is combined with the peripheral velocity u_0 (AC), giving the relative velocity U_0 (CB) and the inlet angle BCA of the first ring of blades. The steam issues from the first ring of blades with a velocity V_1 (DE) relative to the blades, the absolute velocity being AE. The tangential component of the absolute velocity at outlet is

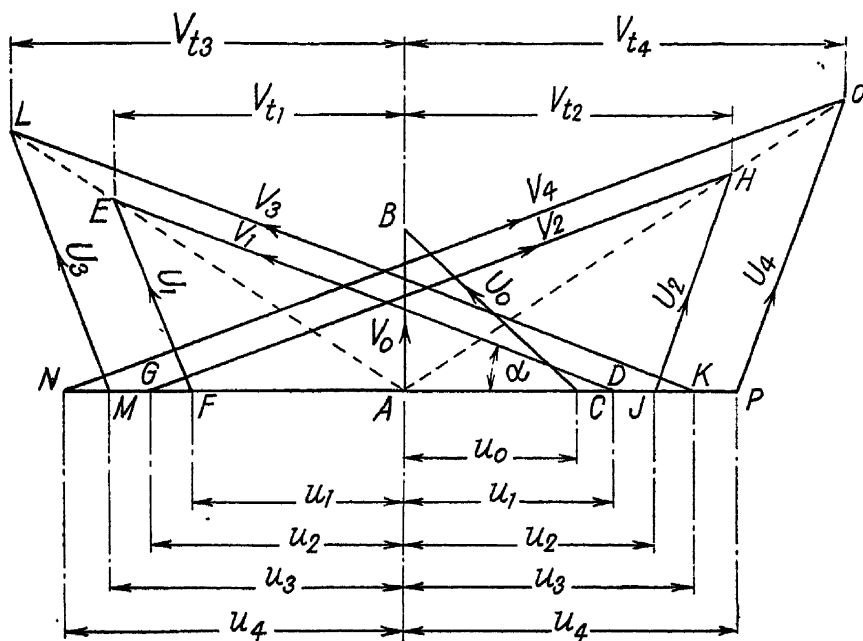
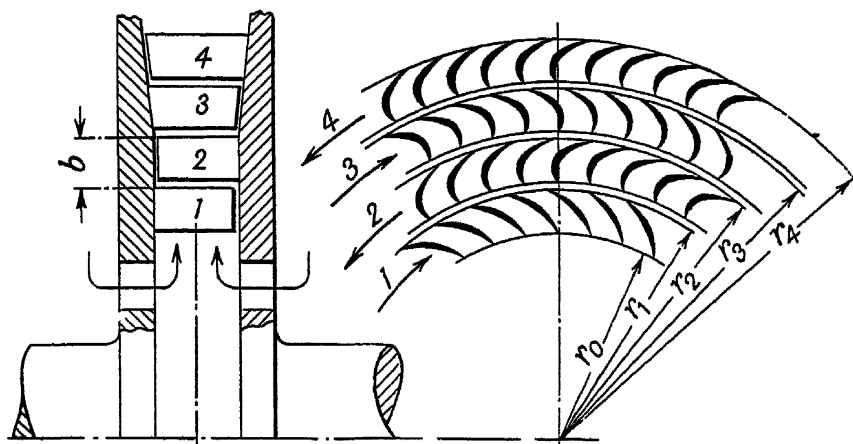


FIG. 319

V_{t1} , and since the initial absolute velocity is radial, the work done in the innermost blade ring per lb. of steam is

$$\begin{aligned} W_1 &= \frac{u_1 V_{t1}}{gJ} \text{ B.Th.U.} \\ &= \frac{1}{gJ} (u_1 V_1 \cos \alpha - u_1^2) \end{aligned}$$

By the steady-flow equation (37), and assuming no loss of heat from the stage—

$$\begin{aligned} \text{Work done} &= \text{Initial kinetic energy} + (I_0 - I_1) \\ &\quad - \text{Final kinetic energy} \\ &= \frac{V_0^2}{2gJ} + (I_0 - I_1) - \frac{AE^2}{2gJ} \\ &= \frac{V_0^2}{2gJ} - \frac{V_1^2 + u^2 - 2u_1 V_1 \cos \alpha}{2gJ} + I_0 - I_1 \end{aligned}$$

Equating these two expressions for the work done, we have

$$\begin{aligned} I_0 - I_1 &= \frac{1}{2gJ} (V_1^2 - V_0^2 - u_1^2) \\ &= \frac{1}{2gJ} \{V_1^2 - U_0^2 - (u_1^2 - u_0^2)\} \quad . \quad . \quad (220) \end{aligned}$$

If the flow is frictionless, then $I_0 - I_1$ is the required adiabatic heat drop. If not, then the adiabatic heat drop

$$h_1 = \frac{1}{2gJ \cdot \eta_n} \{V_1^2 - k^2 U_0^2 - (u_1^2 - u_0^2)\} \quad . \quad (221)$$

where η_n is the efficiency of the blades considered as nozzles (see Art. 75) and k has the same meaning as the blade velocity coefficient in impulse turbines.

Consider now the expansion of steam in the second ring of blades. The relative velocity at inlet is U_1 (FE) at r_1 , and at outlet is V_2 (GH). The tangential components of the absolute velocities at inlet and outlet are, respectively, V_{t1} and V_{t2} . Hence the work done per lb. of steam is numerically equal to

$$\frac{1}{gJ} (u_2 V_{t2} + u_1 V_{t1})$$

since V_{t1} and V_{t2} are of opposite sign.

$$\therefore \text{Work done} = \frac{1}{gJ} (u_2 V_2 \cos \alpha - u_2^2 + u_1 V_1 \cos \alpha - u_1^2)$$

Then, by the steady-flow equation—

$I_1 + (\text{Initial kinetic energy referred to absolute velocity}) = (\text{Work done}) + I_2 + (\text{Final kinetic energy referred to absolute velocity}).$

$$\therefore I_1 - I_2 = (\text{Work done}) + \frac{(AH)^2}{2gJ} - \frac{(AE)^2}{2gJ}$$

$$I_1 - I_2 = \frac{1}{gJ} (u_2 V_2 \cos \alpha - u_2^2 + u_1 V_1 \cos \alpha - u_1^2)$$

$$+ \frac{1}{2gJ} (V_2^2 + u_2^2 - 2u_2 V_2 \cos \alpha)$$

$$- \frac{1}{2gJ} (V_1^2 + u_1^2 - 2u_1 V_1 \cos \alpha)$$

$$I_1 - I_2 = \frac{1}{2gJ} (4u_1 V_1 \cos \alpha - u_2^2 - 3u_1^2 + V_2^2 - V_1^2)$$

But $V_1^2 = U_1^2 - 4u_1^2 + 4u_1 V_1 \cos \alpha$

$$\therefore I_1 - I_2 = \frac{1}{2gJ} \{V_2^2 - U_1^2 - (u_2^2 - u_1^2)\}. \quad (222)$$

If the flow is frictionless, then $I_1 - I_2$ represents the *adiabatic* heat drop in the blades of the second row. If the flow is not frictionless, then the adiabatic heat drop is

$$h_2 = \frac{1}{2gJ\eta_n} \{V_2^2 - k^2 U_1^2 - (u_2^2 - u_1^2)\}. \quad (223)$$

The term $\frac{u_2^2 - u_1^2}{2gJ}$ in the above equation represents a reduction in the value of $I_1 - I_2$ due to the forced vortex action in the blade ring.

Writing down the adiabatic heat drops in the several rings of blades, we have

$$h_1 = \frac{1}{2gJ\eta_n} \{V_1^2 - k^2 U_0^2 - (u_1^2 - u_0^2)\}$$

$$h_2 = \frac{1}{2gJ\eta_n} \{V_2^2 - k^2 U_1^2 - (u_2^2 - u_1^2)\}$$

$$h_3 = \frac{1}{2gJ\eta_n} \{V_3^2 - k^2 U_2^2 - (u_3^2 - u_2^2)\}$$

$$h_{n-1} = \frac{1}{2gJ\eta_n} \{V_{n-1}^2 - k^2 U_{n-2}^2 - (u_{n-1}^2 - u_{n-2}^2)\}$$

$$h_n = \frac{1}{2gJ\eta_n} \{V_n^2 - k^2 U_{n-1}^2 - (u_n^2 - u_{n-1}^2)\}$$

If it be assumed that the velocity coefficient k and the efficiency η_n remain constant throughout, then the sum of the adiabatic heat drops for all the stages considered is

$$\Sigma h = \frac{1}{2gJ\eta_n} \{(V_1^2 + V_2^2 + \dots V_n^2) - k^2 (U_0^2 + U_1^2 + \dots U_{n-1}^2) - (u_n^2 - u_0^2)\}$$

Considering the velocities U_1, U_2, U_3 , etc., we may write—

$$U_1^2 = V_1^2 + 4u_1^2 - 4u_1V_1 \cos \alpha$$

$$U_2^2 = V_2^2 + 4u_2^2 - 4u_2V_2 \cos \alpha, \text{ etc.}$$

Furthermore, with negligible error, we may write for the numerical value of U_0^2 —

$$U_0^2 = V_0'^2 + 4u_0^2 - 4u_0V_0' \cos \alpha$$

where V_0' is a fictitious velocity, such that

$$\frac{u_0}{V_0'} = \rho$$

Substituting the above values of U_0^2, U_1^2 , etc., in the above expression for Σh , we have

$$\begin{aligned} \Sigma h &= \frac{1}{2gJ\eta_n} \{(V_1^2 + V_2^2 + \dots V_n^2) \\ &- k^2 (1 + 4\rho^2 - 4\rho \cos \alpha) (V_0'^2 + V_1^2 + \dots V_{n-1}^2) \\ &- \rho^2 (V_n^2 - V_0'^2)\} \end{aligned}$$

But

$$V_0' = \frac{u_0}{\rho} = \frac{\omega r_0}{\rho}$$

$$V_1 = \frac{u_1}{\rho} = \frac{\omega r_1}{\rho}$$

$$V_n = \frac{u_n}{\rho} = \frac{\omega r_n}{\rho}$$

$$\begin{aligned} \therefore \Sigma h &= \frac{\omega^2}{2gJ\eta_n\rho^2} \{(r_1^2 + r_2^2 + \dots r_n^2) \\ &- k^2 (1 + 4\rho^2 - 4\rho \cos \alpha) (r_0^2 + r_1^2 + \dots r_{n-1}^2) \\ &- \rho^2 (r_n^2 - r_0^2)\} \end{aligned}$$

Suppose that in any group of blade rings to which these relations may be applied, the radial depth of the blades, plus the small radial clearance between adjacent blade rings, is a constant dimension b .

Then $r_1 = r_0 + b$; $r_2 = r_0 + 2b$; $\dots r_{n-1} = r_0 + (n-1)b$; and $r_n = r_0 + nb$.

$r_1^2 = r_0^2 + 2br_0 + b^2$; . . . $r_{n-1}^2 = r_0^2 + 2(n-1)b \cdot r_0 + (n-1)^2 b^2$; and $r_n^2 = r_0^2 + 2nbr_0 + n^2 b^2$.

The sum of the series $r_1^2 + r_2^2 + \dots + r_n^2$ is

$$nr_0^2 + n(n+1)br_0 + n(n+1)(2n+1)\frac{b^2}{6}$$

and of the series $r_0^2 + r_1^2 + \dots + r_{n-1}^2$ is—

$$nr_0^2 + n(n-1)br_0 + n(n-1)(2n-1)\frac{b^2}{6}$$

$$\begin{aligned} \therefore \Sigma h = & \frac{\omega^2}{2gJ \cdot \eta_n \cdot \rho^2} \left\{ \left[nr_0^2 + n(n+1)b \cdot r_0 \right. \right. \\ & + \left. \frac{n(n+1)(2n+1)b^2}{6} \right] \\ & - k^2(1 + 4\rho^2 - 4\rho \cos \alpha) \left[nr_0^2 + n(n-1)br_0 \right. \\ & + \left. \frac{n(n-1)(2n-1)b^2}{6} \right] \\ & \left. - \rho^2(2nbr_0 + n^2b^2) \right\} \quad \dots \dots \dots (224) \end{aligned}$$

This expression will give the sum of the adiabatic heat drops in a group of n rings in which the innermost radius is r_0 ft. and the outermost radius r_n ft. The dimensions b must also be given in foot units. If inch units are preferred, then the whole expression must be divided by 144.

234. Optimum Ratio of Blade Speed to Steam Speed. In order to determine this, consider the adiabatic heat drop in any given blade ring, e.g. ring 3, Fig. 319. Then—

$$h_3 = \frac{1}{2gJ\eta_n} \{V_3^2 - k^2 U_2^2 - (u_3^2 - u_2^2)\}$$

In general, the ratio $\frac{b}{r}$ is about 0.03 to 0.04; hence the difference between u_3 and u_2 is small compared with either of them and we may, allowing a small error for the sake of simplicity, neglect the term $u_3^2 - u_2^2$ in the above expression.

$$\therefore h_3 \doteq \frac{1}{2gJ\eta_n} \{V_3^2 - k^2 (V_2^2 + 4u_2^2 - 4u_2 V_2 \cos \alpha)\}$$

By the same approximation, V_3 may be written as V_2 . Then—

$$h_3 = \frac{V_2^2}{2gJ\eta_n} \{1 - k^2(1 + 4\rho^2 - 4\rho \cos \alpha)\}$$

The work done per lb. of steam in the third ring of blades is

$$\begin{aligned}
 W_3 &= \frac{1}{gJ} (u_3 V_{t3} - u_2 V_{t2}) \\
 &= \frac{u_2}{gJ} (V_{t3} - V_{t2}) \text{ approximately} \\
 &= \frac{2u_2}{gJ} (V_2 \cos \alpha - u_2) \\
 &= \frac{2V_2^2}{gJ} (\rho \cos \alpha - \rho^2)
 \end{aligned}$$

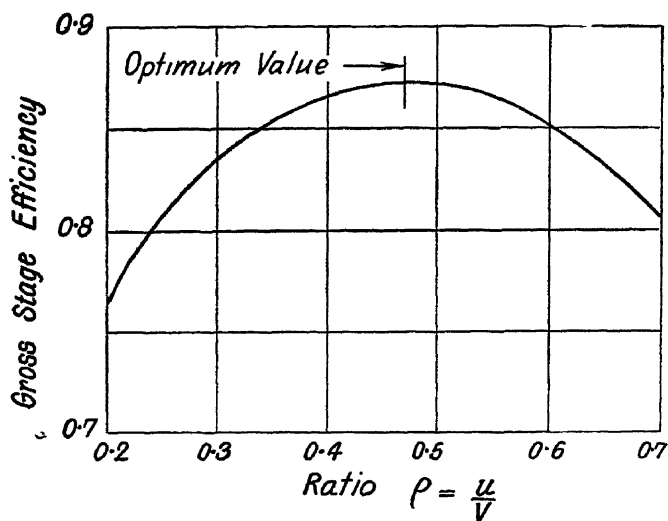


FIG. 320.—Calculated Stage Efficiency of Ljungström Type of Steam Turbine

The gross stage efficiency (neglecting leakage and any rotation losses) is equal to

$$\begin{aligned}
 \eta_{gs} &= \frac{W_3}{h_3} \\
 \eta_{gs} &= \frac{4\eta_n (\rho \cos \alpha - \rho^2)}{1 - k^2 (1 + 4\rho^2 - 4\rho \cos \alpha)} \quad (225)
 \end{aligned}$$

If we assume constant values for the coefficients $\eta_n = 0.90$ and $k = 0.86$, which, it must be stated, are merely assumptions guided by nozzle and blade experiments, then for a uniform blade outlet angle of 20° , the estimated values of the gross stage efficiency are given in Table XVIII and Fig. 320. Inspection of eq. 225 and of Fig. 320 shows that the gross stage efficiency is a maximum when

$\rho = \frac{\cos \alpha}{2} = 0.47$ for 20° blades. It should be noted that if the blade speed ratio is taken to mean the ratio of the *relative* blade velocity to the steam velocity at exit from the blades, then since the relative velocity of the blades is twice the blade velocity in each ring, the most economical ratio of blade speed to steam speed is $\cos \alpha$, or about 0.94 for 20° blades, just as in the Parsons turbine.

TABLE XVIII

ESTIMATED GROSS STAGE EFFICIENCIES AND VALUES OF THE EFFICIENCY FACTOR FOR LJUNGSTRÖM DOUBLE-MOTION TURBINE

Basic Data: Blade outlet angle = 20° ; $\eta_n = 0.90$; $k = 0.86$.

ρ	Gross Stage Efficiency	Values of $f_s(\rho)$			$\frac{K}{H_0}$ when $\frac{b}{r} = 0.04$
		$\frac{b}{r} = 0.03$	$\frac{b}{r} = 0.04$	$\frac{b}{r} = 0.05$	
0.1	0.595	0.598	0.607	0.618	43.2
0.2	0.764	0.794	0.798	0.805	131.8
0.25	0.808	0.868	0.870	0.876	187.8
0.3	0.836	0.927	0.927	0.930	255
0.35	0.854	0.969	0.967	0.968	332
0.4	0.865	0.994	0.993	0.991	422
0.45	0.872	1.007	1.002	0.999	529
0.5	0.872	1.001	0.997	0.995	657
0.55	0.866	0.982	0.973	0.963	815
0.6	0.851	0.943	0.938	0.934	1004
0.65	0.832	0.895	0.885	0.880	1251
0.7	0.807	0.827	0.816	0.811	1574

EXAMPLE 35. The following particulars refer to the inner group of blading in a Ljungström turbine—

Inner diameter of first blade ring		4 in.
Dimension b (Fig. 319)		$\frac{1}{4}$ in.
Number of rings of blades on each rotor		10
Blade outlet angle		20°
R.p.m. of each rotor		3,000

Assuming that $\rho = 0.4$, $\eta_n = 0.9$, and that $k = 0.87$, calculate the total adiabatic heat drop in the group of blades.

$$\omega = \frac{3000 \times 2\pi}{60} = 100\pi. \quad n = 20$$

$$r_0 = 2 \text{ in.} = \frac{1}{8} \text{ ft.} \quad b = \frac{1}{4} \text{ in.} = \frac{1}{48} \text{ ft.}$$

Substituting the given data in eq. 224—

$$\Sigma h = 13.68 (3.261 - 0.312 - 0.050)$$

$$= 39.7 \text{ B.Th.U. per lb. of steam}$$

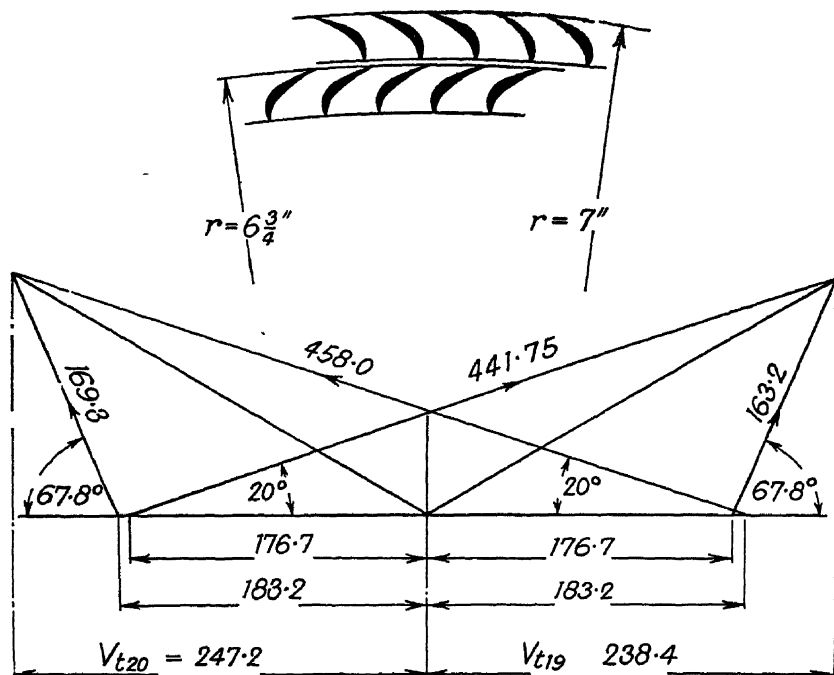


FIG. 321.—Illustrating Example 36

EXAMPLE 36. Calculate for the outermost ring in the blade group mentioned in Example 35 the following—

- Blade inlet angle;
- Velocities of whirl;
- Torque applied by blades to the disc;
- Work done per lb. of steam;
- Adiabatic heat drop in ring;
- Gross stage efficiency;
- Power developed in blade ring.

The steam flow per second is 20 lb. per second.

A simple calculation shows that the external radius of the 19th ring is $6\frac{3}{4}$ in. and of the 20th ring 7 in.

∴ Peripheral velocity of 19th ring = 176.7 ft./sec.

„ „ 20th „ = 183.2 „

$$V_{19} = \frac{176.7}{0.4} = 441.75; \quad V_{20} = \frac{183.2}{0.4} = 458.0$$

The velocity diagrams for the 19th and 20th blade rings are shown in Fig. 321 and require no explanation. The blade inlet angle is 67.8°. The velocities of whirl are respectively 238.4 and 247.2 ft. per sec.

$$\begin{aligned} \text{Torque} &= \frac{w}{g} (V_{t20} \cdot r_{20} - V_{t19} \cdot r_{19}) \\ &= \frac{20}{32.2} \left\{ \left(247.2 \times \frac{7}{12} \right) + \left(238.4 \times \frac{6.75}{12} \right) \right\} \\ &= 172.8 \text{ lb.-ft.} \end{aligned}$$

$$\begin{aligned} \text{Work done per sec.} &= \omega \times \text{Torque} \\ &= 100\pi \times 172.8 = 54,290 \text{ ft.-lb.} \end{aligned}$$

$$\begin{aligned} \therefore \text{Work done per lb. of steam} &= 2714.5 \text{ ft.-lb.} \\ &= 3.49 \text{ B.Th.U.} \end{aligned}$$

Adiabatic heat drop in blade ring

$$\begin{aligned} &= \frac{1}{2gJ \cdot \eta_n} \{V_{20}^2 - k^2 \cdot U_{19}^2 - (u_{20}^2 - u_{19}^2)\} \\ &= \{458^2 - (0.87 \times 163.2)^2 - (183.2^2 - 176.7^2)\} \\ &\div (64.4 \times 777.8 \times 0.9) = 4.156 \text{ B.Th.U. per lb.} \end{aligned}$$

$$\therefore \text{Gross stage efficiency} = \frac{3.49}{4.156} = 84 \text{ per cent}$$

$$\begin{aligned} \text{Power developed} &= \frac{54,290}{550} = 99.8 \text{ h.p.} \\ &= 73.6 \text{ kW} \end{aligned}$$

235. Calculation of Blade Lengths. Suppose that the blade rings have been set down provisionally, i.e. that the blade rings have been divided into groups of a certain radial depth b (Fig. 319) and the inner and outer radii and the number of rings in each group determined. Then the adiabatic heat drop in each group (or part of a group) may be estimated by using eq. 224. If a condition curve similar to Fig. 100 has been drawn in accordance with the specified steam conditions, then, knowing the distribution of adiabatic heat drop, the steam pressures and specific volumes may be obtained at inlet to, and outlet from, each group.

Suppose that at outlet from a particular blade ring v is the specific volume of the steam, V is the relative velocity, l is the axial length of the blades (measured in foot units), and x the "opening" per blade, Fig. 322. Then if there are m blades in the ring—

$$m \cdot x \cdot l = \frac{w \cdot v}{V}$$

or

$$l = \frac{w \cdot v}{m \cdot x \cdot V}$$

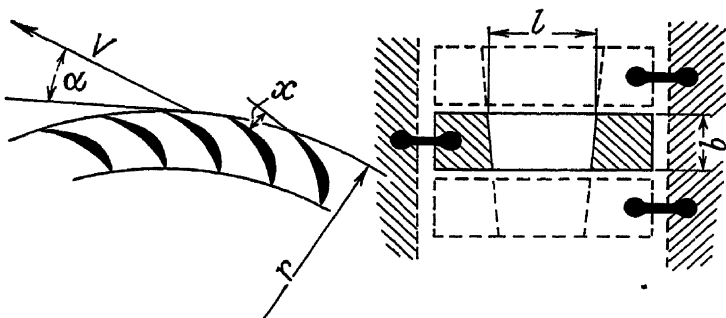


FIG. 322.—Diagram Illustrating Calculation of Blade Lengths

If the thickness of the blade outlet edge be t , then

$$x = \frac{2\pi r \cdot \sin \alpha}{m} - t$$

from which

$$l = \frac{wv}{V(2\pi r \cdot \sin \alpha - mt)} \quad (226)$$

If the thickness t be neglected—

$$l = \frac{wv}{2\pi \cdot r \cdot V \cdot \sin \alpha} \quad (226a)$$

Owing to inertia forces, each blade is in the condition of a beam subjected to a uniformly distributed force and more or less rigidly fixed at the ends in the relatively stiff supporting rings. This produces a bending action, to which must be added the bending due to impulse and reaction. Consequently, if the ratio l/b exceeds a certain value, which will depend mainly on the peripheral velocity of the ring in question, the bending stresses will become excessive and the effective steam-carrying part of the blade must be divided into two or more parts, each suitably strengthened by the supporting rings. Examples of this are given later in this chapter.

236. Relationship between Cumulative Heat Drop, Speed of Rotation, Number of Blade Rings, and Diameter of Blade Rings in the Ljungström Turbine. It has been shown that the adiabatic heat drop in the n th blade ring is

$$\Delta H_n = \frac{1}{2gJ \cdot \eta_n} \{V_n^2 - k^2 \cdot U_{n-1}^2 - (u_n^2 - u_{n-1}^2)\}$$

But $U_{n-1}^2 = V_{n-1}^2 + 4u_{n-1}^2 - 4u_{n-1} \cdot V_{n-1} \cdot \cos \alpha$

$$\therefore \Delta H_n = \frac{1}{2gJ \cdot \eta_n} \{V_n^2 - k^2(V_{n-1}^2 + 4u_{n-1}^2 - 4u_{n-1}V_{n-1} \cdot \cos \alpha) - (u_n^2 - u_{n-1}^2)\}$$

Writing $\chi = 1 - \frac{b}{r}$ (see Fig. 319) and assuming that ρ is uniform throughout the turbine, then—

$$u_{n-1} = \chi \cdot u_n; u_n = \rho \cdot V_n; u_{n-1} = \rho \cdot V_{n-1}; \therefore V_{n-1} = \chi \cdot V_n.$$

$$\therefore \Delta H_n = \frac{V_n^2}{2gJ\eta_n} \{1 - k^2\chi^2(1 + 4\rho^2 - 4\rho \cos \alpha) - \rho^2(1 - \chi^2)\}$$

i.e. $\Delta H_n = \frac{V_n^2}{2gJ} \cdot f_3(\rho)$ (227)

where $f_3(\rho) = \frac{1}{\eta_n} \{1 - k^2\chi^2(1 + 4\rho^2 - 4\rho \cos \alpha) - \rho^2(1 - \chi^2)\}$ (228)

$f_3(\rho)$ is a function of η_n , k , χ , α , and ρ . If η_n , k , χ , and α are given constant values, then $f_3(\rho)$ is a function of ρ only.

It follows that if D is the diameter of the n th ring in inches, and N is the speed of each rotor—

$$\rho = \frac{u_n}{V_n} = \frac{DN}{229} \sqrt{\frac{f_3(\rho)}{2g \cdot J}} \sqrt{\frac{1}{\Delta H_n}}$$

Therefore, for the whole turbine—

$$\rho = \frac{1}{51 \cdot 2} \sqrt{f_3(\rho)} \sqrt{\frac{K}{H_e}} \quad . \quad . \quad . \quad (229)$$

where K is the Parsons coefficient for the turbine and equals

$$\sum_0^n \left(\frac{D}{10}\right)^2 \left(\frac{N}{100}\right)^2$$

where n is now the total number of rings (both wheels).

Values of $f_3(\rho)$ are given in Table XVIII for various values of b/r .

In the same table are given values of $\frac{K}{H_0}$ for values of ρ , based on a value of $b/r = 0.04$, which is about the mean value of this ratio in actual Ljungström turbines. Fig. 323 shows the calculated values of the gross stage efficiency plotted against values of the efficiency factor.

237. General Description. The Ljungström turbine was invented and designed by two Swedish brother engineers, Dr. Birger and Mr. Fredrik Ljungström, about the year 1912. The new machine was a

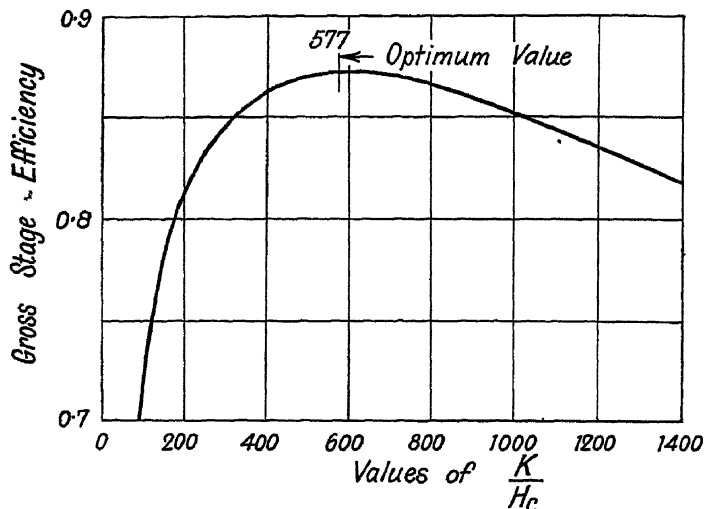
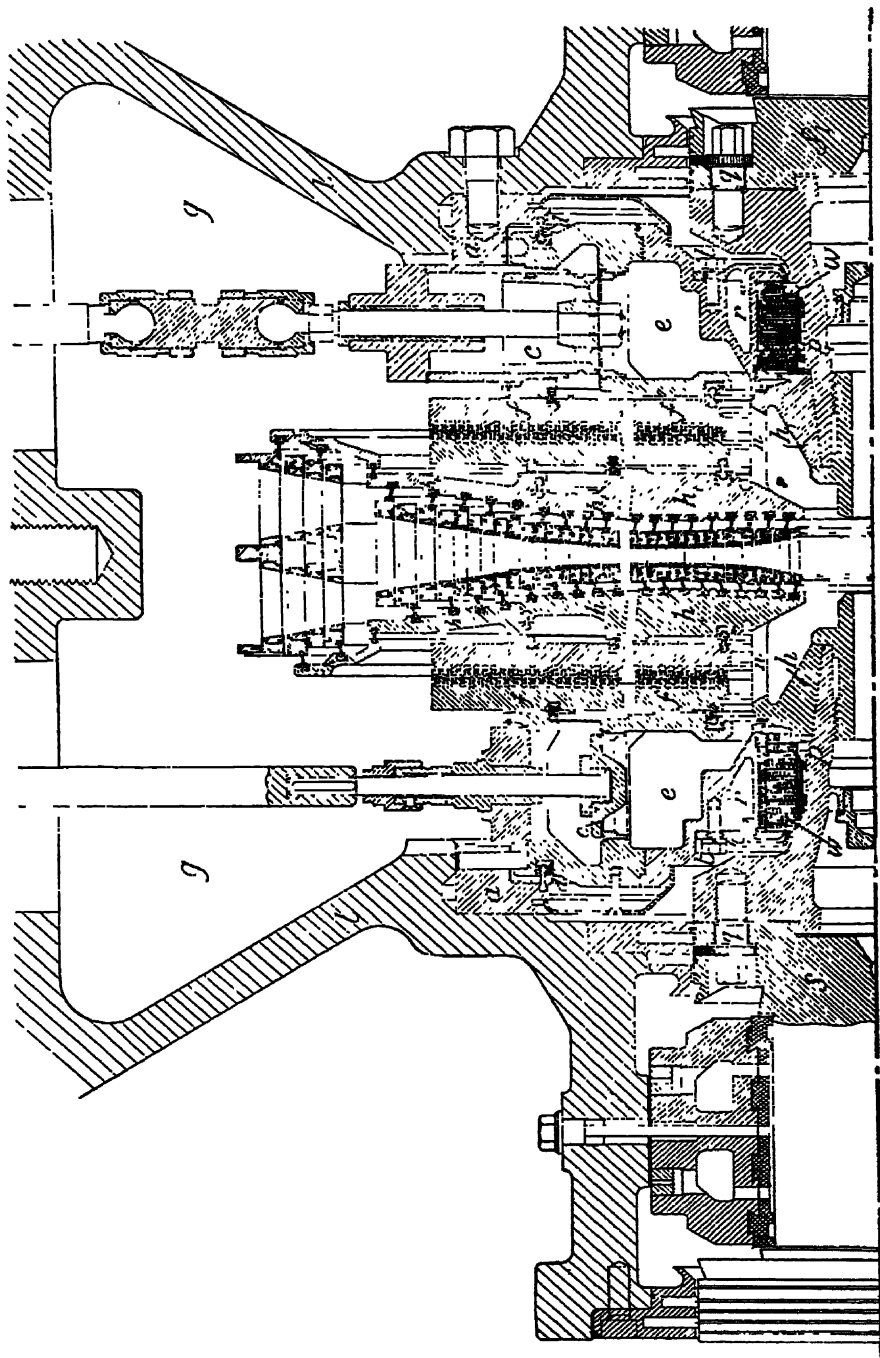


FIG. 323.—Calculated Stage Efficiency of Radial-flow Double-motion Turbine against Efficiency Factor

radical departure from the then existing ideas of steam turbine construction, and embodied several ingenious devices to reduce steam leakage and damage due to differential expansion, and also to minimize heat losses by conduction and radiation. So well conceived was the original design that to-day, after over thirty years' experience, it stands almost unaltered except to meet the needs imposed by its application to larger powers. It is constructed in Sweden by the parent company, the Svenska Turbinfabriks Aktiebolaget Ljungström, and in England by the Brush Electrical Engineering Co., Ltd., of Loughborough.

The reader who is unfamiliar with the general features of the Ljungström turbine is asked to read Art. 9 and study Fig. 7 before reading the following description.

The construction adopted depends, to some extent, on the output of the turbine. Fig. 324 shows a section through the upper half of the blade system of a 1,875 kW turbine. The steam chests *e* (right and left) are flexibly mounted in the flanges *a* by the dumb-bell



joint rings d , symmetrical expansion of the steam chests being further ensured by radial locating pins. The fixed balance discs f , with their labyrinth packing, are secured to the steam chests by expansion rings.

Very short turbine spindles p are spigoted into the alternator rotor shafts s and s_1 and carry the turbine rotors h and h_1 . As will be seen from the figure, the alternate blade rings are carried on each rotor, the blade rings also being attached to the rotor discs by means of dumb-bell expansion rings. The rotating balance discs are connected to the rotor discs by the same method.

Steam enters the turbine through the side of the exhaust branch, Fig. 325, and passes through branch pipes into the lower part of the steam chests. Under normal load conditions the steam flows into the inner part e of the steam chests, through ports n cut in the hub of the rotors and so to the centre of the blade system. The steam, under the difference of pressure between the steam chest and the surrounding exhaust space g , then flows radially outwards through the blade system.

The variation in blade length may be noted. At first, the growth of area due to the increasing diameter is greater than that required due to the increasing volume of steam, bearing in mind that the steam velocity also increases as the diameter increases. In the later stages, however, the rapid increase in steam volume calls for longer blades. As a result the blade length diminishes up to about the eighteenth stage, thereafter it increases. It may be said that the variation in blade length is not directly related to the shape of a convergent-divergent nozzle because, whereas in the nozzle the steam velocity is a continuous function of the pressure, in the turbine it is not.

At the thirty-sixth blade ring the safe limit of blade length has been reached, bearing in mind the stresses due to bending, and in the remaining four stages the blades are each subdivided into two parts of equal length.

For loads above the economical load, either the right-hand overload valve c is automatically opened by the governor or the hand-operated left-hand overload valve c is opened. In either case, steam is admitted, in amount depending on the overload, through openings in the balance discs and rotor discs to the space after the eighteenth blade ring.

Leakage of steam from the steam chest to the atmosphere is reduced to a very small amount by the labyrinth packing w . Details of this will be given in Art. 241.

The employment of two or more blades in parallel in the low-pressure part of the blade system has already been mentioned. Fig. 326 shows a somewhat old design of a 5,000 kW turbine by the Svenska Turbinfabriks Aktiebolaget Ljungström of Finspong, Sweden, in which the blades of the last six stages are subdivided

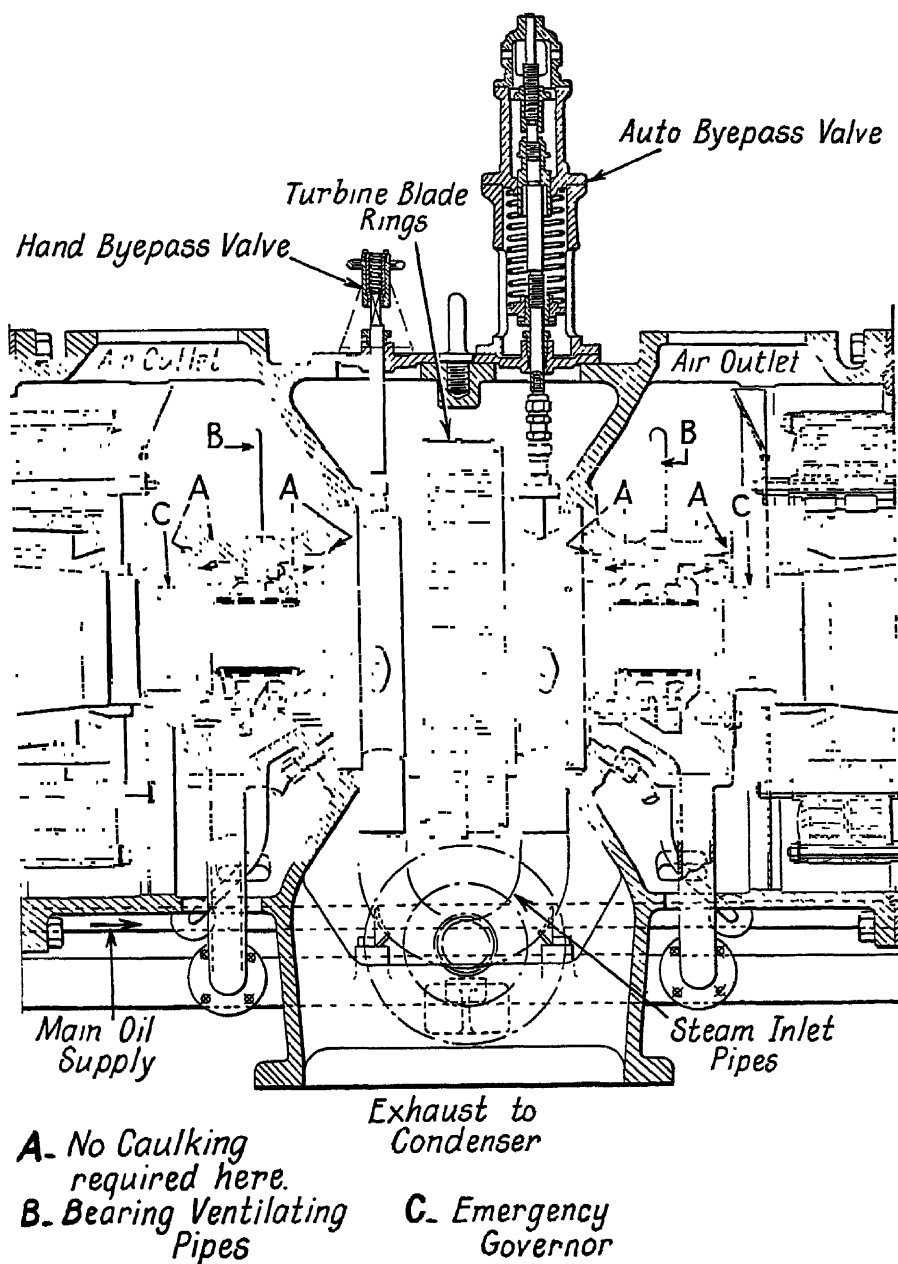


FIG. 325.—Part Section of 1,875 kW Brush Ljungström Turbine

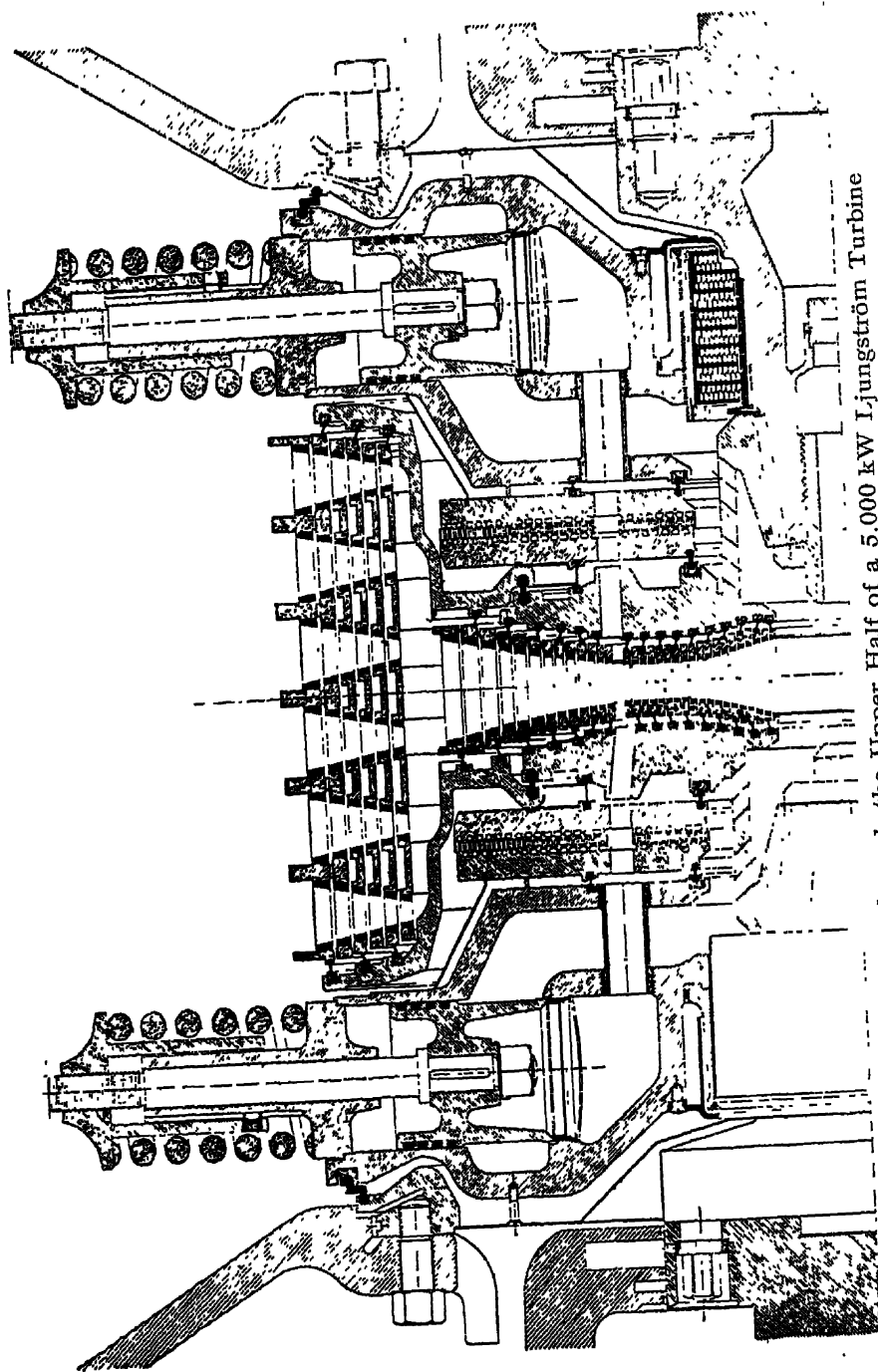


FIG. 326.—Section through the Upper Half of a 5,000 kW Ljungström Turbine

into six sections. The inertia forces on the blades are borne by the intermediate supporting rings to which the blades are attached. These are, of course, continuous rings. It will be observed that in the outer rings where the axial length of the supporting rings is reduced, owing to the greater length of the working blades, the necessary area of the ring is obtained by increasing its depth.

There is obviously a limit to the extent to which the L.P. blades may be subdivided, and in the more recent designs the necessary area through the L.P. blades is obtained by the use of one or two stages of axial-flow blading. Fig. 327 shows a cross-section through a Brush Ljungström turbine of 7,000 kW capacity in which a disc carrying one ring of axial-flow impulse-reaction blading is attached to each rotor by a number of taper pins. The final stage is thus a double-flow reaction turbine.

The general arrangement of the turbine is shown in Fig. 328. The turbine proper is supported on the exhaust flange of the condenser, the only foundations required being those for the condenser. Each turbine rotor is coupled direct to an alternator rotor. The alternator frames are overhung, but the overhung weight is taken by four steel struts which transmit the load to heavy brackets on the condenser. The two alternators operate in parallel. An exciter is arranged at one end of the set, the governor being located at the other.

A further application of the same principle is illustrated in Fig. 329, which shows a section through the upper half of a 15,000 kW 3,000 r.p.m. Brush Ljungström turbine. A low-pressure cylinder A, surrounding the radial-flow blading system, is fitted with two sets of nozzle vanes or fixed blades B and D, whilst each rotor carries a ring of blades C and E mounted on the stiff discs F and G respectively. Sections through the blading and nozzles are shown in Fig. 329.

Steam is tapped off for regenerative feed-heating at three points, namely, H (high-pressure steam), I (intermediate-pressure steam), and from L for low-pressure steam.

Designs have been produced for Ljungström turbines to develop 30,000 kW at 3,000 r.p.m. and 70,000 kW at 1,500 r.p.m.

238. Blade Construction. Two methods of constructing the blades and their supporting rings are employed. That used for turbines up to 4,000 kW is illustrated in Figs. 330 to 333. The blades themselves are milled out of round nickel steel bar, ground up in the direction of steam flow and afterwards cut off to the required length, a tongue of metal being left at each end, as shown by 17, Fig. 330. The blades are fitted into slots punched in discs (15) which have previously been turned to the channel section shown in the same illustration. The blades are held at the correct angle and distance apart by means of thin sheet iron strips (20) and then welded into the discs. After the sheet iron strips have been removed, the welding discs are turned

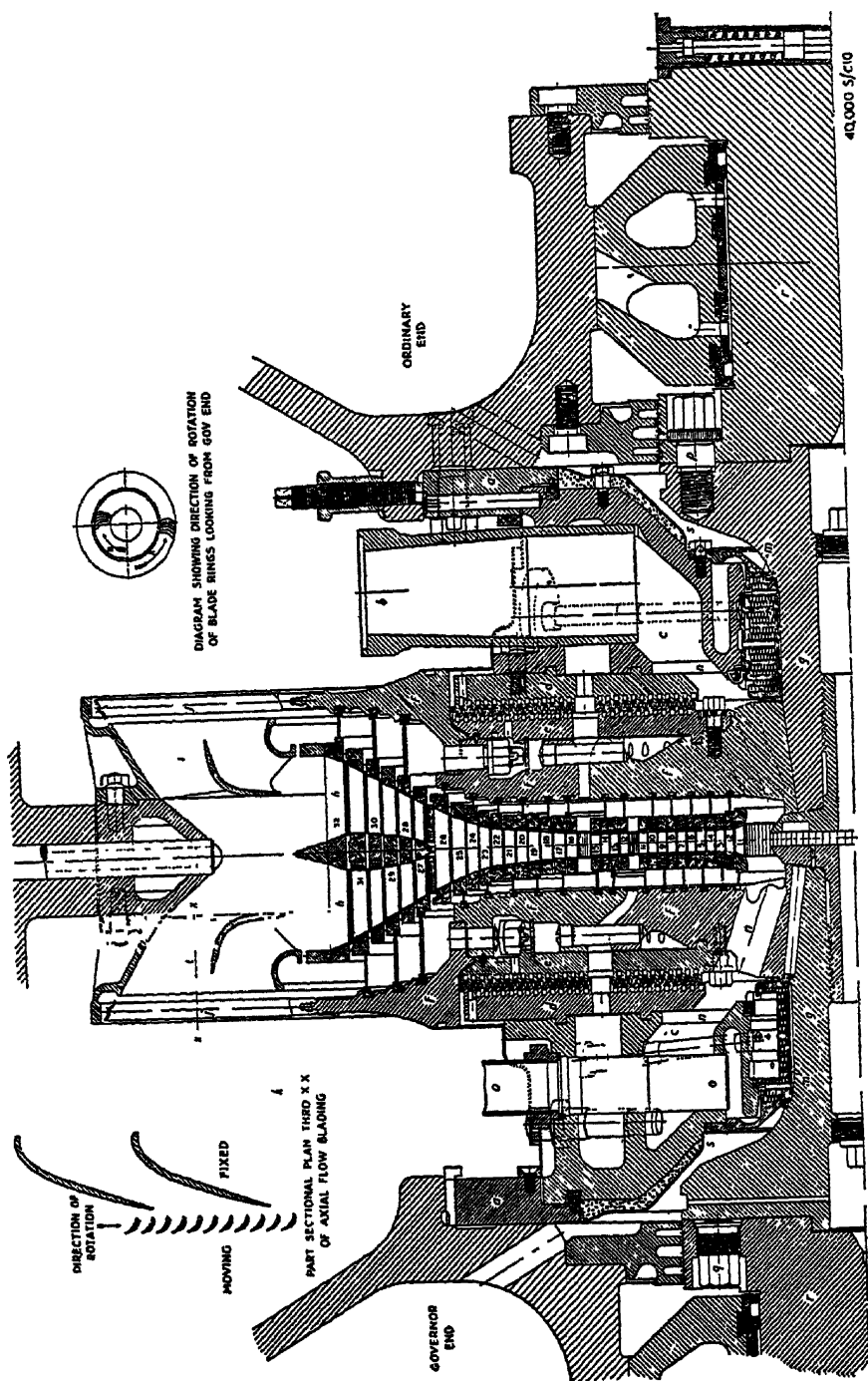


FIG. 327

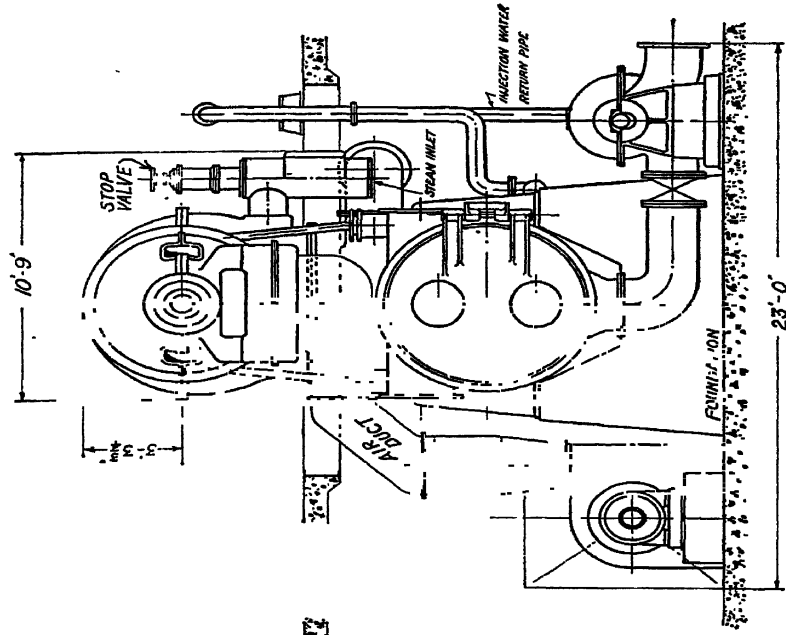
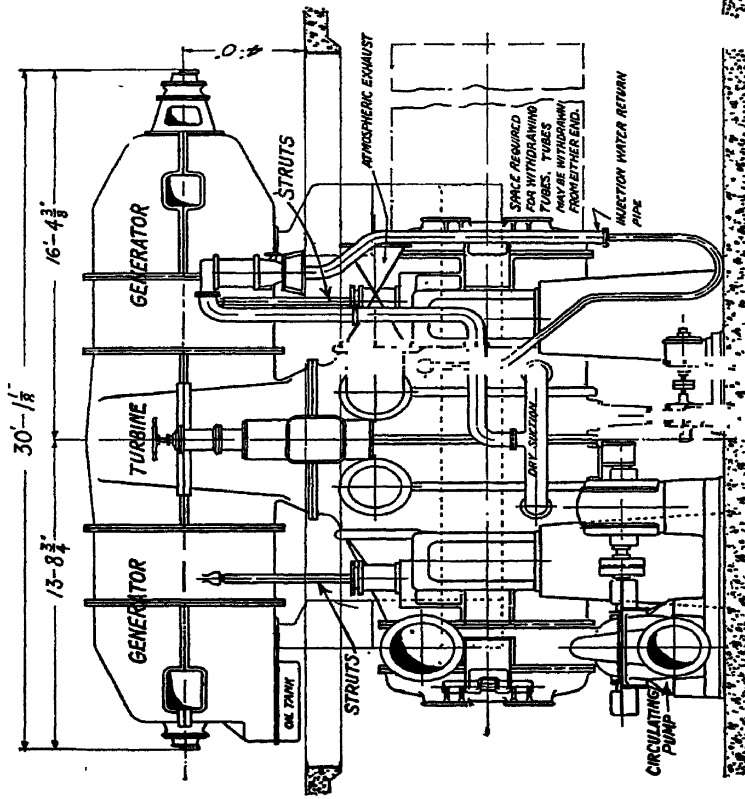


Fig. 328.—General Arrangement of 7,000 kW Ljungström Turbine

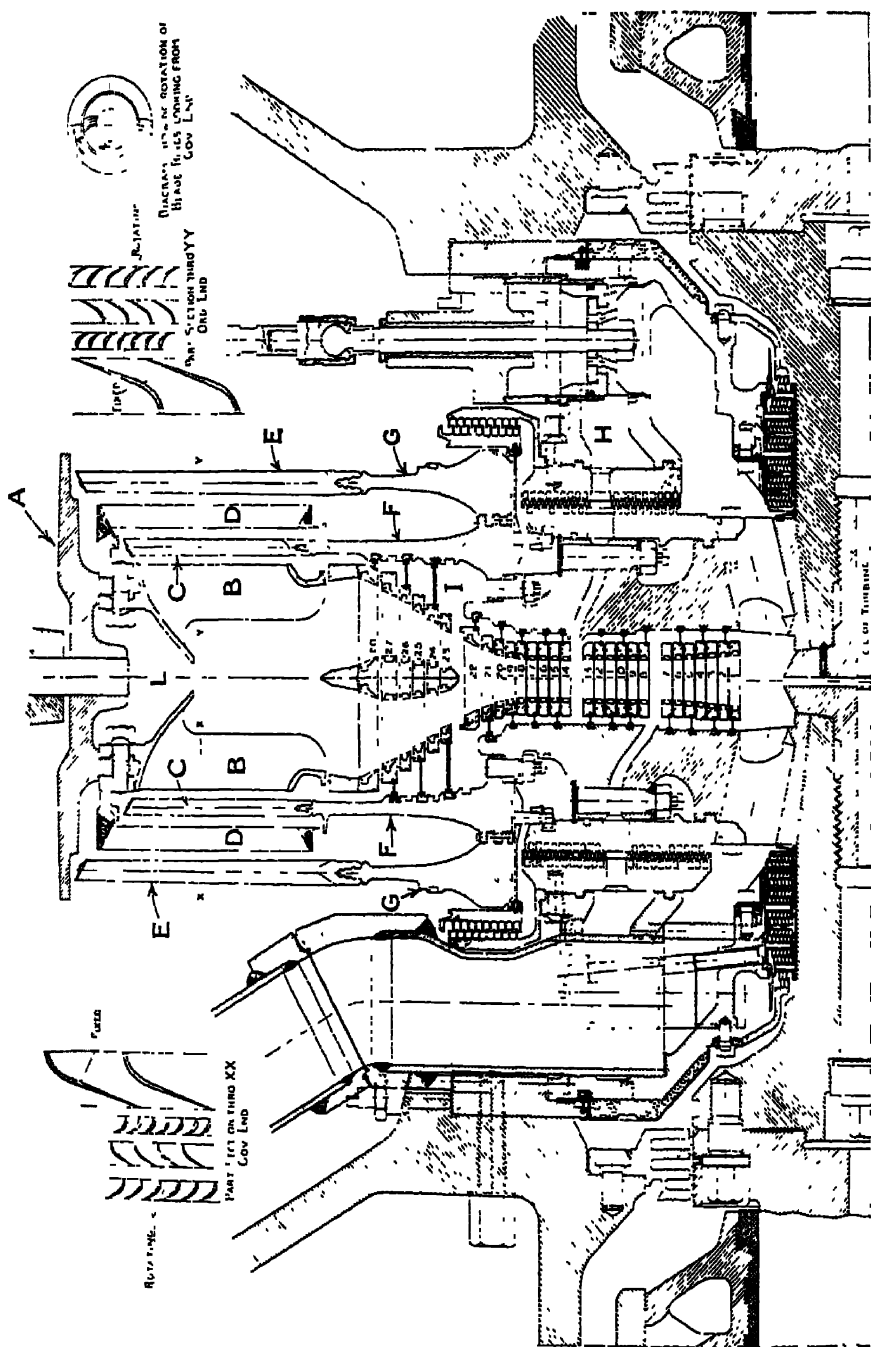


FIG. 329.—Section through Blade System of 15,000 kW Brush Ljungström Turbine

off so as to produce the dovetail profile indicated in Fig. 331. This dovetail consists partly of blade metal, welding ring, and partly of

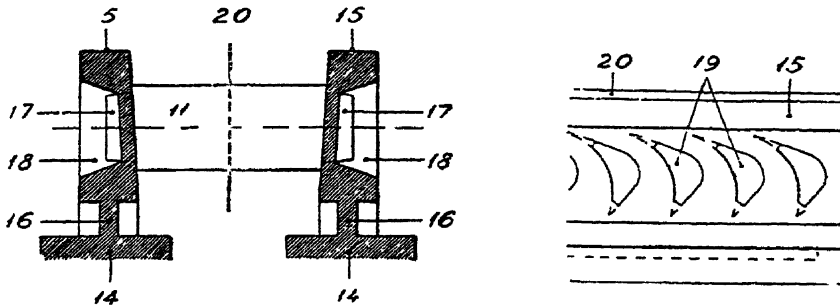


FIG. 330

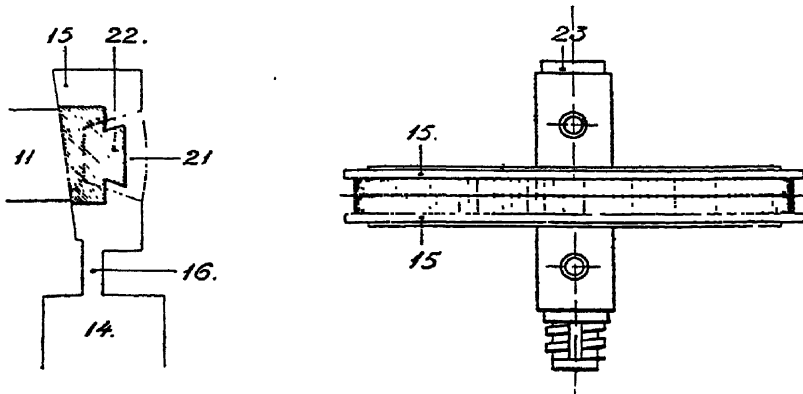


FIG. 331

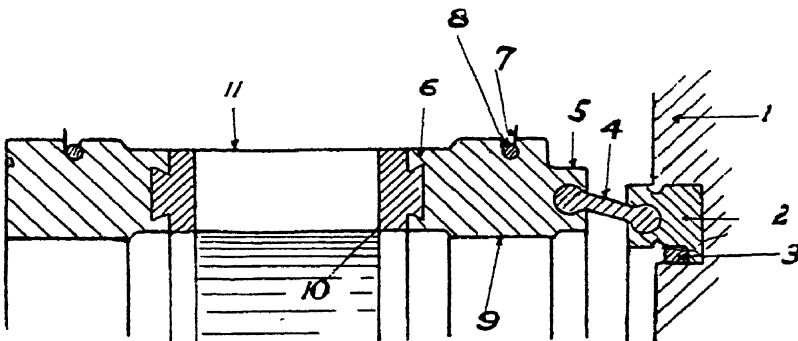


FIG. 332.—Section of Finished Blade Ring

weld metal, and has insufficient strength to resist the inertia forces. In the completed blade ring, Fig. 332, the inertia forces are taken by the supporting rings 9. The complete blade ring is attached to the rotor disc by the dumb-bell ring 4, one end of the latter being gripped

by the supporting ring 9, the other end by the ring 2, which is secured by the caulking strips 3. A section of the supporting ring before rolling is shown in Fig. 333. The projecting edges *a* and *b* are rolled

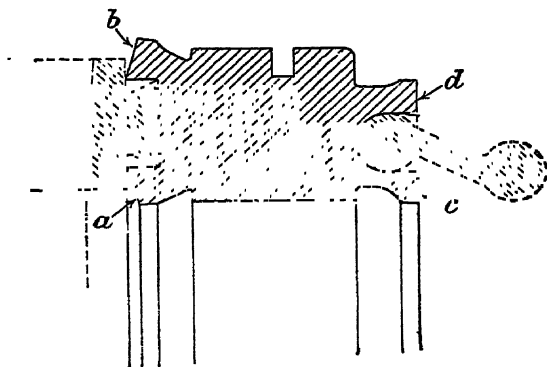


FIG. 333.—Section of Strengthening Ring before Rolling

over to grip the dovetail of the blades; the edges *c* and *d* being likewise rolled over to grip the expansion ring.

Steam leakage from one ring to the next is reduced to the minimum by the packing strips 7, Fig. 332, of nickel or steel, which are held in grooves by the caulking strips 8.

The dumb-bell expansion ring, which is such a feature of the Ljungström turbine, enables contiguous parts of the turbine to

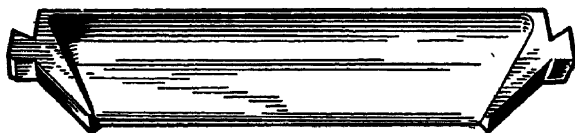


FIG. 334.—New Blade Construction

adjust their positions relative to one another when their dimensions are changed due to the effects of temperature and stress and yet maintain perfect alignment. Moreover, the small cross-section of the ring reduces the heat conduction to small limits.

In turbines for outputs of 5,000 kW and over, the blade and its dovetail are milled either from solid bar or drop forgings (Fig. 334). The blades are afterwards assembled in the supporting rings to form a complete circle (Fig. 335), the dovetails being gripped by rolling. As there is no welding of the blades in this construction, heat-treated materials or stainless steel can be used for the blades, according to requirements. These blade materials make it possible to utilize longer blades and, consequently, a smaller number of blades in parallel in the L.P. region. Thus, even with the largest outputs, only two blades are now used in parallel. If necessary, two or even more axial-flow stages are used to provide the necessary steam way.

239. Disc Construction. In a radial-flow turbine working under steady conditions, the centre portions of the disc may be subjected to a steam temperature of 700° F. to 800° F., while the temperature of the outer part of the disc would normally be in the region of 80° F. to 100° F. In a disc constructed in one piece such temperature differences would produce considerable stresses, quite apart from the stresses due to rotation.

Fig. 336 shows the disc construction employed on the smaller turbines. As will be seen, the complete wheel is comprised of a centre part including the hub, together with two other annular

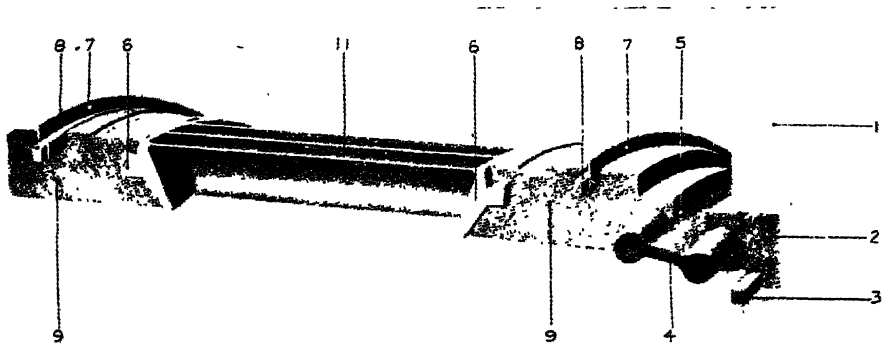


FIG. 335.—Section through Ljungström Turbine Blade Ring

rings attached by dumb-bell expansion rings (1). One side of the disc is grooved to receive the blade rings, while the other side is arranged to receive the labyrinth disc. There are openings (2) in the hub through which steam flows into the centre of the blade system.

Reference to Figs. 324, 326, 327, and 329 will show that the short overhung turbine shaft, which is subjected to the heating action of the live steam, is made hollow so that the fluctuations in temperature of the spindle and hub will be practically simultaneous and relative movement will be avoided.

The disc construction in the larger sets is well shown in Fig. 329. The first 22 stages of radial-flow blading are carried on two discs which are connected together by a series of radial taper bolts. The taper of the bolts has been chosen so as to maintain a practically constant fit, independent of the differential expansion of the connected parts due to stress and temperature variations. The last three pairs of rings are attached to the sides of the discs F, which carry the first stage of axial-flow blading. The discs F are also connected to the radial-flow discs by short radial taper bolts. Alternating with these bolt holes are radial holes which enable bled steam to

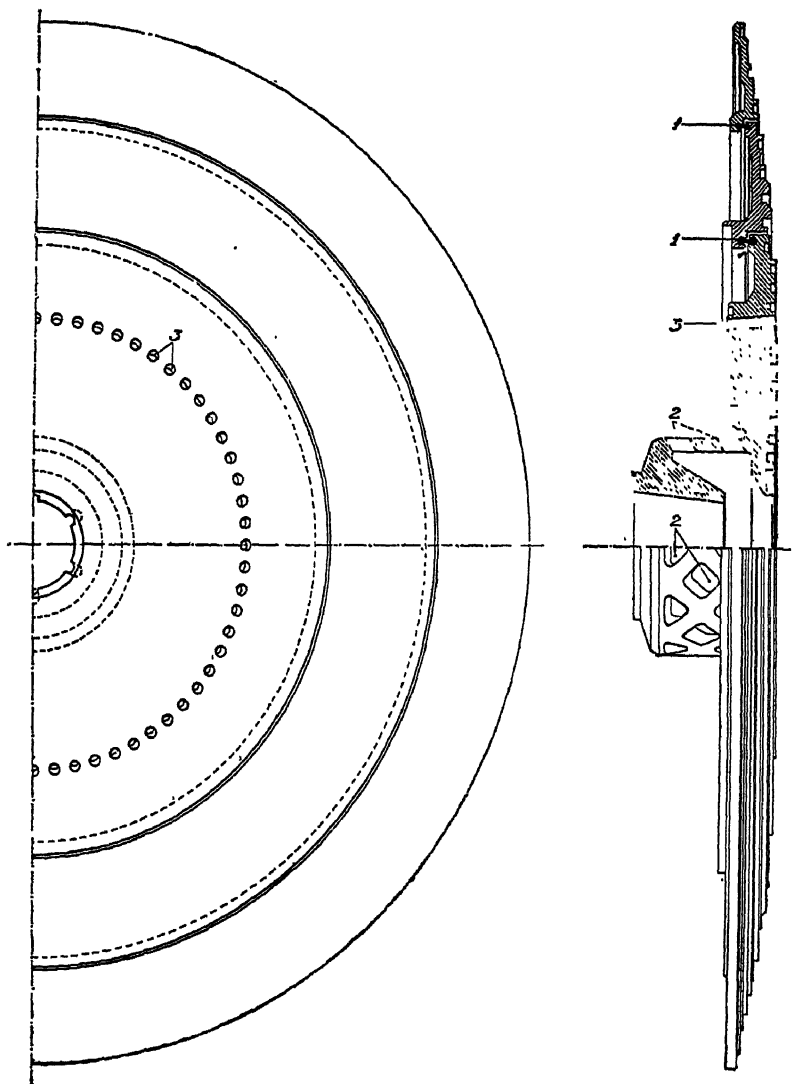


FIG. 336.—Ljungström Turbine Disc

be drawn off from the space I. The last disc on each rotor is connected to its neighbour by triple registers and a series of parallel driving pins.

240. Balancing Arrangements. Reference has already been made in Art. 9 to the balancing discs attached to the steam chests and rotors. The labyrinth packing for these discs is shown in detail in

Fig. 337. A number of grooves are turned in the fixed and moving discs and a projecting ring between each pair of grooves fits into a groove on the opposite disc. Such projecting rings are furnished with a groove into which is fitted a thin strip of nickel having a fine clearance. The total number of constrictions is obviously twice the number of grooves. The temperature conditions are practically the same for the fixed and moving dummy discs, and

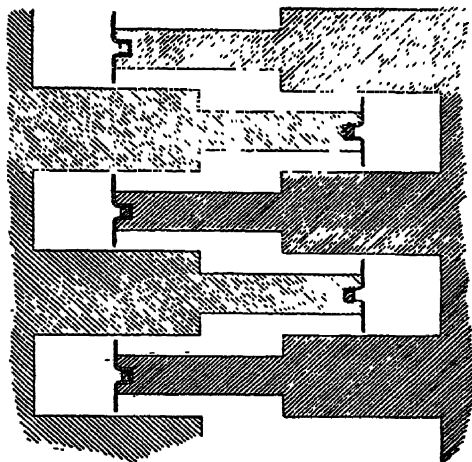


FIG. 337.—Labyrinth Packing for Ljungström Turbine Balancing Discs

hence the clearances will hardly vary and may, therefore, be made very small.

241. Shaft Glands. The shaft glands have to be packed against the full steam pressure and are one of the most ingenious features of the turbine. Each gland comprises a number of fixed and rotating elements finished to the section shown in Fig. 338. The rings are turned out of solid steel, the rotating and fixed rings being mounted on feathers 6 and 7 respectively. The edges of the rings are turned to a thin section and are then rolled inwards at an angle of 45° , so that when the rings are in position, the rolled edges make contact with the neighbouring rings. The first few revolutions of the turbine are sufficient to wear a very slight clearance, which has no tendency to increase. In the 7,000 kW Brush Ljungström turbine, the gland provides for no less than 107 throttlings of the steam, and yet its axial length is only $2\frac{3}{8}$ in.

The steam escaping from the last constriction is arranged to blow into a special pocket, such as *r* in Fig. 324 (see also Fig. 327). The escaping steam, leaving the last constriction of the packing with a high velocity, produces an ejector action and so draws air into the said pocket. The mixture of air and steam is led away to a separate

condenser through the tubes of which the main condensate flows. The object of this arrangement is to prevent the escaping steam from leaking into the restricted space between the gland and the main bearing. In some cases, arrangements are made for supplying

live steam for sealing purposes when starting up and when operating at very light loads.

242. General Notes. The Ljungström turbine has been constructed in units up to 15,000 kW with a speed of 3,000 r.p.m. for each rotor, and in units up to 50,000 kW with a rotor speed of 1,500 r.p.m.

There are no heavy castings subjected to high pressures and temperatures which would require careful warming up in order to avoid distortion. The blade rings are of comparatively small mass and will quickly respond to changes in steam temperature. Moreover, the difference in diameter between adjacent rings is small, so that the relative movements due to temperature and stress are very small indeed. As a result, the

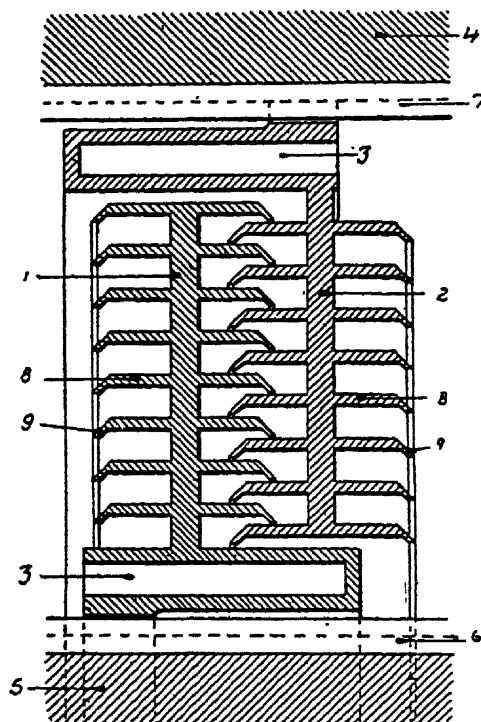


FIG. 338.—Labyrinth Shaft Packing for Ljungström Turbine

turbine may be started and put on load without any preliminary heating. The 7,000 kW turbine has been run up in less than five minutes from cold, and the 12,500 kW turbine has been started from cold and been ready for load in about twelve minutes. Consequently, the Ljungström turbine is particularly suitable for a peak-load and reserve power station. There is such a station at Västerås in Sweden, and by special design of the auxiliaries to facilitate quick starting, the 50,000 kW turbine there may be started from cold and ready for load in four minutes.

A few results of tests on Ljungström turbines will be found in Chapter XXVII.

HIGH-PRESSURE TURBINES

243. Limitations Imposed by Properties of Materials. The desire of power engineers to attain to still higher thermal efficiencies in steam plants has led to the use of higher steam pressures and temperatures. In general, the theoretical thermal efficiency of a steam turbine is increased as the initial temperature of the steam is increased, provided the lower limit is kept constant. If we consider the somewhat impractical case of initially dry and saturated steam, however, the theoretical thermal efficiency increases as the initial temperature is increased until a temperature of about 650° F. is reached; beyond that temperature the efficiency diminishes as the temperature is further elevated.

The lower limits of temperature and pressure depend chiefly on the temperature and quantity of the available supply of cooling water, and for any given place this may be assumed to be roughly constant.

The upper limit of temperature, on the other hand, is circumscribed mainly by the mechanical properties, at elevated temperatures, of the materials used for superheater tubes, steam pipes, turbine throttle valves, and cylinders. The behaviour of these materials at high temperatures is a matter of the greatest importance. Research work is being done in all interested countries, and knowledge on this subject is being slowly and laboriously accumulated. Much has been done, much more remains to be done.

The ultimate strength of mild steel increases with increasing temperature up to about 400° F., but at higher temperatures it falls more or less rapidly, as shown by the curve in Fig. 339. The yield point also falls off rapidly as the temperature increases and, according to one investigation, shrinks to zero at 1,200° F., which means that very small stresses would produce a visible extension in the material. Certain alloys have physical properties which are superior to those of ordinary mild steel at high temperatures. Some of these have, however, presented difficulties in tube making. Test results for two such alloys are shown in Fig. 340,* one being Firth's heat-resisting chromium steel and the other known as Heraeus B.7.M.

Brief reference must now be made to another property of metals

* Reproduced from a paper, "High Pressures and Temperatures for Steam Prime Movers," by O. Wiberg. *Congrès International de Mécanique Générale, Liège, 1930, Vol. II, p. 118.*

when subjected to stress at a high temperature. It is the slow deformation of the metal. If a specimen of steel, for example, is kept at a given temperature, and a certain steady load is applied, it will be found that the extension will increase rapidly at first and then less rapidly. The subsequent behaviour of the specimen will

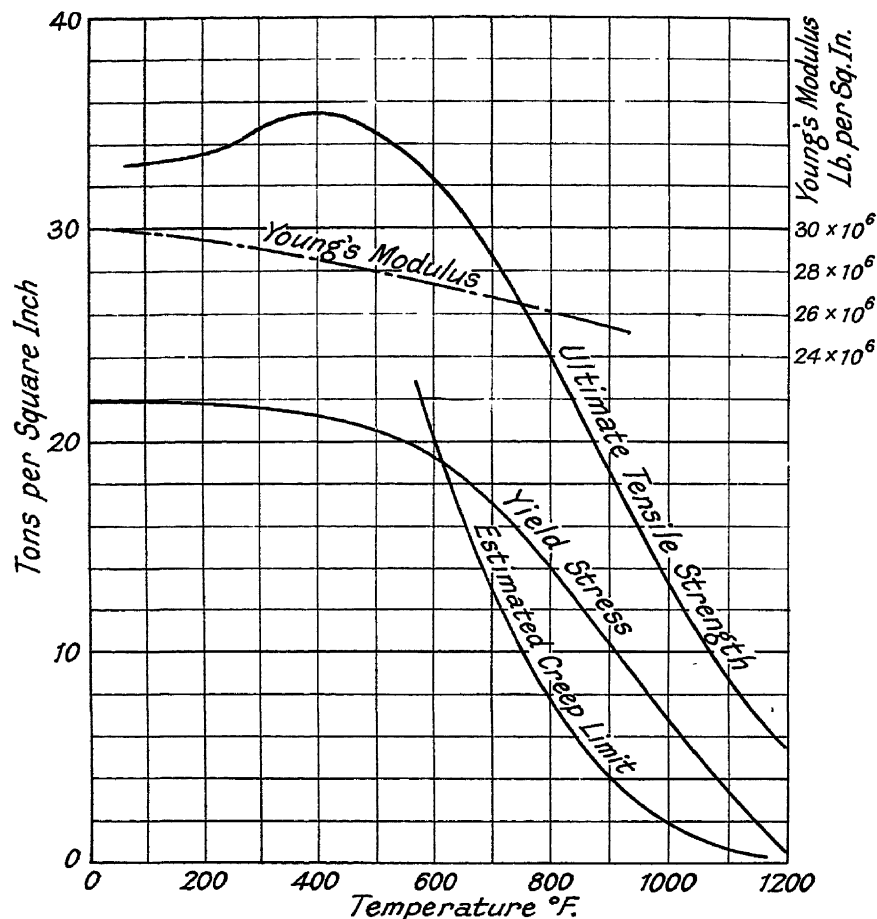


FIG. 339.—Mechanical Properties of 0.15 per cent Carbon Steel

depend on the temperature and on the stress. If the stress is above a certain amount, then the extension will continue, at a greater or lesser rate, until the specimen finally breaks. If the stress is lower than this particular amount, the extension will in time be retarded and may *appear* to cease. The phenomenon of slow continuous extension is usually termed "creep," and at the particular temperature considered the particular stress below which the extension

finally appears to cease has been termed the "creep limit." If such a creep limit actually existed, then it is clearly a very significant stress for the designer of high-temperature power plant; for then, provided the working stresses are kept below the creep limit, no continuous straining of the material will take place. Curves of estimated creep limit are shown in Figs. 339 and 340.

As the methods of measurement used have been made more and

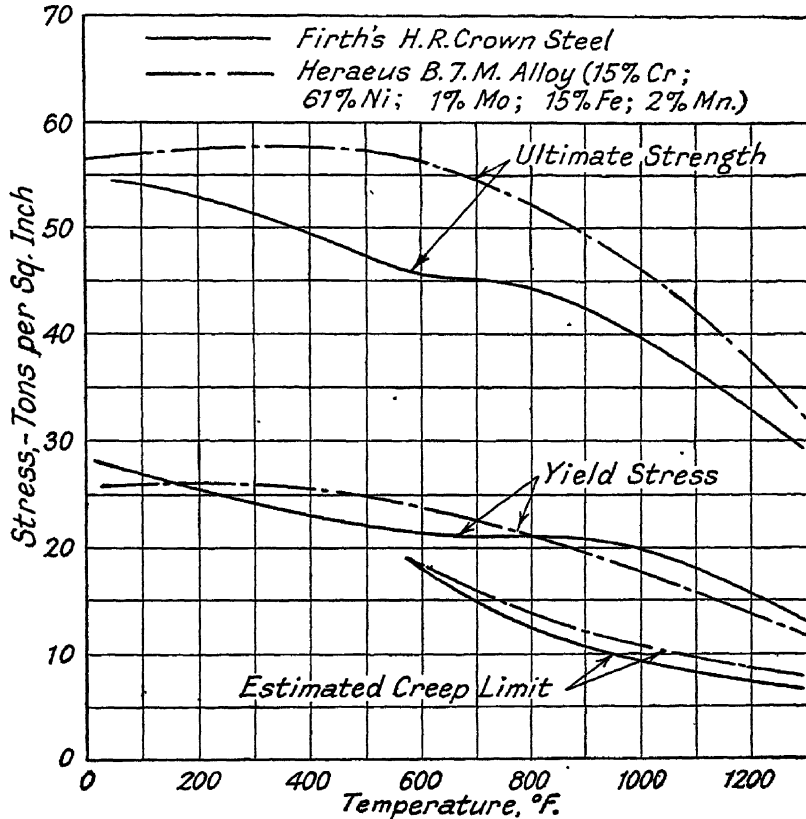


FIG. 340.—Mechanical Properties of Alloy Steels

more refined, and when the tests have been carried out for much longer times, it has become evident that creep is a continuous process, and therefore it seems doubtful whether a "creep limit" even exists. This is the view put forward by Dr. R. W. Bailey. In these circumstances, the conception of a creep limit must be replaced by that of a "creep rate." In other words, if a given material is maintained at a given temperature indefinitely, then at a given stress after the period of initial strain it will deform at a definite, if extremely small, rate. A typical set of such creep-rate-stress

curves* is shown in Fig. 341. Superheater tubes and steam pipes and even the high-pressure turbine cylinder will go on increasing in size with passing time, extremely slowly at the more moderate steam temperatures, but more rapidly at the higher temperatures.

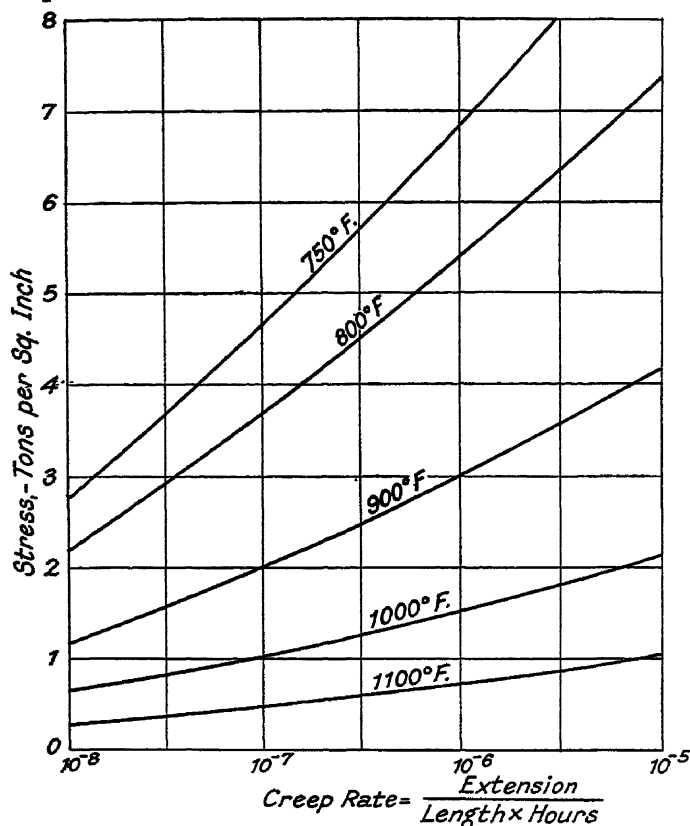


FIG. 341.—Creep Properties of 0.15 per cent Carbon Steel

Bolts which are used for tightening up high-pressure high-temperature steam pipe joints will, under the stress due to tightening, gradually increase in length until that stress is appreciably reduced and, unless the nuts are again tightened up, the joint will commence to leak.

For a masterly treatment of the application of creep data to the design of power plant parts the reader is referred to a paper (1) by Dr. R. W. Bailey.

Development in the use of high-pressure steam has proceeded along somewhat different lines in different countries. In the

* Reproduced from paper "Some Considerations affecting the Future Development of the Steam Cycle," by K. Baumann, *Proc. Inst. Mech. Eng.*, 1930, Vol. II, p 1305.

United States the tendency some years ago was to adopt higher steam pressures and moderate temperatures (about 750° F.), and in order to obviate difficulties due to the resulting high wetness of the exhaust steam, reheating was adopted. In Great Britain the tendency was towards higher steam temperatures and moderate

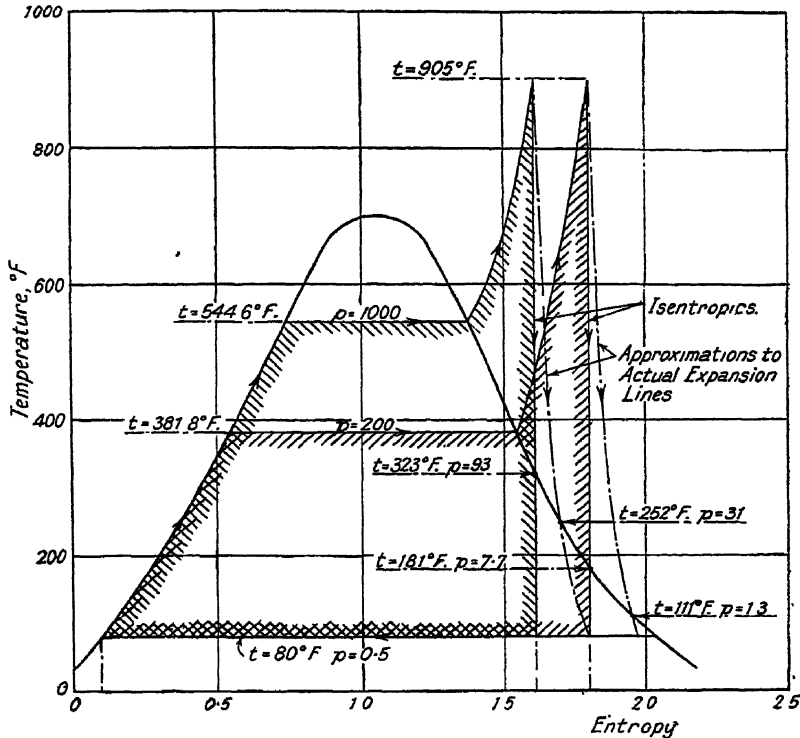


FIG. 342.—T- ϕ Diagram for Rankine Cycles with the same Initial Temperature and Initial Pressures of 200 and 1,000 lb. per sq. in. abs.

pressures without the added complication of reheating. In all cases regenerative feed-heating is employed. There are to-day many power stations in the United States operating at 1,200 lb. per sq. in. and with steam temperatures ranging from 850 to 950° F. Some years ago, the British Thomson-Houston Co., Ltd., constructed a 10,000 kW turbine to operate with steam at 365 lb. per sq. in. gauge and 1,000° F. This turbine was installed at Detroit. Further particulars, together with test results, will be given in Art. 254 of Chapter XXVII. In Art. 246 of the present chapter will be found particulars of a large turbine unit operating at 1,900 lb. per sq. in. and 930° F.

244. Effect of Steam Pressure on Thermal Efficiency for a given Upper Limit of Temperature. With the ideal working substance

visualized in the Carnot Cycle, all the heat absorption occurs at the upper limit of temperature, and the heat rejection takes place entirely at the lower limit of temperature, the thermal efficiency depending merely on the upper and lower temperature limits. With practical working substances, such as water, temperature is not the sole criterion of efficiency. Considering the usual steam turbine cycle, and ruling out the use of initially dry and saturated steam on practical grounds, it will be clear that it is possible to use various initial steam pressures with a given initial steam temperature. Thus, we may use a moderate pressure and a high initial superheat, or a very high initial pressure and a moderate superheat.

Thermodynamic considerations lead us to the conclusion that the greatest theoretical thermal efficiency will be obtained in that process in which the greatest possible amount of heat is supplied at the highest possible temperature. With a moderate pressure, owing to the larger latent heat of vaporization and the relatively small specific heat of the superheated steam, the proportion of the heat supplied at the high temperatures is less than in the case of the high initial pressure. This is borne out by calculations, of which an example is illustrated by Fig. 342.

The following data refer to two steam turbines operating with an initial temperature of 905° F. and an exhaust temperature of 80° F.

	Case I	Case II
Initial pressure . . . lb. per sq. in. (abs.)	200	1,000
Initial temperature . . . ° F.	905	905
Final pressure . . . lb. per sq. in. (abs.)	0.5	0.5
Final temperature . . . ° F.	80	80
Initial total heat . . . B.Th.U. per lb.	1,476	1,450
Final total heat after adiabatic expansion . . . B.Th.U. per lb.	972	866
Dryness fraction after adiabatic expansion . . .	0.884	0.784
Pressure at which steam becomes dry and saturated . . . lb. per sq. in. (abs.)	7.7	93
Adiabatic heat drop . . . B.Th.U. per lb.	504	584
Sensible heat of water at 80° F. . .	48	48
Heat supplied per lb. of steam . . .	1,428	1,402
Theoretical thermal efficiency . . .	0.353	0.416
Percentage gain due to increased pressure . . .	—	17.8
Efficiency of Carnot Cycle $\frac{(T_1 - T_2)}{T_1}$. . .	0.605	0.605
When turbine efficiency ratio is 0.82—		
Final dryness fraction . . .	0.97	0.884
Pressure at which steam becomes dry and saturated . . . lb. per sq. in. (abs.)	1.3	31

The gain in thermal efficiency due to the adoption of the higher pressure is at once apparent from a study of the figures in the above table. What is perhaps at first astonishing is that the heat necessary to generate a pound of steam is actually less at the higher pressure than it is at the lower; moreover, the heat drop is appreciably greater.

This increase in thermal efficiency is partly counterbalanced by two other effects of increasing the pressure. In the first place, the higher the pressure for a given initial temperature, the earlier does the steam become dry and saturated during its expansion, and the greater is the wetness fraction of the exhaust steam. This is evident from the $T-\phi$ chart, Fig. 342, and from the data set out above. In the lower pressure case, the steam remains dry almost to the end of the expansion, whereas in the other case it becomes wet in the actual turbine when the pressure has reached 31 lb. per sq. in. abs. This increased wetness at the exhaust end of the turbine is more serious still at more moderate initial temperatures, and is the cause of increased erosion of turbine blading and also of a certain reduction in turbine efficiency.

In the second place, the losses due to disc friction are likely to be higher with higher pressures owing to the increased density of the steam, unless the design is suitably modified to counteract it. The power absorbed in rotating a turbine disc in steam has been shown in Art. 80 to be—

$$H.P._d = \frac{c \cdot D^2 \cdot u^3}{v} = \frac{c_1 D^5 N^3}{v}$$

where the symbols have the usual meanings.

Now, if the ratio of blade speed to steam speed remains constant, then the heat drop per stage is equal to $c_2 \cdot u^2$, and if H is the total heat drop in the H.P. section of the turbine, then the number of stages is equal to

$$n = \frac{H}{c_2 u^2}$$

Then assuming, for the sake of simplicity, a constant specific volume v , the total disc friction horse-power—

$$\begin{aligned} \Sigma H.P._d &= \frac{H}{c_2 u^2} \cdot \frac{c \cdot D^2 \cdot u^3}{v} \\ &= \frac{c_3 \cdot D^2 \cdot u \cdot H}{v} \\ &= \frac{c_4 \cdot D^3 \cdot N \cdot H}{v} \end{aligned}$$

where c , c_1 , c_2 , etc., are appropriate constants.

Hence, if the speed of rotation N is kept constant, and if other things remain sensibly constant, the power absorbed by disc friction will vary directly as the cube of the disc diameter. In other words, if the blade speed is reduced to one-half, the total disc friction will be reduced to one-eighth, in spite of the fact that the number of wheels is increased four times.

Since $D \propto \frac{u}{N}$, we may also write

$$\Sigma H.P_d = c_5 \cdot \frac{u^3}{N^2} \cdot H$$

Thus the total disc friction loss, for a constant value of the blade velocity u , is inversely proportional to the square of the rotational speed, other things being sensibly constant.

It is clear, then, that in order to reduce the loss due to disc friction—

(a) in constant speed machines, the disc diameters must be reduced and the number of stages increased to cope with the heat drop;

(b) in machines designed for very high steam pressures, the rotational speed may be increased, either with or without an attendant reduction in blade speed.

The above somewhat rough treatment is not strictly accurate because where one stage is replaced, say, by four, the smaller heat drops in the latter case involve somewhat higher stage pressures and therefore higher steam densities; but the treatment is correct as a guide.

Further advantages follow the use of small diameters in super-high-pressure turbines. Complete admission, with consequently reduced vane-action losses, may be obtained. The casing is smaller and may sometimes be made from a steel forging so that it is more suited to withstand the high temperatures and pressures, and is less liable to give trouble due to distortion. Also in many cases the rotor may be machined from a single forging, thus enabling smaller diaphragm glands to be used for a given shaft critical speed. For examples of this construction see Figs. 345, 346, and 348.

245. Primary Turbines. Many of the super-high-pressure turbines so far installed are so-called "primary turbines." The more or less tentative adoption of high pressures in existing power stations has frequently taken the form of the installation of a high-pressure boiler supplying steam to the primary turbine, while the latter exhausts into the ordinary steam main of the station. In this way, experience could be gained in the generation and utilization of high-pressure steam without the necessity of scrapping the existing plant, and at the same time the capacity of the station could be increased. In such cases the high-pressure turbine is used as a base-load unit,

the "normal-pressure" turbines only being loaded as required; by this procedure the thermal efficiency of the station is maintained at a high value.

In newer stations, or in the case of large extensions, complete and independent high-pressure units are being installed. As an example of the latter, the 53,000 kW Metropolitan-Vickers turbine, described in Art. 246, may be cited.

In 1925 Messrs. Brown Boveri & Co. installed in the Langerbrugge power station of the Centrales Electriques des Flandres a

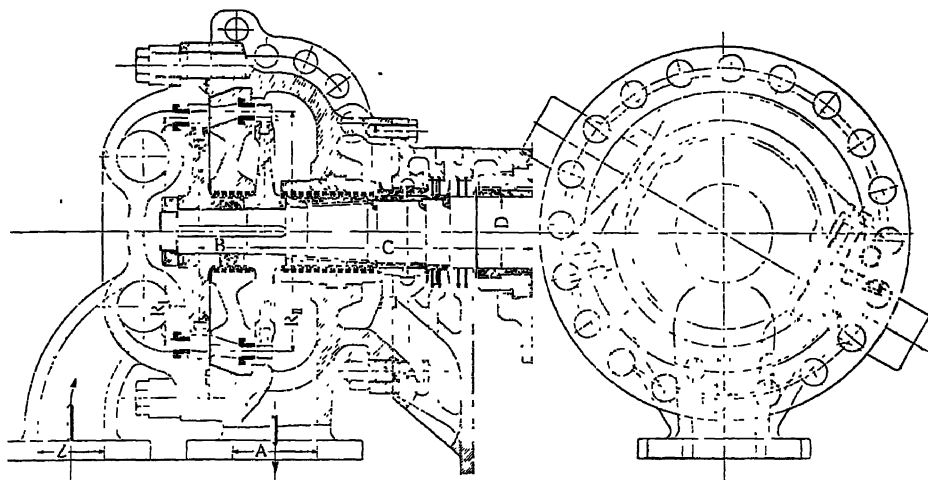


FIG. 343.—Two-stage H.P. Primary Turbine

Initial pressure—725 lb. per sq. in. 1,675 kW

primary turbine to develop 1,675 kW. Steam is supplied at a pressure of 725 lb. per sq. in. abs. and a temperature of about 830° F., while it is exhausted at 306 lb. per sq. in. abs. to be used in the existing turbines. The primary unit comprises two small turbines in series, each similar to that shown in Fig. 343. Each rotor carries two single-row impulse wheels 14.65 in. mean diameter. These run at 8,000 r.p.m., and drive an alternator at 1,500 r.p.m. through reduction gearing. A view of the set before lagging is shown in Fig. 344.

By using an overhung shaft, half of the high-pressure glands are eliminated. In order to cool the shaft, water is supplied under pressure to a chamber on the outlet side of the shaft gland. A leak-off and ample baffling prevent this water from finding its way into the bearing.

The more usual methods of construction have been followed in a later turbine built by the same firm for the same power station and utilizing the highest steam pressure so far used. The steam pressure

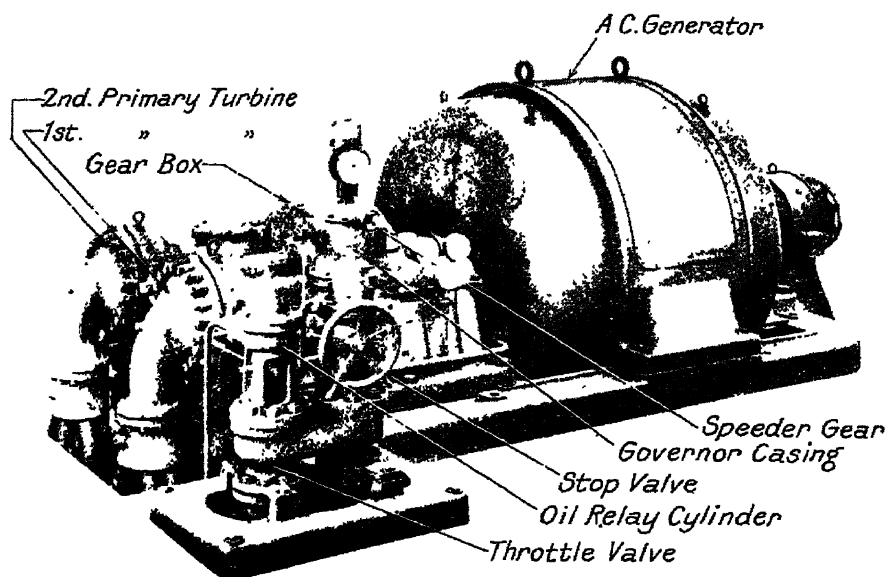


FIG. 344.—Brown-Boveri H.P. Primary Turbine

Initial pressure = 725 lb. per sq. in. abs.
 Exhaust pressure = 306 lb. per sq. in. abs.
 1,675 kW

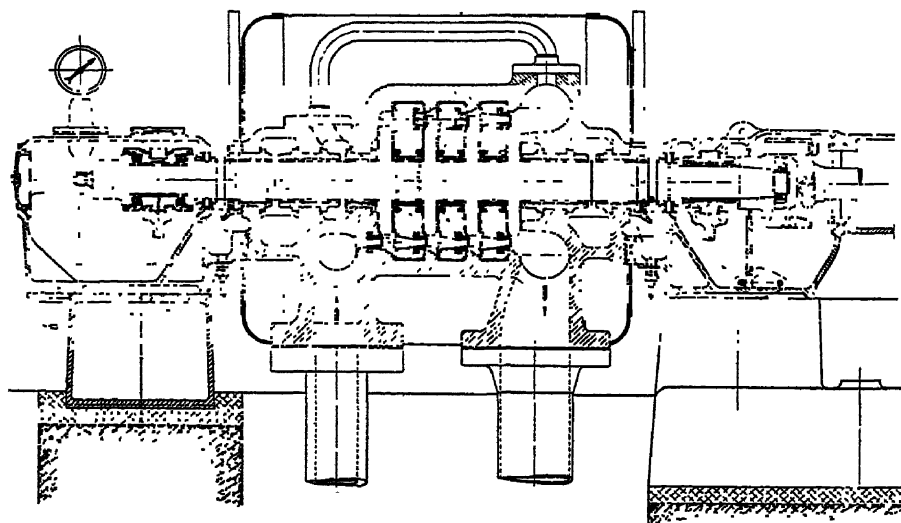


FIG. 345.—Section through Brown-Boveri High-pressure Primary Turbine in Langerbrugge Power Station of the Compagnie des Centrales Electriques des Flandres et du Brabant. 4,000 kW, 2,940 lb. per sq. in. abs., 806° F.

is 2,940 lb. per sq. in. abs. and the temperature 806° F. The turbine, Fig. 345, expands the steam in four stages to a final pressure of 882 lb. per sq. in. abs., after which the exhaust steam is reheated and admitted to the 800 lb. steam system. Steam is generated in a Benson boiler with a capacity of about 250,000 lb. per hour. The four impulse wheels have a mean diameter of 14.75 in. and develop 4,000 kW at 7,000 r.p.m.

A further example of a primary turbine is illustrated in Fig. 346. This machine was constructed by Messrs. Escher Wyss & Co., Zürich, for the Siemens-Schuckertwerke, Berlin, to operate in conjunction with an experimental Benson boiler. The following particulars relate to the operating conditions, etc.—

Output (D.C. machine)	1,000 kW
Initial steam pressure	1,415 lb./sq. in. (abs.)
Initial steam temperature	750° F.
Exhaust pressure	200 lb./sq. in. (abs.)
Turbine speed	10,000 r.p.m.
Generator speed	3,000 r.p.m.
Mean diameter of 1st stage	6.7 in. ($u = 293$ ft./sec.)
Mean diameter of 9th stage	10.6 in. ($u = 464$ ft./sec.)
Nozzle height, 1st stage	0.12 in.
Nozzle height, 9th stage	0.27 in.

There are many interesting features about this turbine. The casing is made in two parts; the main part is one piece of forged steel without any horizontal joint; the exhaust end is also a single piece steel casting. Such a cylinder construction eliminates the difficulty of sealing the horizontal joint and also gives a symmetrical casing which is not likely to give trouble due to distortion. To insert the turbine rotor and diaphragms, the casing is placed vertically, as shown in Fig. 347, and these inner parts are inserted from the top. The casing is tapered to facilitate assembly in this manner.

The small height of the nozzles called for very careful machining. The nozzles are made in single pieces, machined on all surfaces and held together by means of rings. To avoid leakage, the joints between the individual pieces are welded. The diaphragms are provided at their periphery with resilient supports in order to permit of their expansion in the cold casing when the turbine is being warmed up. The supports may be seen in Figs. 348 and 349.

Attention is drawn to the curved shape of the longitudinal section of the nozzles (see Fig. 348). This shape has been adopted to ensure that the steam issuing from the nozzles will fill the blade channel at inlet and flow in at right angles to the inlet edge of the blade. The theory underlying this construction is already given in Art. 139.

The H.P. gland has to pack against the exceptionally high pressure of 1,220 lb. per sq. in., and the gland loss on test proved to be 925 lb.

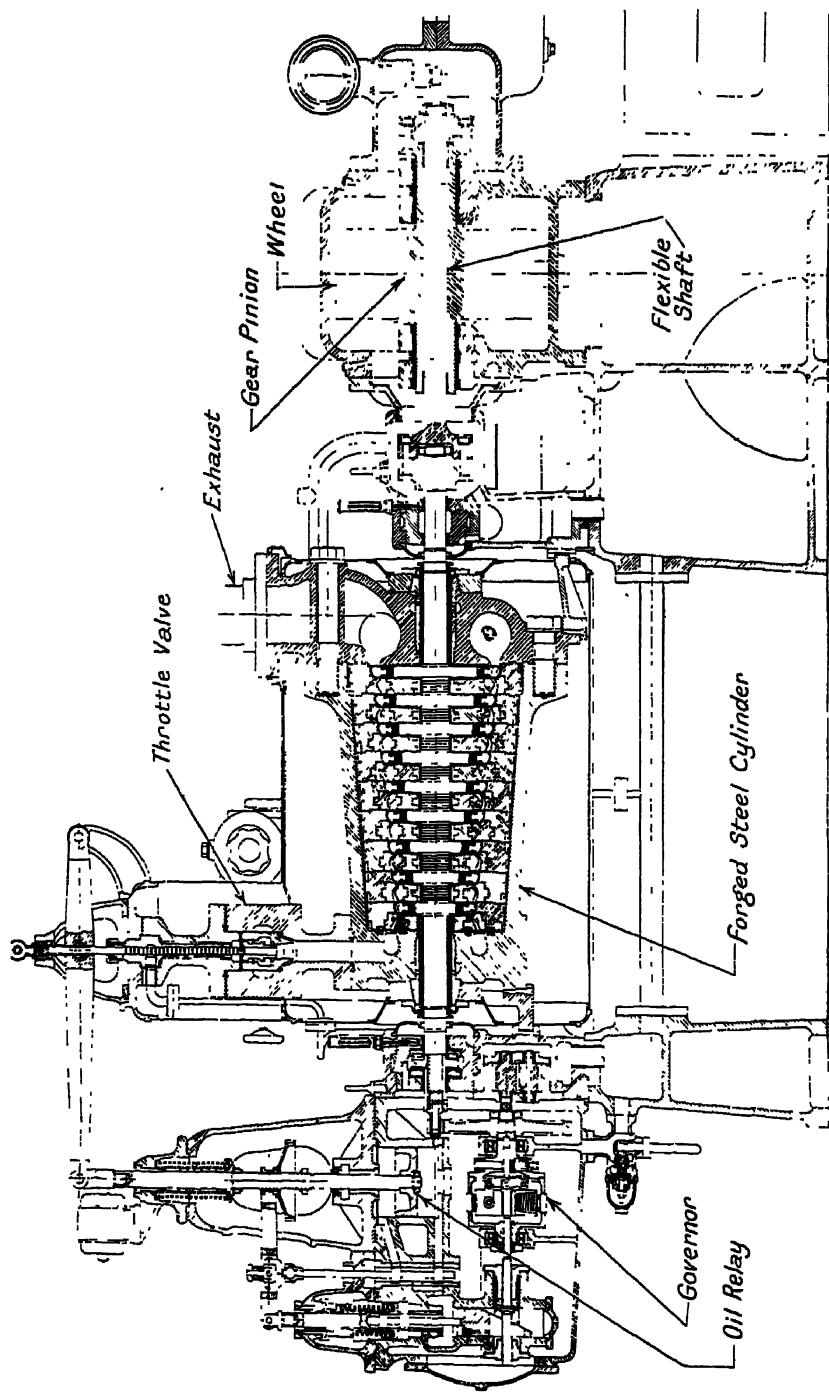


FIG. 346.—Sectional Elevation of Escher Wyss H.P. Primary Turbine

Initial pressure = 1,415 lb. per sq. in. abs.

Exhaust pressure = 200 lb. per sq. in. abs.
1,000 kW

per hour. In order to limit the gland leakage losses to a reasonably small value, the shaft diameter had to be kept as small as possible,

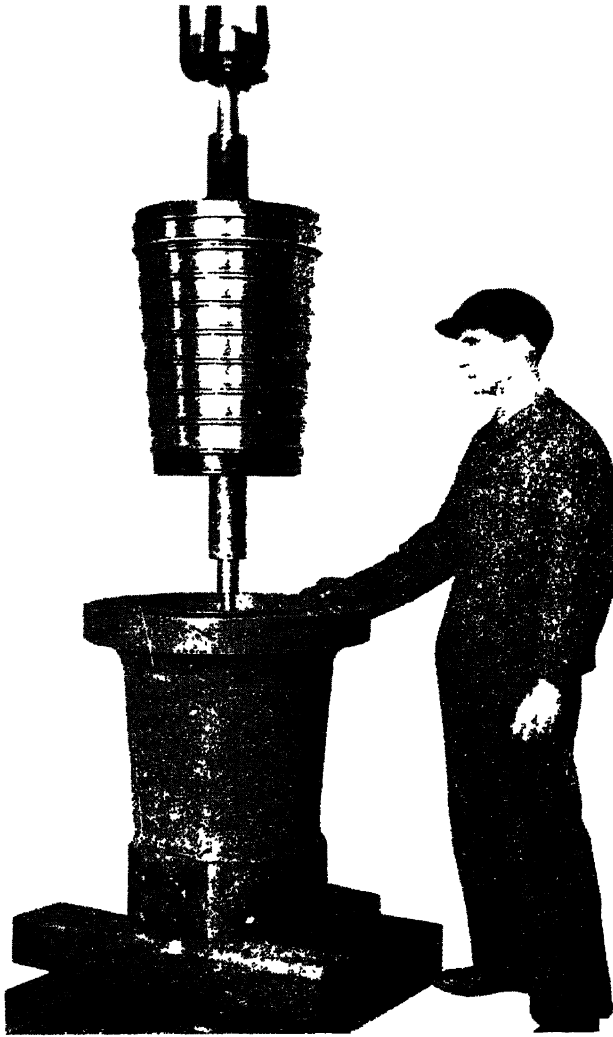


FIG. 347.—Showing Insertion of Rotor and Diaphragms in Conical Steel Casing

and to this end a flexible shaft is used having a critical speed of 2,800 r.p.m. The operating speed is therefore 3.57 times the first critical speed. To give the shaft the adequate support necessary for satisfactory running, each bearing is provided with two short

bearing surfaces. This is used in preference to a long bearing which might cause excessive heating of the oil. At the high-pressure end of the turbine, the Michell adjusting block is conveniently arranged between the two halves of the journal bearing. It is said that the transition through the first critical speed is hardly discernible.

Very comprehensive tests were carried out on this experimental

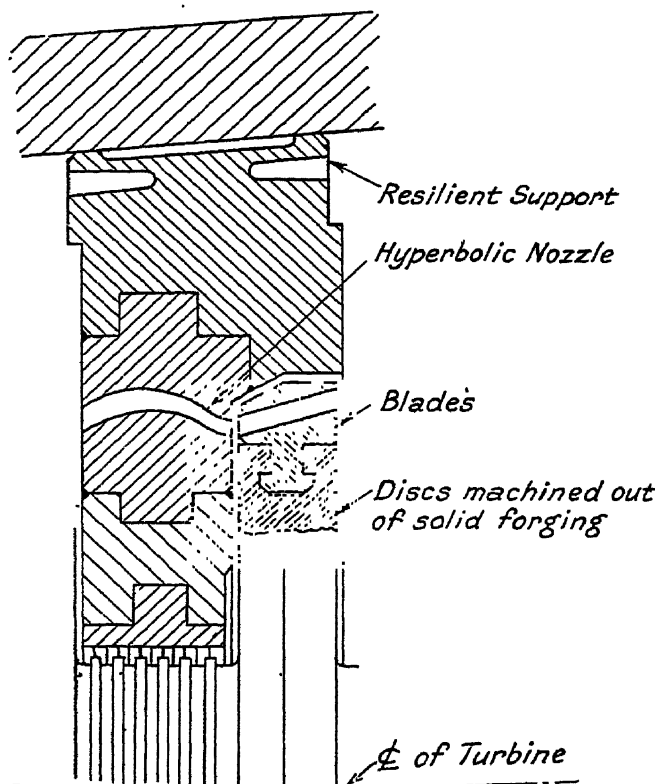


FIG. 348.—Half Section through One Stage of Escher Wyss Primary Turbine

turbine, the efficiency ratio being determined both by measuring the turbine output and by deducing the actual heat drop from observations of the steam admission and exhaust pressures and temperatures. An internal efficiency ratio of 64 per cent was recorded, and deducting 7 per cent for gearing and gland losses, the overall efficiency ratio of about 57 per cent was obtained, which, in view of the extremely small nozzle and blade heights and the high pressure, must be regarded as a satisfactory result. Some idea of the small dimensions of the turbine may be obtained from Fig. 349.

Messrs. Escher Wyss & Co. have later constructed a further

turbine to operate with a Benson boiler at a pressure of 2,500 lb. per sq. in. gauge and a temperature of 790° F.

246. 53,000 kW Turbine Operating at 1,900 lb. per sq. in. This turbine was installed in 1938 at the Brimsdown A Station of the North Metropolitan Electric Power Supply Company. It takes steam from two Löffler boilers, each capable of generating 210,000 lb.

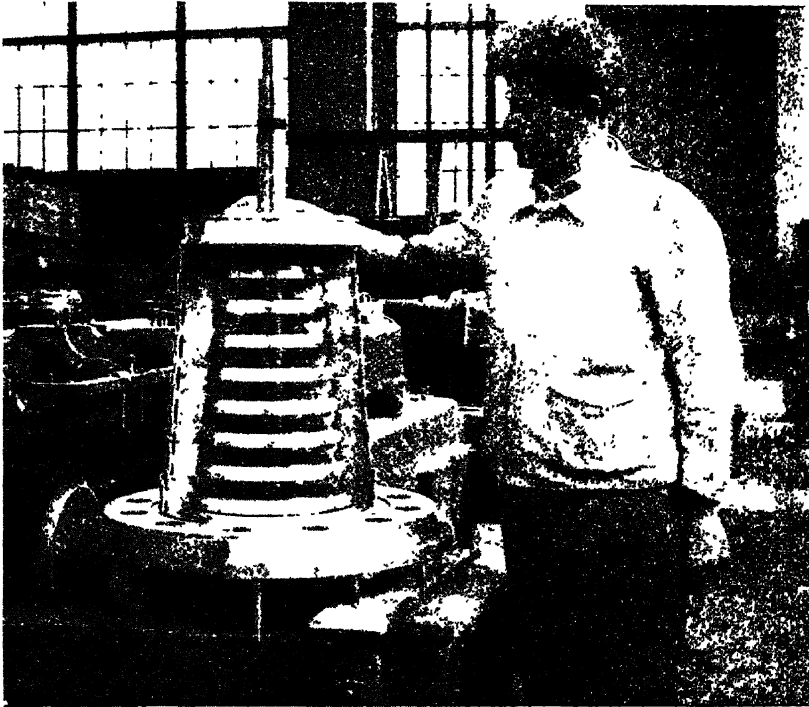


FIG. 349.— View of Partially Assembled Escher Wyss Primary Turbine

of steam per hour at a pressure of 2,000 lb. per sq. in. and a temperature of 940° F. The Löffler boiler is of the indirectly-heated type in which superheated steam is employed as the heat-conveying medium. For reasons of economy involved in the process of steam generation the operating pressure should be at least 1,400 lb. per sq. in. and preferably higher.

The turbine, which was constructed by the Metropolitan-Vickers Electrical Co., Ltd., comprises two units—

H.P. Unit. This has two cylinders and is supplied with steam at 1,900 lb. per sq. in. and 930° F. and exhausts at 200 lb. per sq. in. It generates 19,000 kW at a speed of 3,000 r.p.m. The casing of No. 1 turbine comprises two 0.5 per cent molybdenum steel forgings and contains one two-row wheel followed by 12 single-row wheels.

The wheels are turned out of a single forging. The cylinder is divided by a horizontal joint and the flanges are 12 in. thick. The casing of No. 2 turbine is made of cast molybdenum steel and is furnished with 12 impulse stages, the wheels being machined from a single forging. The H.P. gland of No. 1 turbine is provided with 150 constrictions.

L.P. Unit. This is a condensing turbine and receives reheated steam from the H.P. unit. The initial pressure is about 200 lb. per sq. in., and the steam temperature after reheating is about 810° F. The unit generates 34,000 kW at a speed of 3,000 r.p.m. There are two cylinders, No. 1 cylinder having 13 single-row wheels, and No. 2 cylinder 5 stages arranged for double flow. The tip speed in the last stage is 1,060 ft. per sec. It was originally intended to reheat the steam in two stages, firstly by live steam, and secondly by flue gases. So far, however (1942), it has not been necessary to use the live steam reheater as the specified reheat temperature has been obtained without it.

BIBLIOGRAPHY

1. BAILEY, R. W. "The Utilization of Creep Test Data in Engineering Design." *Proc. Inst. Mech. Eng.*, 1935; 131; 131.

The primary object of commercial testing is to determine whether or not the guaranteed performance is actually achieved. Practically all such tests must be carried out on site, as it is not generally possible either to provide sufficient load or sufficient steam to carry out a test in the maker's works, except in the case of relatively small units.

In the course of such acceptance tests, additional information is often obtained which is useful to the designer in enabling him to check the internal performance of the machine or examine the performance of some section of the turbine. Such analyses will be given in detail in later paragraphs.

248. Test Apparatus. The layout for a typical turbine test is shown in Fig. 350. The valves in the steam pipe are (1) the isolating valve A, (2) the stop valve B, and (3) the governor, or throttle valve C. The pressure and temperature of the steam are measured before the stop valve, and when the information is desired, after the throttle valve, i.e. at inlet to the nozzle box of the first stage in an impulse turbine or the steam belt in a reaction turbine. The vacuum gauge is shown diagrammatically and, of course, is not to scale. A glass tube D, which must be at least 30 in. long, is connected at the top to a point on the exhaust branch or, alternatively, to a perforated pipe inserted through the exhaust branch and extending across it. The lower end of D dips into a vessel containing mercury, the surface of which is exposed to the atmosphere. E is a barometric tube which also dips into the same vessel. The height h_v is the so-called "vacuum" and may be expressed in inches or cm. of mercury. The height h_b is, of course, the barometric height. As the pressure on the surface of the mercury in the barometer is the negligibly small vapour pressure of mercury at the air temperature (0.0000142 lb. per sq. in. abs. at 60° F.), the difference of pressure h_e is, to all intents and purposes, the absolute pressure of the exhaust steam, expressed in inches or cm. of mercury. The corresponding pressure p_e in lb. per sq. in. abs. is

$$p_e = 0.491 \times (h_e \text{ in. Hg.})$$

$$p_e = 0.1934 \times (h_e \text{ cm. Hg.})$$

The steam consumption of the turbine may be measured by any one of several methods. The most accurate is by weighing the quantity of condensate in a given time. One method is indicated in Fig. 350. Three tanks are arranged as shown. The valve F in the feed pipe is closed, or a blank flange is inserted. The condensate is first allowed to flow into a storage tank. At approximately regular intervals of time a certain quantity of water is weighed in the weighing tank and then allowed to pass into the feed tank, from whence it is withdrawn by a special test pump. Other methods of measuring condensate by weighing tanks, calibrated tanks, and venturi meters

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are described in the author's book *Steam Turbine Operation*. Notes on steam meters are given in the same book. Various tests should be made to determine the condenser leakage, if any, and any other water leakage into the turbine. It is outside the scope of this book to deal with these methods. Full particulars will be found in the

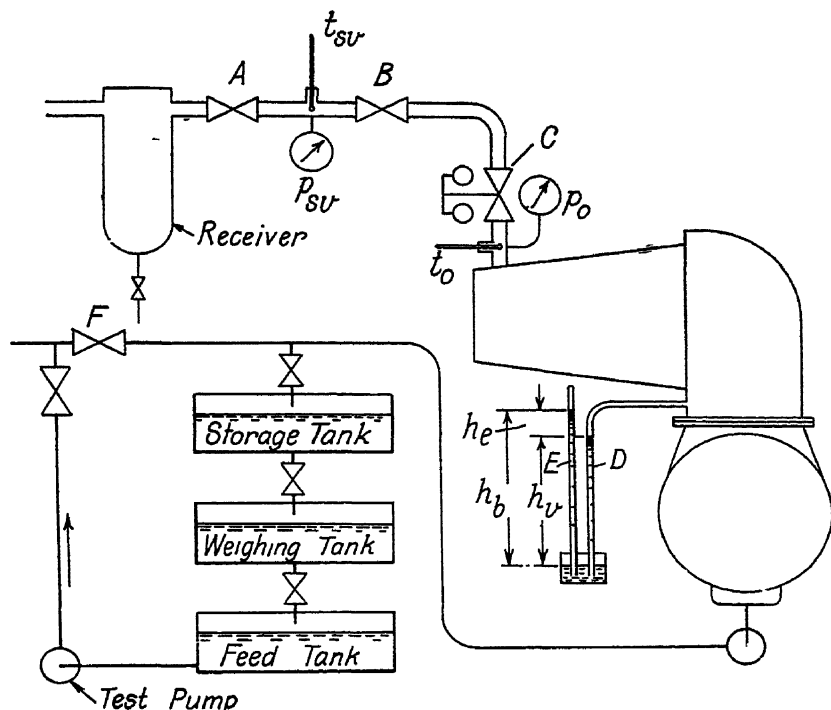


FIG. 350.—Diagrammatic Arrangement of Test Apparatus

British Standard Test Code for Acceptance Tests for Steam Turbines No. 752—1940.

If the turbine is coupled to a D.C. or A.C. generator, the generator output is determined by established methods. In the case of a marine turbine, the power is calculated from a measurement of the torque in the propeller shaft by means of a torsionmeter together with a measurement of the rotational speed obtained by means of an integrating revolution counter.

249. Correction Factors. It is almost impossible to regulate the operating conditions during a test so that the steam pressure and temperature before the stop valve and the vacuum in the exhaust branch have the precise values specified in the guarantee. This is especially true of the initial steam temperature. Consequently, it becomes necessary to apply certain corrections

to the measured steam or heat consumption to allow for these deviations. A discussion of these correction factors will be found in the author's *Steam Turbine Operation*.

TEST RESULTS

The object of the present chapter is not to explain the technique of turbine testing but to present to the reader a few selected reports of tests. The object is to show to what stage of perfection the modern steam turbine has attained. So far as possible, the tests reported have been carried out by independent authorities of repute. For obvious reasons, the heat drop data and the resulting efficiencies are given exactly as in the original reports; as these have been based on different Steam Tables they are not strictly comparable, but the differences are probably small. In some cases the author has had regretfully to convert Continental units to British units in order to preserve similarity in presentation.

Before giving test results for turbines of moderate and large output a few results for the single-wheel de Laval turbine will be given.

250. Tests on Single-wheel de Laval Turbines. Some test results for de Laval turbines of the original single-wheel design are given in Table XIX. Test No. 1 gives results of a test carried out on a small de Laval turbine by the author, while the remainder are given by the De Laval Steam Turbine Co., whose figures are given in columns 1-7; while column 8, giving the efficiency ratio, has been calculated from the data supplied. The losses due to bearing friction, disc friction, and blade windage are comparatively small, and the principal reason for the differences in the efficiency ratio is the fact that, for economic reasons, the ratio of blade velocity to steam velocity is further removed from the theoretical optimum value in the smaller turbines than in the larger ones. From approximate data regarding the dimensions of the turbines in question, the ratio of blade speed to steam speed is given in column 12. Tests show that the multi-stage de Laval turbines reach much higher efficiency ratios than those given in Table XIX.

251. Tests on 15,000 kW Impulse Turbine. Table XX gives results (1) obtained during tests made on February 12, 1924, on a 15,000 kW impulse turbine, operating at 1,500 r.p.m., constructed by the B.T.-H. Company, Ltd., and installed in the Grove Road Power Station, London, of the Central Electric Supply Company. Great care was exercised in carrying out the tests and in making all measurements. The steam consumption was determined by weighing the condensate in special water-weighing tanks placed on weighbridges. The conditions were kept as near guarantee conditions as possible so that very small corrections had to be applied to the observed steam consumptions, as may be seen from Table XX.

TABLE XIX
TEST RESULTS FOR DE LAVAL STEAM TURBINES

1	2	3	4	5	6	7	8	9	10	11	12
Ref. No.	Rated B.H.P.	Steam Pres- sure lb. per sq. in. abs.	Initial Super- heat ° F.	Vacu- um per cent	Back- Pres- sure lb. per sq. in. abs.	Steam Cons. lbs. per B.H.P. hour	Effy. Ratio per cent	Adia- batic Heat Drop B.Th.U. per lb.	Theo. Steam Velocity V	Ap- prox. Blade Velocity u	Ap- prox. $\frac{p}{u}$ $= \bar{V}$
1	18.19*	105.6	51.5	87.8	1.79	25.7	36.1	275.2	3,720	627	0.169
2	50	141.7	—	84	2.35	23.1	40.6	271.5	3,690	860	0.233
3	100	167	40	83	2.5	18.0	49.5	286	3,790	1,135	0.300
4	150	126.6	148	84	2.35	16.82	52.3	290	3,820	1,135	0.297
5	150	115.7	207	94.4	0.82	15.2	47.8	351	4,200	1,135	0.27
6	250	132	—	93	1.03	14.1	58.7	308	3,930	1,280	0.326
7	300	208	69	88	1.765	13.94	56.3	325	4,040	1,375	0.341
8	300	137.2	26	90	1.47	15.68	54.7	297	3,860	1,375	0.357
9	330	153	204	94	0.88	12.65	55.0	367	4,300	1,375	0.32
10	500	104	—	93.5	0.955	15.2	56.5				
11	500	200	118	88	1.765	13.35	57.5				

* Actual test B.H.P.

TABLE XX
TESTS ON 15,000 kW IMPULSE TURBINE

No. of Test	1	2	3	4	5
Duration . . . min.	60	60	30	60	60
Load	Full	$\frac{3}{4}$	$\frac{1}{2}$	Full	$\frac{1}{4}$
p_{10} lb. per sq. in. gauge	193.1	194.1	193.7	195.3	191.0
t_{10} ° F.	634.9	628.4	635.1	620.3	623.3
Superheat ° F.	249.8	242.9	249.8	234.3	239.0
Vacuum at exhaust flange in. Hg. (30 in. Bar.)	27.93	28.22	28.46	27.92	27.62
Net condensate lb. per hr.	165,030	125,555	85,110	167,280	207,025
Net output . . . kW	15,189	11,509	7,623	15,279	18,855
Steam consumption at test conditions lb. per kWh	10.87	10.91	11.17	10.95	10.98
Corrected steam consumption	10.80	10.82	11.12	10.76	10.76
Overall efficiency ratio of turbine and alternator under test conditions					
per cent	79.06	77.50	74.04	79.14	80.46
Estimated efficiency of alternator	96.50	—	—	96.50	97.0
Efficiency ratio of turbine alone under test condi- tions	81.93	—	—	82.02	82.95
percent					

The tests were carried out under the supervision of the consulting engineers, Messrs. Kennedy and Donkin.

The tests were worked-out on the basis of Callendar's Steam Tables. Bearing in mind that the efficiency ratio is based on the steam conditions before the stop valve, the result is very satisfactory.

252. Tests on 35,000 kW Impulse Turbine. The following particulars relate to tests on one of three 35,000 kW turbo-generators constructed by the Metropolitan-Vickers Electrical Co., Ltd., and installed in the Deptford West Power Station, London.

The economic load is 28,000 kW, i.e. 80 per cent of the maximum continuous load. The turbines are designed to operate with steam at 350 lb. per sq. in. gauge and at 750° F. with the following vacua—

At 100 per cent full load	28.75 in. Hg.
„ 80 „ „ „	29.00 „
„ 60 „ „ „	29.12 „
„ 40 „ „ „	29.18 „

The turbine has two cylinders in tandem, the particulars being as follows—

Cylinder	No. of Stages	Mean Dia. in.	Mean Blade Velocity
H.P.	24	42	275
L.P.	15	62 $\frac{1}{4}$ —84	411—550

Speed: 1,500 r.p.m.
Total K value: 273,250

Steam is bled from the L.P. turbine at three points for feed-heating, the final feed temperature being 250° F. Evaporators are incorporated in the feed-heating system of each turbine. The test results (2) are given in Table XXI.

TABLE XXI
TEST RESULTS FOR 35,000 kW IMPULSE TURBINE

Test No.	1	2	3	4
Nominal load: . . . per cent	40	60	80	100
<i>Turbine Readings—</i>				
p_{sv} . . . lb per sq. in. gauge	346.8	341.7	342.9	350.4
t_{sv} . . . ° F.	759.6	749.8	761.4	784.9
Superheat at stop valve . . . ° F.	324.7	316.3	327.6	349.1
Vacuum at exhaust . . . in. Hg.	29.114	28.950	28.873	28.581
Final feed temperature . . . ° F.	211.7	220.3	239.6	257.8
<i>Electrical Readings—</i>				
Net load . . . kW	14,346	21,011	28,382	35,701
Power factor . . .	0.94	0.94	0.95	0.945
<i>Water Measurements—</i>				
Total water weighed . lb. per hr.	145,545	208,335	271,170	346,420
Ejector steam . . .	1,500	1,500	1,500	1,500
Raw water evaporated . . .	5,609	8,318	10,913	14,050
Net steam consumption . . .	138,436	198,517	258,757	330,870
Raw water evaporated in per cent of net steam consumed .	4.05	4.18	4.23	4.25
Test consumption (including feed heaters and evaporators) lb. per kWh	9.650	9.448	9.117	9.268
<i>Heat Consumption and Thermal Efficiency—</i>				
Enthalpy at stop valve . . . B.Th.U. per lb.	1400.0	1395.1	1401.1	1413.0
Enthalpy of feed leaving last feed heater . . . B.Th.U. per lb.	179.7	188.3	207.9	226.3
Net heat supplied . . .	1220.3	1206.8	1193.2	1186.7
Test heat consumption at generator terminals (feed-heating and evaporation) . . . B.Th.U. per kWh	11,776	11,402	10,878	10,998
Test thermal efficiency at generator terminals . . . per cent	28.97	29.92	31.36	31.02
Generator efficiency . . .	94.57	96.00	96.75	97.08
Test thermal efficiency at turbine coupling (feed-heating and evaporation) . . . per cent		31.17	32.41	31.95

253. Tests on 11,000 kW Impulse Turbine. The calculation of thermal efficiency, efficiency ratio, etc., from test data will now be illustrated by data obtained by Professor Stodola during a test (3) on an 11,000 kW turbine constructed by the Escher Wyss Engineering Works, of Zürich. The leading particulars of the turbine are given in Table XXII. The following are additional particulars—

$$K = 270,500$$

BEARINGS:

Steam End. Diameter: 180 mm. = 7.09 in.

Length: 360 mm. = 14.17 in.

Projected area = 100.2 sq. in.

Exhaust End. Diameter: 200 mm. = 7.87 in.

Length: 400 mm. = 15.75 in.

Projected area = 124 sq. in.

Glands. Packed with carbon rings. Shaft diameter at glands = 220 mm. = 8.66 in. Radial clearance = 0.4 mm. = 0.016 in. Leakage through both glands estimated to be 265 lb. per hour.

TABLE XXII
PARTICULARS OF 11,000 kW STEAM TURBINE
Speed: 3,000 r.p.m.

Stage No.	Mean Diameter in.	Mean Blade Velocity ft./sec.	Nozzle Angle		Blade Angle	
			Inlet	Outlet	Inlet	Outlet
1	47.2	618	90	14	30	20
2	48.0	629	120	14	30	20
3	49.0	641	115	14	30	20
4	51.2	670	120	14	30	20
5	53.1	696	120	12	30	20
6	55.5	727	120	11	28	20
7	57.5	753	125	10	28	20
8	59.8	783	125	10	28	20
9	61.4	804	130	12.5	28	20
10	63.0	825	120	18	33	35

The test results are given in Table XXIII. Tests 1 and 2 were preliminary tests, while 3 and 4 were the principal tests.

Detailed Analysis of Full Load Test. Fig. 351 shows the performance of the turbine at full load, Test No. 4, set down on the $\phi - I$ chart, while the full working out of the test data is given in Table XXIV.

The adiabatic heat drops are the values given by Professor Stodola, converted into British units. Point A, Fig. 351, represents the steam condition on the boiler side of the stop valve, while D

TABLE XXIII

TEST RESULTS FOR 11,000 kW IMPULSE TURBINE

Notes.— p_{gv} and p_0 are the steam pressures before the governor valve and before the first-stage nozzles respectively.

t_{gv} and t_0 are the steam temperatures before the governor valve and first-stage nozzles respectively

p_c = pressure in condenser.

p_s = pressure behind the last wheel.

All pressures in lb. per sq. in. abs.

All temperatures in deg. F.

Test No.	1	2	3	4
Duration min.	51	109	45	120
Bar. height in. Hg.	29.20	29.07	29.80	29.80
p_{gv}	212.1	210.4	192.0	197.6
t_{gv}	616.6	636.9	663.4	662.9
p_{gv}	206.3	209.0	191.4	195.1
t_{gv}	614.0	634.0	661.2	659.7
p_0	14.28	93.28	131.6	178.6
t_0	586.4	617.3	653.5	657.2
p_c	0.646	0.343	0.427	0.606
p_s	0.653	0.395	0.484	0.664
Steam rate lb. per hr.	10,380	61,550	83,870	113,000
Gland loss (estimated)	265	265	265	265
Total steam rate	10,645	61,815	84,135	113,265
R.p.m.	3,045	3,040	3,041	3,041
Power at switchboard kW	0	5,477	7,966	10,975
Total loss of alternator	300	479	543	596
Power at turbine coupling	300	5,956	8,509	11,571
Efficiency of alternator per cent	—	92.0	93.6	94.9
Steam consumption, referred to power at turbine coupling lb. per kWh	35.46	10.38	9.888	9.788

shows the condition in the nozzle box at inlet to the first-stage nozzles. At full load, the pressure at the outlet side of the last wheel had a mean value 0.664, while the pressure in the condenser was 0.606 lb. per sq. in. abs. The various adiabatic heat drops may be readily obtained from a good ϕ —I chart, or by calculation. These are set down in Table XXIV, the reference letters on Fig. 351 being given a more general interpretation.

Since the power at the turbine coupling is 11,571 kW, and the

total steam rate is 113,265 lb. per hour, the work done at the coupling is

$$\frac{11,571 \times 3413}{113,265} = 348.5 \text{ B.Th.U. per lb. of steam}$$

As the heat drop AC at full load is 427.1, the efficiency ratio of the turbine based on the coupling power, the steam conditions before the stop valve, and the pressure in the condenser is

$$\eta_r = \frac{348.5}{427.1} \times 100 = 81.59 \text{ per cent}$$

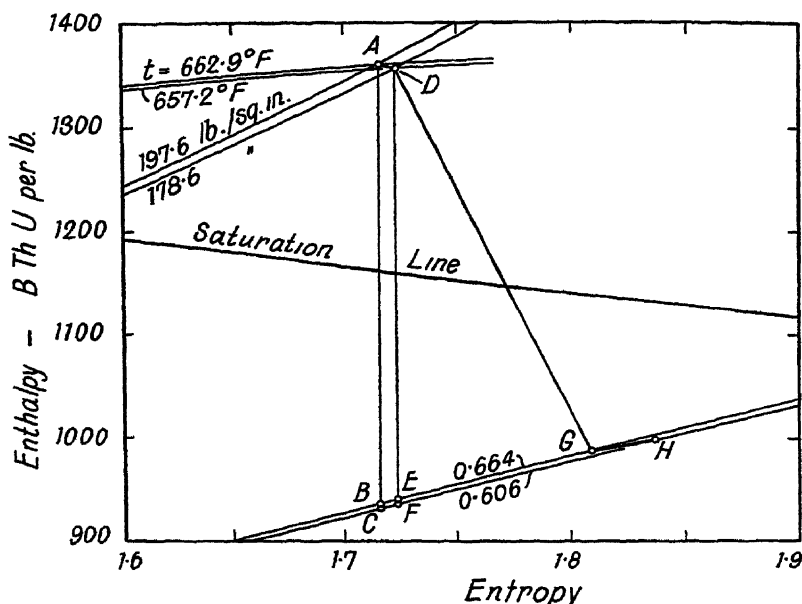


FIG. 351.—Mollier Diagram for Full-load Test of 11,000 kW Turbine

Similarly, the efficiency ratio based on the coupling power, the steam conditions in the nozzle box, and the steam pressure at outlet from the last stage wheel is

$$\frac{348.5}{414.7} \times 100 = 84.03 \text{ per cent}$$

From the experience of the builders and from special tests, the mechanical losses were estimated to be made up as follows—

High-pressure bearing	34.9 kW
Low-pressure bearing	49.5 "
Adjusting block, oil pumps, and governor drive	30.6 "
Total external loss	<u>115.0</u> "

This was assumed to be the same in all tests, and, added to the coupling power, gives the internal (shaft) output of the turbine. The heat drop corresponding to the internal power is found as above, together with the corresponding efficiency ratios. Deducting the

TABLE XXIV

DEDUCTIONS FROM DATA OBTAINED DURING TEST OF 11,000 kW
STEAM TURBINE

Note.—All heat drops are in B.Th.U. per lb. of steam. All efficiencies are expressed as percentages.

Test No.	1	2	3	4
Adiabatic heat drop AC (Fig. 351) .	—	449·7	442·1	427·1
„ „ „ DE „ .	252·5	395·7	413·3	414·7
„ „ „ DF „ .	253·3	402·5	417·8	419·3
Heat drop corresponding to power at coupling	—	328·7	345·0	348·5
Efficiency ratio based on coupling power, steam condition before stop valve, and pressure in condenser .	—	73·11	78·05	81·59
Efficiency ratio based on coupling power, steam condition in nozzle box, and pressure at outlet from last wheel	—	83·08	83·49	84·03
Internal power kW	415	6,071	8,624	11,686
Heat drop corresponding to internal power	—	335·1	349·7	351·9
Internal thermal efficiency ratio based on the steam condition in the nozzle box and the steam pressure at outlet from the last wheel	—	84·68	84·61	84·85
Internal thermal efficiency ratio based on the steam condition in the nozzle box and the pressure in the condenser	—	83·25	83·68	83·93
Reheat referred to adiabatic heat drop based on steam condition in nozzle box and the pressure at outlet from the last wheel . . . per cent	—	3·85	3·79	3·69
Final velocity of steam at outlet from the last wheel . . . ft. per sec.	—	565	627	670
Corresponding loss . B.Th.U. per lb.	—	6·37	7·85	8·97
Terminal velocity loss referred to adiabatic heat drop DE . . . per cent	—	1·61	1·90	2·16

heat drop corresponding to the internal power from the enthalpy at D, we obtain the enthalpy at G, the point G representing the condition of the steam at outlet from the last wheel.

From the velocity diagrams for the last stage of the turbine, Professor Stodola was able to calculate the final velocity of the steam from that stage. These values are given in Table XXIV, together with the outlet velocity loss.

Suppose that H, Fig. 351, represents the condition of the steam entering the condenser, and—

Let W_i = internal work, B.Th.U. per lb. of steam

V_c = velocity of steam entering condenser

V_0 = outlet velocity from last wheel.

Then, by the steady-flow equation and assuming that the steam velocity in the nozzle box is small—

$$I_D = I_G + W_i + \frac{V_0^2}{2gJ}$$

$$\text{and} \quad I_D = I_H + W_i + \frac{V_c^2}{2gJ}$$

$$\therefore \quad I_H - I_G = \frac{V_0^2 - V_c^2}{2gJ}$$

This enables the point H to be obtained.

The steam consumption curve is given in Fig. 352. With simple throttle governing, this curve is a straight line and corresponds to the well-known Willans line for steam engines. The steam consumption at zero coupling power is 8,000 lb. per hour, which is about 7 per cent of the full-load consumption. Another important relation, which has already been discussed in Art. 110, is shown by Fig. 353. To all intents and purposes, the test points lie on a straight line which passes through the origin, or more accurately, slightly above it, owing to the effect of gland leakage.

From an analysis of the test data, Professor Stodola estimated that the efficiency of the stage working under the best conditions (stage 6, with blades about 2.4 in. long), excluding wheel friction and leakage, and without deducting the loss due to ejector action, would be about 90 per cent.

254. Tests on 10,000 kW Impulse Turbine Operating with Steam at 1,000° F. The following brief account is of tests made on a steam turbine supplied to the Detroit Edison Company in the year 1931 by the British Thomson-Houston Co., Ltd., of Rugby. The plant comprises a two-cylinder tandem turbine, a 12,500 kVA generator, and three regenerative feed heaters. The turbine was designed for a pressure of 380 lb. per sq. in. abs., a steam temperature of 1,000° F. at the throttle, and an exhaust pressure of 1 in. Hg. abs. It operates at 3,600 r.p.m. and is of the impulse type with

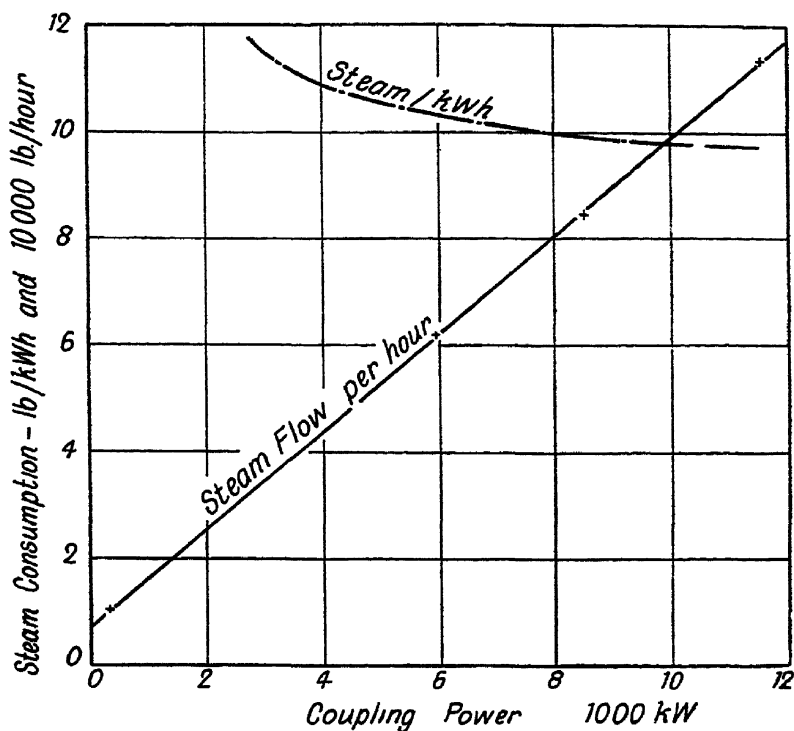


FIG. 352.—Steam Consumption Curves for 11,000 kW Turbine

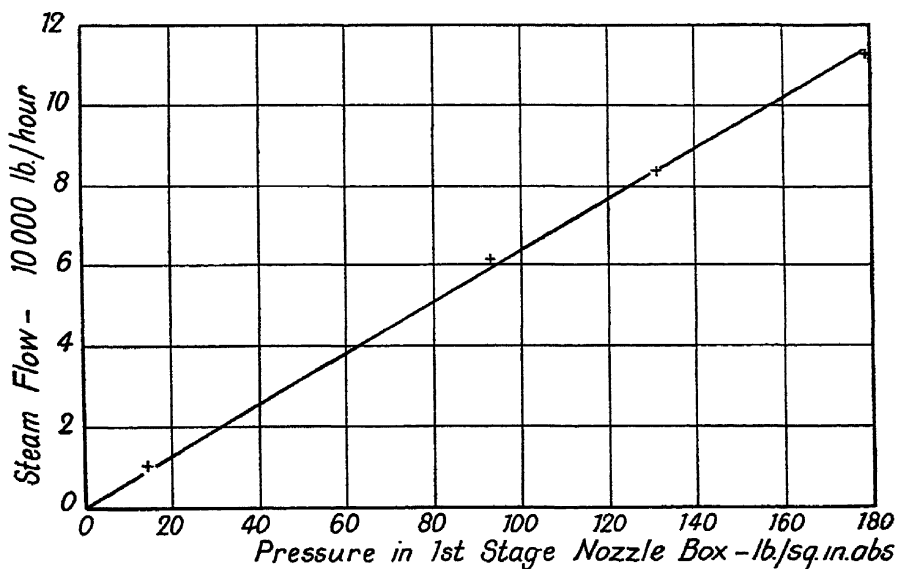


FIG. 353

9 stages in the H.P. cylinder and 11 in the L.P. cylinder. The mean diameter of stages 1, 2, 9, 10, and 20 are 32·5, 23, 23·5, 32, and 50 in. respectively.

The three feed heaters are of the horizontal 4-pass type with the following heating surfaces—

Heater	Connected to Turbine Stage No.	Heating Surface sq. ft.
H.P.	9	284
I.P.	14	303
L.P.	17	463

The arrangement of the heater drains is shown in Fig. 354. It will be seen that the drain water from the H.P. heater is cascaded into the

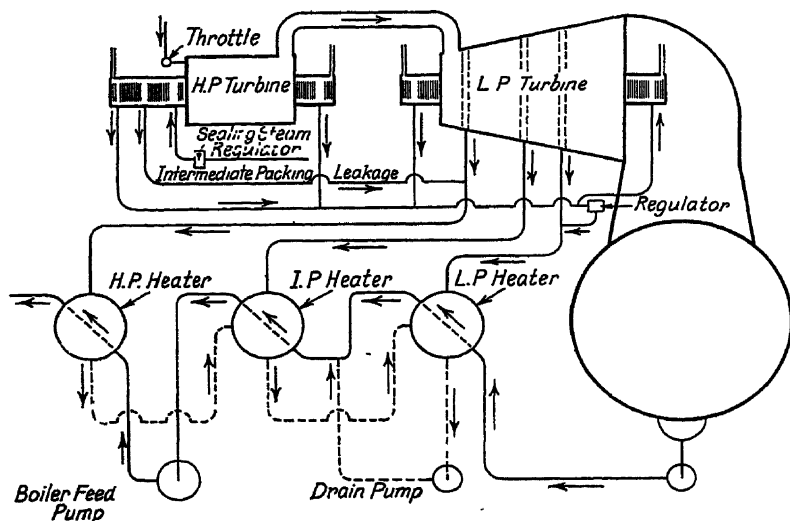


FIG. 354.—Diagrammatic Arrangement of 10,000 kW. B.T.-H. Turbine

I.P. heater; the combined drains from the latter are then cascaded into the L.P. heater, while the combined drains from the three heaters are extracted by a drain pump from the L.P. heater and delivered to the feed pipe after that heater.

Special attention was paid to the steam-end glands of the H.P. turbine, because leakage of the high-temperature steam through the gland might cause serious heating of the shaft and bearing. The axial length of the H.P. gland is greater than that taken up by the

wheels in the H.P. cylinder. The steam leaving the first-stage wheel has a temperature of about 930° F., and is prevented from passing through the gland by using saturated steam of slightly higher pressure as sealing steam. When the sealing steam regulator is set to give a pressure at inlet to the packing 0.5 lb. per sq. in. above that in the first-stage wheel chamber, the leakage of saturated steam past the inner packing into this chamber amounts to about 10 per cent of the total sealing steam, or about 0.6 per cent of the total flow to

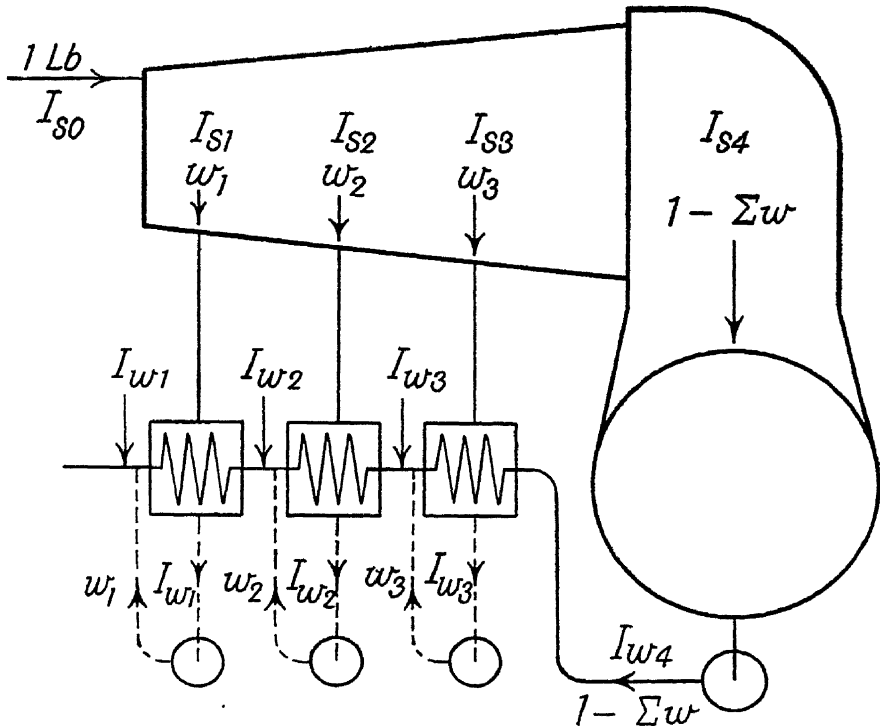


FIG. 355

the turbine for a load of 10,000 kW. As shown in Fig. 354, part of the sealing steam from the intermediate H.P. packing passes into the H.P. feed heater, while the remainder passes on to the outer packing. Part of the leakage through the outer packing seals the gland at the exhaust end of the L.P. turbine while the remainder passes through a pressure regulator into the L.P. feed heater.

Live steam for the turbine was taken from the boilers at 700° F. and further superheated in an oil-fired superheater. Thus steam at any temperature between 700° F. and 1,000° F. could be supplied to the throttle by controlling the fuel supply to the secondary superheater. A summary of the tests (4) at 1,000° F. is given in Table XXV.

In calculating the thermal efficiency of the ideal "unit" (the term "unit" being used to represent the combination of turbine and feed heaters) which corresponds to the actual unit, it is assumed that the steam expands isentropically and that the number of feed-heating stages is the same as in the actual unit, the temperature reached by the feed water in each stage being assumed equal to the saturation temperature of the steam extracted from the turbine at the stage in question. Then following the method of Art. 101 (but neglecting the slight effect of the drain pumps on the water enthalpies) and referring to Fig. 355, the following relations may be obtained—

$$w_1 = \frac{I_{w1} - I_{w2}}{I_{s1} - I_{w2}}$$

$$w_2 = \frac{(1 - w_1)(I_{w2} - I_{w3})}{I_{s2} - I_{w3}}$$

$$w_3 = \frac{(1 - w_1 - w_2)(I_{w3} - I_{w4})}{I_{s3} - I_{w4}}$$

The available energy due to the expansion of the steam through the turbine, expressed in B.Th.U. per lb.

$$= I_{s0} - I_{s4} - w_1(I_{s1} - I_{s4}) - w_2(I_{s2} - I_{s4}) - w_3(I_{s3} - I_{s4}) = \Sigma h$$

Since the heat supplied in the cycle $= I_{s0} - I_{w1}$, the thermal efficiency of the ideal unit is

$$\eta_c = \frac{\Sigma h}{I_{s0} - I_{w1}}$$

Normally, a steam turbine has only one supply of steam, i.e. the amount coming through the throttle valve, but in this case the turbine has two sources of supply. The chief supply is, of course, that through the throttle valve; the secondary supply is the steam supplied to the regulator for the purpose of sealing the high-pressure glands. As already stated, the sealing steam is saturated. In determining the equivalent steam conditions for the corresponding ideal cycle, the following method was adopted.

Let W_a = actual flow of throttle steam, lb. per hour

W_b = actual flow of sealing steam, lb. per hour

$W_1 = W_a + W_b$ = actual total flow

I_a = enthalpy of steam at entrance to throttle

I_b = enthalpy of sealing steam at entrance to regulator

I_{s0} = enthalpy of steam entering the corresponding ideal unit at state 0.

Then

$$I_{s0} = \frac{W_a \cdot I_a + W_b \cdot I_b}{W_1}$$

TABLE XXV

SUMMARY OF TESTS ON 10,000 kW TURBINE WITH STEAM AT 1,000° F.

1. Net load at generator terminals, kW	10,068	8,034	5,986	4,042
2. Temperature, ° F.—				
(a) Steam entering throttle	1,003	1,006	1,004	1,005
(b) Sealing steam entering regulator	445	448	450	452
(c) Feed water leaving 9th-stage heater	337	322	304	282
3. Abs. pressure, lb. per sq. in.—				
(a) Steam entering throttle	392	390	393	387
(b) Sealing steam entering regulator	391	405	415	420
(c) Steam at 9th-stage bleeder nozzle	128	103	78	55
(d) Steam at 14th-stage bleeder nozzle	40.5	32.5	24.3	17.4
(e) Steam at 17th-stage bleeder nozzle	10.5	8.4	6.3	4.5
(f) Steam entering condenser	0.50	0.49	0.46	0.52
4. Steam rate, lb. per kWh—				
(a) To throttle	8.388	8.403	8.519	8.965
(b) To sealing-steam regulator	0.541	0.530	0.523	0.559
5. Generator efficiency (including bearings, ventilation, and excitation), per cent	95.58	95.07	94.11	91.90
6. Energy consumption rate, B.Th.U. per kWh—				
(a) Complete unit (turbo-generator and heaters)	10,731	10,890	11,186	11,995
(b) Turbine and heaters	10,257	10,353	10,527	11,023
7. Thermal efficiency, per cent—				
(a) Complete unit	31.81	31.34	30.51	28.45
(b) Turbine and heaters	33.28	32.97	32.42	30.96
(c) Corresponding ideal unit	41.86	42.00	42.28	41.59
8. Engine efficiency, per cent—				
(a) Complete unit	75.99	74.62	72.16	68.41
(b) Turbine and heaters	79.50	78.49	76.68	74.44

I_{s1} , I_{s2} , I_{s3} , and I_{s4} are the enthalpies after isentropic expansion, and I_{w1} , I_{w2} , I_{w3} , and I_{w4} are the enthalpies of saturated water at the bleed pressures p_1 , p_2 , p_3 , and p_4 respectively.

The heat consumption of the plant is

$$H = \frac{W_1(I_{s0} - I_f)}{K} \text{ B.Th.U. per kWh}$$

where I_f is the enthalpy of water at the actual feed temperature on test.

Then the thermal efficiency of the actual unit is

$$\eta_{th} = \frac{3413}{H}$$

The ratio η_{th}/η_c is described as the engine efficiency of the complete unit (turbine-generator and heaters).

If η_g = efficiency of generator, then—

$$\frac{\eta_{th}}{\eta_c \times \eta_g} = \text{engine efficiency of the turbine and heaters only.}$$

The test data were worked out on the basis of Keenan's Steam Tables.

Tests were also made at steam temperatures of 700°, 800°, and 900° F. The heat consumption under these conditions and at varying

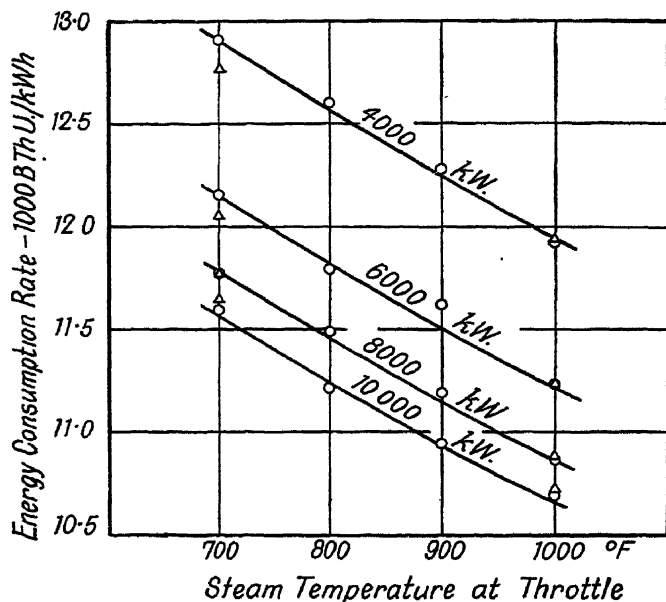


FIG. 356.—Energy Consumption Rate of 10,000 kW B.T.-H. Turbo-generator and Three Heaters

Note.—Triangles show test points when exhaust pressure was regulated to approximately 1 in. Hg. abs. Circles show results of tests with unregulated exhaust pressure.

loads is shown in Fig. 356. The curves show that the heat consumption rate for the unit was decreased by 7 per cent by raising the initial temperature from 700° F. to 1,000° F.

The heat losses by radiation and convection from the turbine, heaters, bled-steam pipes, and feed pipes were all estimated from the test data. Radiation accounts for some 65 per cent of the total

loss, which amounted to 224,900 B.Th.U. per hour. Table XXVI gives the energy balance for the unit for a generator load of 10,068 kW and a steam temperature of 1,000° F. The loss due to radiation and convection is seen to amount to 0.2 per cent of the heat energy supplied, or 0.6 per cent of the turbine output.

TABLE XXVI
ENERGY BALANCE FOR 10,000 kW TURBINE

Energy Distribution	B.Th.U. per lb. of Total Flow to Turbine	Per cent
Energy delivered by turbine shaft . . .	400.0	33.3
Energy lost by radiation and convection . .	2.5	0.2
Energy rejected to condenser (by balance)	798.5	66.5
Total energy supplied . . .	1201.0	100.0

255. Tests on 1,000 kW Back-pressure Combination Turbine. The following tests (5) were made in May, 1925, by Professor Stodola on a Brown-Boveri back-pressure turbine, and are of special interest because of the care taken to determine the performance of the reaction blading. The turbine had a single-row impulse wheel in the first stage (mean diameter not stated) and 28 reaction stages on a drum 460 mm. diameter. The reaction blade height varied from 30 mm. in the first stage to 60 mm. in the last. Thus the mean diameter increased from 490 mm. (19.3 in.) to 520 mm. (20.5 in.) and at a speed of 3,000 r.p.m. the mean blade velocity from 253 to 268 ft. per second. The mean tip clearance was 0.73 mm. (0.029 in.) or 1.62 per cent of the mean blade height.

The test results for the complete turbine at generator outputs of 788 kW and 950 kW are given in Table XXVIIA.

The steam leaking through the balance piston labyrinth packing was condensed and weighed. By deducting this from the main condensate the weight of steam passing through the turbine blades could be determined.

The losses due to bearing friction, oil pump, and governor drive were determined in the following manner. The uncoupled turbine was driven at speed in a rarefied atmosphere by artificial means. At first the vacuum was the highest possible; then more and more air was allowed to leak into the casing. During these experiments the steam pipe was still coupled to the turbine but the valve was shut and a drain pipe on the main steam pipe was kept open so as to ensure that no steam leaked into the turbine. By partial closure of valves leading to the condenser the vacuum was diminished, and so the amount of work absorbed for different densities could be determined. The turbine could be quite filled with air whose density

TABLE XXVIIA
TEST RESULTS FOR COMPLETE 1,000 kW BACK-PRESSURE
COMBINATION TURBINE

	Test No.	6	7
Item No.	Duration min. Barometric height in. Hg.	60 28·78	60 28·78
	Steam pressures, lb. per sq. in. abs.—		
1	Before nozzles	139·9	154·2
2	After impulse wheel	108·6	119·1
3	Exhaust belt	39·75	39·46
	Steam temperatures, ° F.—		
4	Before nozzles	534·1	545·9
5	After impulse wheel	491·0	500·0
6	Exhaust belt	308·2	302·5
	Steam flow, lb. per hour—		
7	Through reaction blading	29,990	32,990
8	Through dummy piston	1,000	1,150
9	Total	30,990	34,140
10	R.p.m.	3,000	3,000
	Output, kW—		
11	At alternator switchboard	788·0	950·0
12	Total loss in alternator	53·5	60·8
13	Turbine coupling power	841·5	1010·8
14	Efficiency of alternator per cent	93·6	94·0
15	Total steam consumption in lb. per kWh at coupling	36·84	33·78
16	Bearing friction, oil pump, and governor power kW	22·8	22·8
17	Disc friction and blade windage of impulse wheel kW	12·65	13·75
18	Internal power of turbine (13) + (16)	864·30	1033·60
19	Blade power of turbine (17) + (18)	876·95	1047·35

could be determined by the pressure and temperature. The plotting of the test points against density with constant speed yielded a straight line. The intercept on the power axis is the power required to overcome bearing friction and the governor and oil-pump drive. The bearings were brought to the same temperature and the oil pump delivery at the same pressure and quantity as during the power tests.

The heat balance and internal efficiency ratios are worked out in Table XXVIIb, which does not call for any further explanation.

In order to test the reaction blading separately and so avoid roundabout calculations of its efficiency, the impulse wheel was stripped of blades and the nozzles were built out so that the steam could pass over the impulse wheel to the reaction blading with only a slight drop in pressure. The results of these tests are set out in

TABLE XXVIIb
HEAT BALANCE FOR COMPLETE 1,000 kW BACK-PRESSURE
COMBINATION TURBINE

Note.—Item Nos. are continued from Table XXVIIA

Item No.	Test No.	6	7
20	Adiabatic heat drop for impulse wheel B.Th.U. per lb.	26-273	27-016
21	Internal heat drop for impulse wheel B.Th.U. per lb.	18-788	20-021
22	Theoretical power of impulse wheel, based on total steam flow kW	239	270
23	Internal power of impulse wheel, based on total steam flow and on condition of steam kW	171	200
23a	Blade power of impulse wheel (17) + (23) „	183-65	213-75
24	Adiabatic heat drop for reaction blading B.Th.U. per lb.	91-316	99-993
25	Internal heat drop for reaction blading B.Th.U. per lb.	80-435	86-578
26	Theoretical power in reaction section, based on working steam only kW	803-3	967-3
27	Internal power in reaction section, based on working steam only and on condition of steam kW	707-6	837-6
28	Total theoretical power of working steam (22) + (26) kW	1042-3	1237-3
29	Total internal power of working steam (23) + (27) kW	878-6	1037-6
30	Efficiency = $\frac{\text{Coupling power}}{\text{Theo. power of working steam}}$ ratio = $\frac{(13)}{(28)}$ per cent	80-7	81-7
31	Internal efficiency of turbine calculated from press. and temp. of working steam = $\frac{(29)}{(28)}$ per cent	84-3	83-9
32	Internal efficiency of turbine calculated from measured power of working steam per cent	82-9	83-5
32a	Turbine efficiency = $\frac{\text{Blade power}}{\text{Theo. power of working steam}}$ = $\frac{(19)}{(28)}$ per cent	84-1	84-6
33	Internal efficiency of impulse stage, based on steam condition per cent	71-5	74-1
33a	Impulse wheel efficiency = $\frac{\text{Blade power}}{\text{Theo. power of total steam}}$ = $\frac{(23a)}{(22)}$ per cent	77-0	79-2
34	Internal efficiency of reaction blading based on steam condition per cent	87-9	86-6
35	Total adiabatic heat drop . B.Th.U. per lb.	116-627	125-588
36	Theoretical power based on total steam flow and on steam condition at nozzle inlet and in exhaust branch kW	1,058	1,257
37	Efficiency based on the total steam flow, on the coupling power, and on the steam condition at inlet to the nozzles and in the exhaust belt per cent	79-5	80-4

Table XXVIII_A, while the heat balance, given in Table XXVIII_B, leads to an analysis of the internal efficiency of the reaction blading. This was determined in two ways: firstly, by the ratio of the actual heat drop (determined from the initial and final condition of the steam) to the adiabatic heat drop, and secondly, by the ratio of the actual work done to that which would be available in a perfect turbine. In estimating the internal work done by the reaction blading, an allowance was made for the disc friction of the impulse wheel.

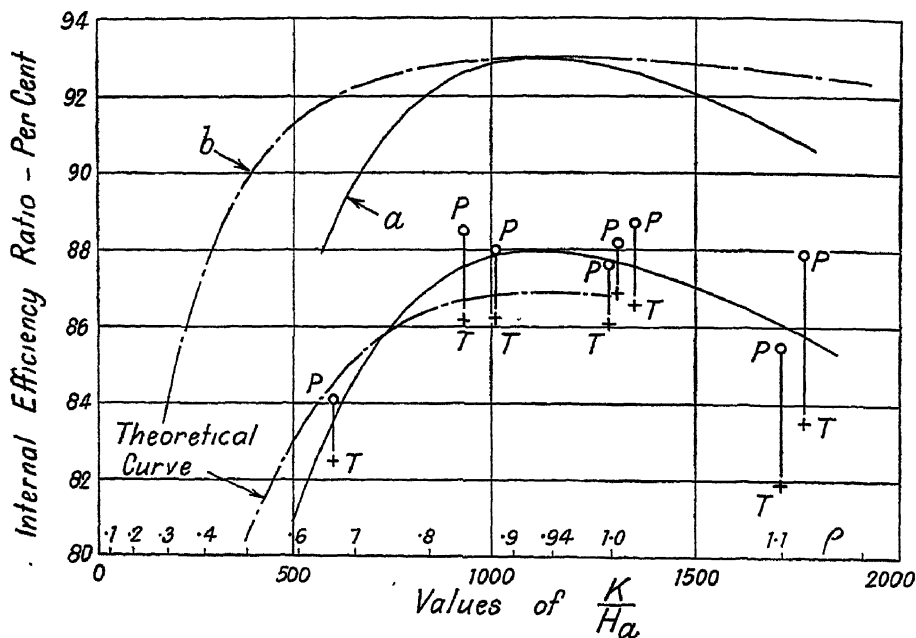


FIG. 357.—Results of Tests on 10,000 kW Back-pressure Turbine

The results of these tests are shown in Fig. 357, efficiency being plotted against values of $\frac{K}{H_a}$. Points marked P are based on the estimated internal power; those marked T are based on the condition (temperature) of the steam. The author's theoretical curve, taken from Fig. 303, is shown for comparison. It must be remembered that the theoretical curve is drawn on the assumption of constant coefficients, which implies that for each value of $\frac{K}{H_a}$ a different blade angle would be used. If, as in the actual turbine, a deviation from the optimum value of $\frac{K}{H_a}$ involved shock losses due to incorrect blade angle, then the two curves would be in better agreement.

It should be noted that the internal efficiency reaches the relatively

TABLE XXVIII

TEST OF REACTION BLADING OF 1,000 kW BACK-PRESSURE COMBINATION TURBINE

Item No.	Test No.	.	.	.	.	1	2	3	4	5	1a	2a
	Duration.	.	.	.	.	60	45	45	45	40	90	85
	Bar. height	.	.	.	.	28.73	28.73	28.73	28.73	28.73	28.47	28.47
		.	.	.	.	in. Hg.						
1	Steam pressures, lb. per sq. in. abs.—											
2	Before reaction blading	.	.	.	.	92.51	90.20	111.5	96.20	75.65	94.19	93.22
3	Back pressure	.	.	.	.	39.46	28.13	48.01	39.88	39.53	39.21	39.30
4	Steam temperatures, ° F.—											
5	Before reaction blading	.	.	.	.	452.0	450.4	453.5	441.2	433.5	449.9	431.9
6	In exhaust space	.	.	.	.	302.6	254.2	305.9	293.7	325.2	296.7	285.1
7	Steam condensed in main condenser	.	.	.	lb. per hr.	26,140	26,590	31,180	27,340	19,750	26,420	26,170
	Steam leakage through dummy piston	.	.	.	"	917	922	1,107	955	721	946	988
	Total steam consumption	.	.	.	"	27,057	27,512	32,287	28,295	20,471	27,366	27,158
8	R.p.m.	.	.	.	.	3,000	3,000	3,000	3,500	3,000	3,000	2,500
9	Power at alternator switchboard	.	.	.	kW	436.8	608.0	526.3	443.1	217.4	455.5	448.5
10	Total loss in alternator	.	.	.	"	41.8	47.3	44.3	53.2	41.1	42.5	34.5
11	Power at turbine coupling	.	.	.	"	478.6	655.3	570.6	496.3	258.5	498.0	483.0
12	Alternator efficiency	.	.	.	per cent	91.3	92.7	92.3	89.3	84.1	91.5	92.9
13	Steam consumptions, lb. per kWh											
	Steam used in turbine	.	.	.	.	54.63	40.58	54.64	55.08	76.40	53.06	54.18
14	Coupling power	.	.	.	.	56.54	41.99	56.58	57.01	79.18	54.95	56.23
	Total steam consumption	.	.	.	.							
	Coupling power	.	.	.	.							
15	Power for bearing friction, etc.				kW	27.8	27.7	28.7	41.94	27.8	27.9	18.4
16	Internal power = (11) + (15)				"	506.4	683.0	599.3	538.24	286.3	525.9	501.4

TABLE XXVIIIb

HEAT BALANCE FOR TESTS ON REACTION BLADING OF 1,000 kW BACK-PRESSURE COMBINATION TURBINE

Note.—Items Nos. are continued from Table XXVIIIa

Item No.	Test No.	.	.	.	.	.	.	1	2	3	4	5	1a	2a
17	Adiabatic heat drop	.	.	.	.	.	.	75.497	99.562	73.897	76.414	57.836	76.979	73.831
18	Internal heat drop	.	.	.	.	.	.	65.018	85.858	64.049	63.765	47.322	66.879	63.671
19	Theoretical power of working steam	$= \frac{\text{B.Th. U. per lb.} \times (5) \times (17)}{3413} \text{ kW}$						578.3	776.4	675.5	612.2	335.0	596.5	566.5
20	Theoretical power of total steam	$= \frac{(7) \times (17)}{3413} \text{ "}$						598.5	803.0	699.5	633.8	347.4	617.5	587.7
21	Efficiency ratio $= \frac{(11)}{(20)}$	.	.	.	.	.	.	80.0	81.6	81.6	78.3	74.4	80.6	82.2
22	Efficiency ratio $= \frac{(11)}{(19)}$	.	.	.	.	.	.	82.8	84.4	84.5	81.1	77.2	83.5	85.3
23	Internal efficiency of the working steam based on the steam conditions at inlet and outlet							86.1	86.24	86.6	83.5	81.9	86.9	86.2
24	Internal efficiency of the working steam based on the internal power relative to the state of the steam at inlet and outlet							87.6	88.0	88.7	87.9	85.5	88.2	88.5

high value of 88 per cent. Professor Stodola estimated that by extrapolating the blade clearance to zero, the blade efficiency (corresponding to our gross stage efficiency) would reach the very high value of 93 per cent. The estimated gross stage efficiency is shown by the curve *a*. Taking η_{gs} as 0.93 when $\rho = 0.9$, and assuming $\alpha = 20$ degrees and $\eta_n = 0.944$ (based on the Institution of Mechanical Engineers Steam Nozzle Research Committee's work), the author estimates that the value of the carry-over coefficient ϕ in eq. 94 is 0.888. The corresponding gross stage efficiency is then given by the curve *b*.

256. Tests on 36,000 kW Steam Turbine Operating with Steam at 1,700 lb. per sq. in. abs. and with Live Steam Reheating. The following report is an abridgment of an article published (6) in the *Brown Boveri Review* describing tests made by Professor E. Josse. The tests are of interest because of the high initial steam pressure and because of the comparative data with and without reheating by live steam.

The turbine, constructed by Messrs. Brown Boveri & Co., Ltd., is of the three-cylinder type, the L.P. cylinder being of the double-flow type. It is designed for steam at 1,700 lb. per sq. in. abs. and at 896° F. at the inlet nozzles, for intermediate superheating up to 680° F. at inlet to the I.P. cylinder, and with two extraction branches for feed-heating, of which, usually, only the low-pressure extraction branch is used in service. The speed of the turbine is 3,000 r.p.m. Particulars of the wheels and blading are given in Table XXIX.

TABLE XXIX

PARTICULARS OF WHEELS AND BLADING OF 36,000 kW TURBINE

D = mean dia. of blades in inches; u = mean blade velocity, ft. per sec.; l = blade heights, inches.

Turbine	No. of Stages	D	u	l	$\sum (\frac{D}{10})^2$
H.P.	1 Curtis (2-row wheel) 32 Reaction	39.37	515	1st row—1.10	31.0
		21.26	279	2nd row—1.38 1.61—3.38	144.8
I.P.	18 Reaction	40.16	525.5	2.16—8.47	290.4
L.P.	6 Reaction	56.5	739	3.46—14.36	191.8

$$\text{Total } \sum \left(\frac{D}{10} \right)^2 = 658.0$$

$$\text{Hence } K = 658 \left(\frac{3,000}{100} \right)^2 = 591,000$$

$$\text{and } \frac{K}{H_o} = \frac{591,000}{640.6} = 922$$

If, in Art. 222, the effect of carry-over from one stage to the next be neglected, and if the nozzle efficiency be taken as unity, then the quantity F_1 becomes unity. Consequently, by equation 206—

$$\rho = \sqrt{\frac{K_p}{2620H_c}}$$

in which we write K_p to represent the Parsons number of a pressure-compounded impulse turbine with single-row wheels. Even when carry-over and nozzle losses are taken into account the value of F_1 is nearly unity at the optimum ρ and for the usual nozzle angles.

Considering next the corresponding expression for the reaction turbine deduced in Art. 223, the value of the quantity F_2 is practically equal to 2 at the most economical value of ρ , and we may write

$$\rho = \sqrt{\frac{K_r}{1310 \cdot H_c}}$$

where K_r represents the Parsons number for a reaction turbine. Thus, for comparative values, $K_p = 2K_r$. It is shown in Art. 224 that $K_p = \mu^2 \cdot K_v$. Hence for a two-row Curtis wheel, $K_p = 4 \cdot K_v$ and $K_r = 2K_v$. This simple relation has been used in calculating the K value of the whole turbine. It will be seen from Fig. 303 that the value of $\frac{K}{H_c}$ is relatively high for this turbine.

Owing to circumstances, it was impossible to test the turbine at its most economical load of 30,000 kW. Tests were made at 25,000 kW and 19,000 kW. The tests at 25,000 kW are alone reported here. Three tests were carried out, the turbine being operated—

- (a) With reheating and single-stage feed-heating.
- (b) With reheating but without feed-heating.
- (c) Without reheating and without feed-heating.

The arrangement of the plant, showing the test points, is shown in Fig. 358. The principal test results are given in Table XXX. It will be seen that the gain due to reheating alone is 0.52 per cent on 32.65 per cent, i.e. 1.6 per cent. The gain due to feed-heating alone is 1.55 per cent on 33.17 per cent, i.e. 4.7 per cent.

Calculation of Efficiency Ratio. The results of the tests without feed-heating, and with and without reheating, are given in Table XXXI. The H.P. turbine uses about 37 per cent of the total heat drop, and of this about half is used in the Curtis stage. From temperature measurements, the efficiency of the Curtis stage was estimated to be from 60 to 62 per cent. The efficiency ratio of the H.P. turbine as a whole is about 73 per cent; thus the efficiency ratio of the reaction stages is about 80 per cent. The internal efficiency ratio of the I.P. cylinder is 88 per cent.

The internal efficiency ratio of the L.P. turbine could not be

determined from the condition of the steam at exhaust because the steam was wet. The output of the H.P. and I.P. turbines was calculated and deducted from the estimated total internal power of the turbine; this then gave the internal power of the L.P. turbine. In Test No. 3 without reheating, no separate estimate of power for the I.P. and L.P. cylinders is made because the steam at outlet from the I.P. turbine is saturated and, therefore, its enthalpy cannot be

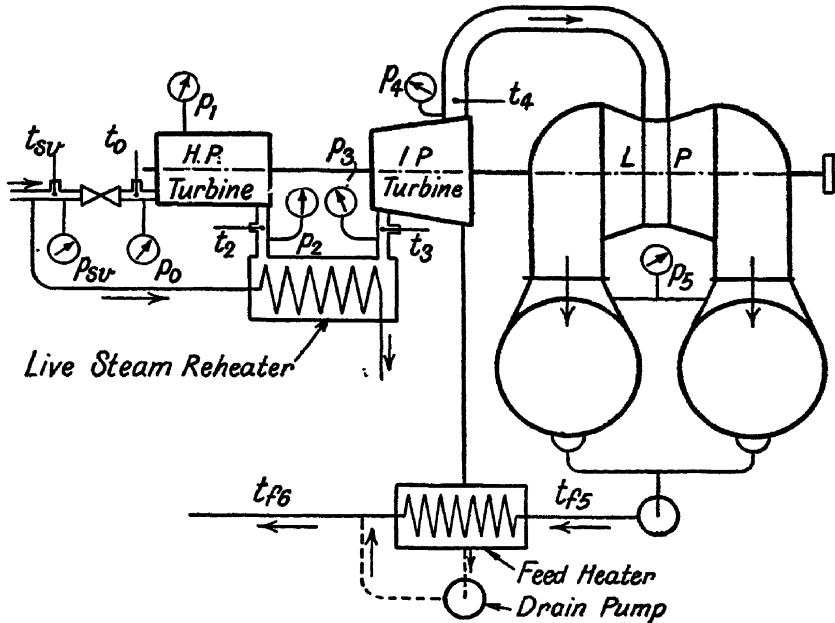


FIG. 358.—Diagrammatic Arrangement of 36,000 kW Steam Turbine with Live Steam Reheating

evaluated. The separate calculation has been omitted from Table XXXI for both tests. It should be observed that the live steam reheating reduces the wetness of the exhaust steam from 12.8 to 5.9 per cent and raises the combined efficiency ratio of the I.P. and L.P. turbines from 77.9 to 80.7 per cent.

It should be stated that in estimating the efficiency ratio of the whole turbine, the adiabatic heat drop is the sum of the three partial drops in the three turbines, and this is appreciably greater than the straight adiabatic heat drop from the initial condition. Usually, the overall efficiency ratio of a condensing turbine is referred to the straight adiabatic heat drop, but this is not possible in the present case because of the intermediate reheating. If the heat regained in the H.P. turbine owing to the non-isentropic expansion is taken into account, and if the I.P. and L.P. turbines be taken

TABLE XXX

TEST MEASUREMENTS OBTAINED DURING TESTS ON 36,000 kW
TURBINE WITH REHEATING BY LIVE STEAM

Test No.	1	2	3
Conditions	Reheating and Feed- heating	Reheating No Feed- heating	No Reheating No Feed- heating
Output at generator terminals . kW	24,940	25,411	22,656
Output at turbine coupling . . „	26,050	26,522	23,822
Total steam consumption lb. per hr.	184,900	184,800	183,100
Specific steam consumption, lb. per kWh			
(a) referred to generator output .	7.416	7.271	8.083
(b) referred to turbine output .	7.101	6.968	7.688
<i>Steam Pressures</i> lb. per sq. in. abs.			
Before stop valve p_{sv}	1,716	1,722	1,730
After governor valve 1 and in nozzle box 1 p_0	1,696	1,701	1,701
H.P. 1st-stage wheel chamber . . p_1	573	572	559
H.P. cylinder exhaust p_2	162	157.3	122.2
I.P. cylinder inlet p_3	135	135	117
I.P. cylinder exhaust p_4	11.2	12.5	11.5
Exhaust branch (mean) p_5	0.768	0.768	0.740
<i>Steam Temperatures</i> ° F.			
Before stop valve t_{sv}	891.0	889.7	886.5
In nozzle box 1 after throttling at con- stant enthalpy t_0	888.8	888.1	884.7
H.P. cylinder exhaust t_2	441.5	441.9	399.2
I.P. cylinder inlet t_3	662.0	662.7	399.2
I.P. cylinder outlet t_4	218.3	235.9	199.4
<i>Condensate Temperatures</i> ° F.			
Inlet to feed heater t_{fs}	92.7	92.7	90.5
Outlet from feed heater t_{fe}	187.2	—	—
<i>Enthalpy of Steam</i> B.Th.U. per lb. (Knoblauch, 2nd Edition)			
Before stop valve	1422.2	1421.1	1418.4
H.P. cylinder exhaust	1240.6	1241.3	1223.6
I.P. cylinder inlet	1357.5	1357.9	1223.6
I.P. cylinder exhaust	1154.3	1162.3	—
Specific heat consumption of turbine referred to coupling power B.Th.U. per kWh	9,828	10,290	10,460
Thermal efficiency of turbine referred to coupling power per cent	34.72	33.17	32.65

turbine is a double-flow machine. Each line generates 20,000 kW at 3,000 r.p.m. The steam is reheated after the H.P. turbine to the initial temperature of 700° F. Three-stage regenerative feed-heating is employed. The results given in Table XXXII were published in *The Engineer* (7).

Without feed-heating and assuming frictionless and adiabatic expansion, the heat consumptions without and with reheating are

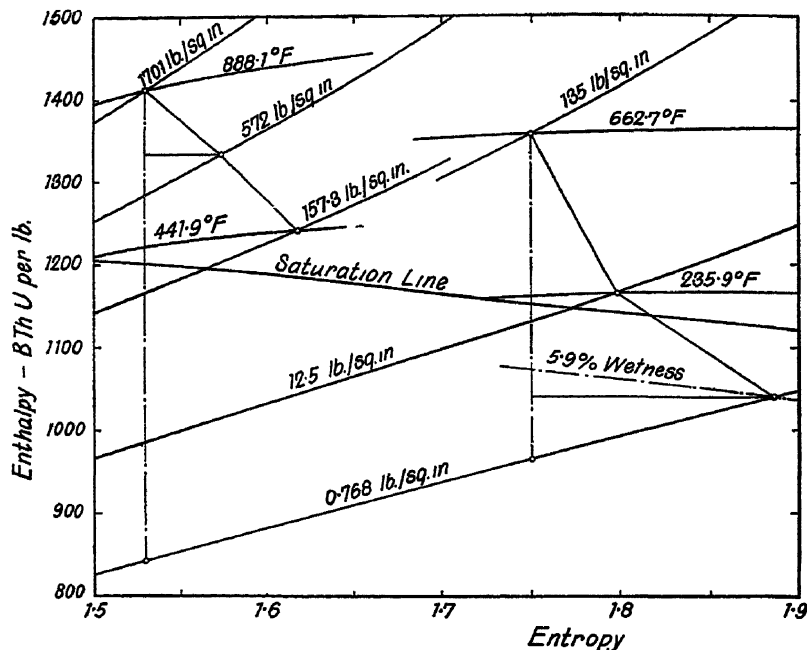


FIG. 359.— ϕ - I Diagram for Test No. 2, with Live Steam Reheating

9,445 and 9,223 B.Th.U. per kWh respectively. Thus the theoretical saving due to reheating is only 2.35 per cent. The difference between this and the actual saving of 7.53 per cent is undoubtedly due to the reduction of the effects of moisture in the intermediate and low-pressure cylinders which is brought about by reheating. The smaller saving under feed-heating conditions is due, of course, to the smaller steam flow in the I.P. and L.P. cylinders, especially in the low-pressure stages of the latter.

258. Test Results for Ljungström Turbines. A few test results for Ljungström turbines are given in Table XXXIII. These results were published in a paper (8) by Mr. O. Wiberg of Finspong, Sweden. It should be pointed out that the turbine efficiency ratio given in the table is referred to the condition of the steam after the steam inlet valve and the net steam consumption of the turbine, i.e. excluding the steam consumption of the ejectors.

TABLE XXXII
SUMMARY OF RESULTS OF TESTS ON PARSONS 40,000 kW TURBINE
WITH FLUE-GAS REHEATING

Note. The following tests were made at the economical load.

	Specified Condi- tions with Reheat- ing and Feed- heating	As Run			
		No Feed-heating		Feed-heating	
		No Reheat	Reheat	No Reheat	Reheat
Load . . . kW	35,000	31,230	37,978	30,172	35,930
<i>Steam Pressures</i> lb. per sq. in. g.					
Before stop valve . . .	350	353.6	356.6	354.5	357.7
H.P. exhaust . . .	—	37.1	49.3	34.4	47.1
I.P. inlet . . .	40	36	45.2	33.7	42.2
Vacuum . . in. Hg.	1	1.39	1.40	1.15	1.19
<i>Steam Temperatures</i> ° F.					
Before stop valve . . .	700	697	708	700	693
H.P. exhaust . . .	360	334	373	327	353.5
I.P. Inlet . . .	700	352	700	318	698
<i>Feed Temperature</i> ° F.					
Entering drain cooler . .	85	—	—	92.16	93.4
Leaving H.P. heater . .	250	—	—	251.5	258
<i>Steam Consumption</i> lb. per kWh					
As run	—	9.81	8.01	10.25	8.54
Corrected	—	9.70	7.97	10.22	8.44
<i>Heat Consumption</i> B.Th.U. per kWh					
As run	—	12,857	11,857	11,878	11,239
Corrected	—	12,826	11,859	11,828	11,179
Saving due to reheating per cent	—	—	7.53	—	5.48

TABLE XXXIII
TESTS ON LJUNGSTRÖM TURBINES

Power Station	Year	Max. Output kW	Test Output kW	Press. after Stop Valve lb./sq. in. abs.	Temp. after Stop Valve ° F.	Vacuum in. Hg. (30 in. Bar.)	Steam Consumption lb./kWh	Turbine Efficiency Ratio per cent
Lisbon	—	9,000	3,367	92.5	639	28.84	11.25	83.2
			7,099	192.0	653	28.20	10.30	86.0
			9,336	197.6	640	27.51	11.58	79.0
Leghorn	1928	15,000	5,822	190.5	617	28.93	10.35	82.8
			11,598	357.0	680	28.47	9.33	84.0
			15,490	375.5	651	28.06	10.19	79.2
Homburg	1928	14,000	8,450	156.4	635	29.10	9.81	85.6
			11,931	210.0	675	28.78	9.44	85.8
			14,436	213.2	653	28.47	9.95	85.1
Stuttgart	1929	15,600	6,072	204.7	738	29.02	9.42	82.9
			9,307	301.5	738	28.84	9.02	83.5
			12,362	374.0	739	28.58	8.91	84.0
			15,015	443.5	748	28.61	8.73	82.8
			15,575	460.5	748	28.23	9.00	82.1

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The majority of speed governors are of the fly-ball type, but fluid-pressure governors, usually in the form of a centrifugal pump impeller on the turbine spindle, may also be used for speed control. The fly-ball type of governor will now be considered.

260. The Governor. Fig. 360 shows the essential parts of the type of governor mostly used in steam turbines. It comprises a yoke A mounted on a shaft B and supporting two weights C. The

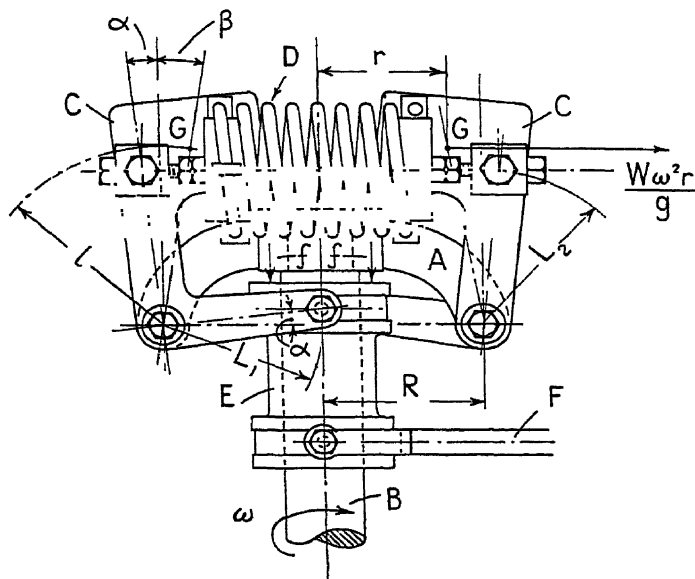


FIG. 360.—Turbine Governor with Weights Connected Directly by Side Springs

centrifugal force on the weights is balanced by the tension in the side springs D. The movement of the governor weights is transmitted through bell-crank levers to a sleeve E and thus through the lever F to the governor valve or relay gear, as the case may be. This type of governor is nearly frictionless and is therefore relatively sensitive. Owing to the fact that the centrifugal force on the weights is more or less directly opposed by the tension in the springs, the only force to be transmitted through the fulcrum pins is that necessary to move the sleeve. In many cases the fulcrum pins are supported by ball bearings.

Consider the mechanics of such a governor.

Let W = mass of each weight (centroid G).

ω = angular velocity in radians per sec.

$2f$ = total force due to friction in the governor mechanism transferred to the sleeve and acting in the direction

opposite to that of sleeve motion. Note: It is a single force, but the two is taken for convenience.

Let P = force due to springs.

Then, considering a vertical governor in the position shown in Fig. 360, and taking moments about either of the fulcrum pins,

$$\frac{W}{g} \omega^2 r l \cdot \cos \beta \pm f L_1 \cos \alpha = P L_2 \cos \alpha + W l \cdot \sin \beta$$

$$\therefore \frac{W}{g} \cdot \omega^2 r \pm \frac{f L_1}{l} \cdot \frac{\cos \alpha}{\cos \beta} = \frac{P L_2}{l} \cdot \frac{\cos \alpha}{\cos \beta} + W \tan \beta.$$

If friction be neglected,

$$\frac{W}{g} \omega^2 r = \frac{P L_2}{l} \cdot \frac{\cos \alpha}{\cos \beta} + W \tan \beta.$$

The force $\frac{W}{g} \omega^2 r$ is termed the "disturbing force" and the force

$\frac{P L_2}{l} \cdot \frac{\cos \alpha}{\cos \beta} + W \tan \beta$ is termed the "controlling force." In a

horizontal governor the effect of gravity on one weight just counterbalances that on the other, and the term $W \tan \beta$ vanishes. In any case it is small compared with the other term and may generally be neglected. As the cosines of the small angles α and β are sensibly equal to unity, the controlling force is given very nearly by $\frac{P L_2}{l}$.

It is clear that for a constant speed ω , the disturbing force is simply proportional to the radius r , and that the disturbing force at various speeds will be represented by straight lines through the origin (Fig. 361). If N_0 is the normal speed of the turbine, and r_1 and r_2 are the extreme radii of the governor weights, then the disturbing forces at these radii are F_1 and F_2 , respectively.

If the governor is to be *isochronous*, i.e. if the disturbing force at the constant speed N_0 is to be equal to the controlling force for all positions of the governor weights, then it is clear that the line AB represents the force to be exerted by the spring.

Such a governor would be useless in practice, because any slight changes in speed due to change of load would cause the weights to move to either extreme position, and so cause hunting.

Since the radius of the governor weights naturally tends to increase when the speed increases, the controlling force exerted by the springs must increase at a greater rate than the disturbing force for constant speed if the governor is to be stable. In other words, the rotational speed at which there is equilibrium must increase with increase of radius, and vice versa. Under these conditions, the controlling force would be represented by a straight line such as CD. N_1 would be the lowest speed corresponding to the radius

r_1 of the governor weights and N_2 would likewise be the highest speed corresponding to the radius r_2 .

If the effects of friction and gravity be ignored then

$$\frac{W\omega^2 r}{g} = mP, \text{ where } m = \frac{L_2}{l} \text{ very nearly.}$$

The force P exerted by the two springs may be written $P = a + br$, where a and b are constants. Hence ma is the negative intercept

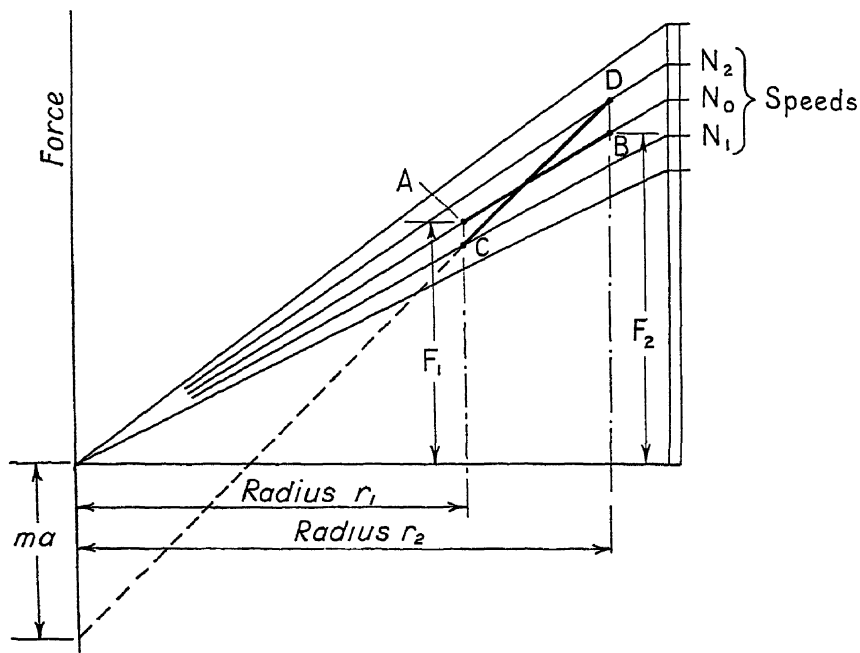


FIG. 361.—Force Diagram for Governor, Neglecting Friction

on the force axis (Fig. 361) and mb is the positive slope of the straight line CD .

$$\text{Thus } \frac{W\omega^2 r}{g} = m(a + br) \text{ (very nearly)}$$

$$\text{and } \omega^2 = \frac{mg}{W} \left(\frac{a}{r} + b \right).$$

This expression enables the relationship between speed and radius, and hence between speed and sleeve travel, to be found in any given case. Such a relationship is shown in Fig. 362. The curve is very flat and is nearly a straight line.

If the weights of a stable frictionless governor are displaced from their equilibrium position and then suddenly released they

will oscillate about the position of equilibrium. The period of oscillation will depend on the restoring force and on the mass of the governor weights. In general, a high-speed governor with small masses will have a shorter period than a low-speed governor with large masses. Moreover, other things being equal, the greater the regulation of the governor, i.e. the greater the percentage change in speed corresponding to the full movement of the governor weights,

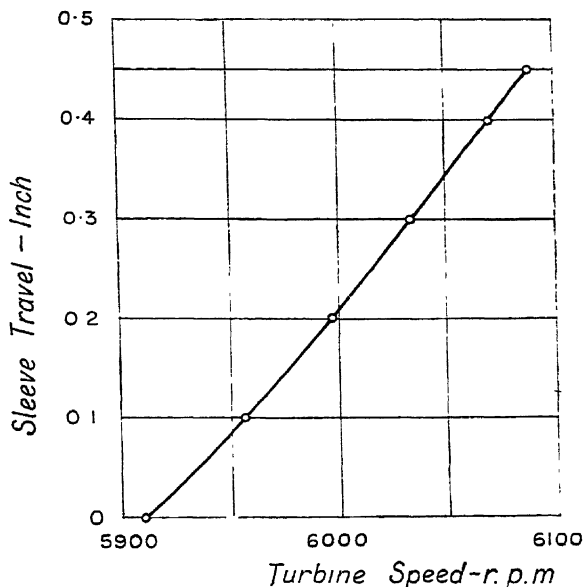


FIG. 362.

the greater the restoring force and hence the shorter the period of oscillation. Considerable damping forces are always present due to friction.

A short period governor is generally desirable so that the response of the turbine to these oscillations will be small. A further advantage of the small-mass high-speed governor is that in view of its low inertia it responds more quickly to change of speed.

261. Simple Governor without Relay. In turbines of small power in which the valves are light and the forces on them due to steam flow are inappreciable, the governor may be arranged to actuate the throttle valve directly. Fig. 363 shows a Terry turbine having a simple and robust governor of this type. The governor is mounted directly on the end of the turbine spindle and is of the Hartnell type in which the centrifugal force on the weights is balanced by the compression of the spring. In order to reduce friction, the governor weights are fulcrumed on hardened steel knife edges and

the governor push rod makes contact with a self-aligning ball bearing in the end of the lever. The governor actuates a double-seated throttle valve.

This turbine contains a single forged steel wheel in the periphery of which are milled semicircular buckets or pockets. The nozzles

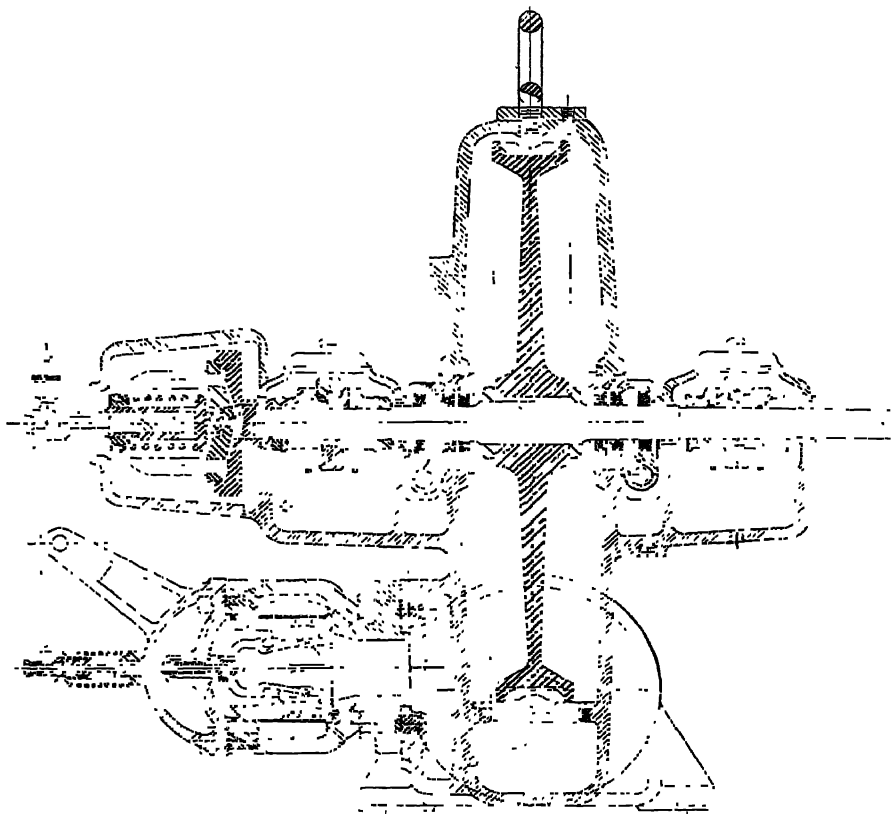


FIG. 363.—Terry Turbine with Direct-acting Governor

are arranged tangentially to the wheel and to one side of it. In addition fixed reversing chambers are arranged close to the nozzles and surrounding the wheel. The steam jet from each nozzle enters the moving buckets at a high velocity, and on leaving at a lower velocity, enters one of the stationary reversing chambers, which re-directs the steam into the pockets of the moving wheel. Thus the steam re-enters the wheel a number of times, always with reduced velocity, until its kinetic energy is expended. It is a form of velocity compounding.

262. Simple Throttle Governor with Oil Relay. Whilst the simple gear described in the previous article is satisfactory for small

turbines, it is not sufficiently sensitive for large machines, or where very close speed control is desired. The frictional forces would require a large and powerful governor to be employed. The difficulty is readily overcome by the use of a relay, a device in which the relatively small force produced by the governor for a very small

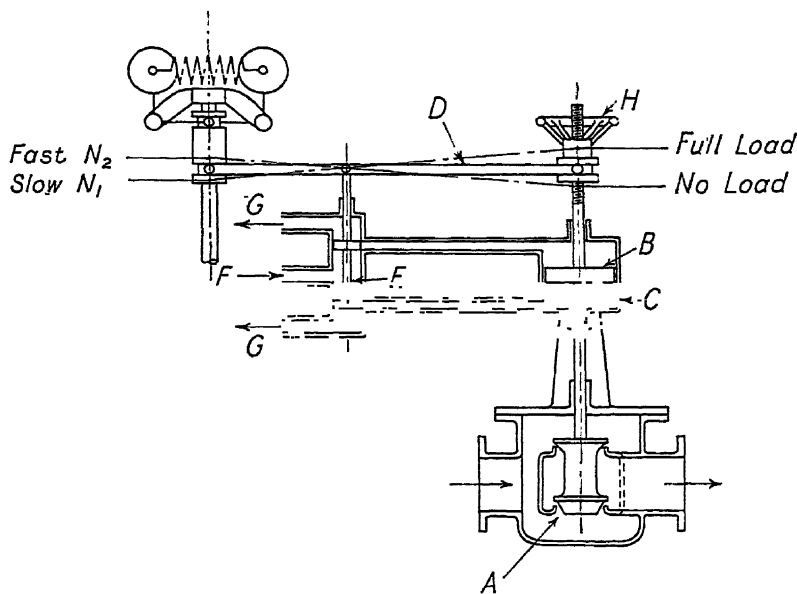


FIG. 364.—Diagrammatic Arrangement of Governor Gear with Oil Relay

change of speed is caused to produce a large force (if such is necessary) to actuate the throttle valve.

A simple differential relay is shown diagrammatically in Fig. 364. The throttle valve A is actuated by the relay piston B sliding in the cylinder C. A "floating" or differential lever D is attached at one end to the governor sleeve, at the other end to the throttle valve spindle, or an extension thereof, and at some intermediate point to a "pilot" valve E which comprises two small piston valves covering ports without any lap, i.e. the length of the valves is just equal to the length of the ports. The operating medium is usually lubricating oil supplied by a pump at a pressure of 40–60 lb. per sq. in. This is supplied to F, the pipes G being open to the oil drain tank.

The operation of the oil relay may be described as follows. We will assume first that the turbine is running at a load less than full load but still an appreciable fraction of that load. The throttle valve A will be open to such an extent that the steam flow is just sufficient to maintain a constant speed under the given load conditions. Suppose now that the load on the turbine is reduced rather

quickly. There is now an excess of energy being supplied to the turbine and the surplus will accelerate the rotor. The turbine and governor speeds will now rise and thus cause a lift of the governor sleeve. For the time being, the throttle valve spindle is stationary and the pilot valve is therefore raised. The upper port is opened to the pressure oil and the lower port to drain. The relay piston B is thus forced downwards and the throttle valve partially closed. The downward movement of the throttle valve lowers the pilot valve and so closes the ports. As soon as the oil ports are covered the relay piston is locked in position. This will occur only when the opening of the throttle valve A is correct for the load on the turbine. It will be clear that the "follow up" movement of the pilot valve, which is provided by the floating lever, effectively prevents the hunting which would occur if the same lever were fixed at the end remote from the governor instead of being attached to the throttle valve spindle.

Since, for equilibrium of the governor mechanism, the pilot valve E must always be in its central position and covering both oil ports, the position of the governor sleeve will vary according to the opening of the throttle valve. The position of the floating lever is indicated by chain-dotted lines in Fig. 364 for no-load and full load. It will be seen that at no-load the governor sleeve is high, the radius of the governor weights is the maximum and the speed N_2 (Fig. 361) is the maximum. Similarly, at full load, the governor sleeve is low, the radius of the weights is the minimum, and the speed N_1 is the minimum. These speeds are steady values for the given load conditions. If the load were suddenly removed entirely, the speed of a single turbine would rise above N_2 but would quickly fall to that value.

If N_2 = no-load speed

N_1 = full load speed

N_0 = mean speed

Then the ratio $\frac{N_2 - N_1}{N_0}$ is defined as the governor regulation. This figure may be as much as 5 or 6 per cent in small turbines, but in turbines of moderate and large output, the regulation is normally about 3 per cent.

The regulation depends upon the design of the governor and also on the throttle valve. A governor gear is said to have straight-line regulation when the speed-load curve is a straight line, in other words when equal increments of turbine load produce equal decrements of speed. This aspect of governing will be considered in greater detail later.

263. Speeder Gear. It is often necessary to adjust the speed of a turbine while it is running. This may be done by the device usually

known as the speeder gear, one form of which is shown diagrammatically in Fig. 364. The end of the floating lever remote from the governor is attached to a bush which is screwed on to the spindle of the throttle valve or of the relay piston. Thus by rotating this bush by the handwheel H, the end of D may be moved relatively to the throttle valve spindle.

If it is borne in mind that for equilibrium the pilot valve must always be in its central position, then it will be immediately clear that to raise the speed of the turbine (high governor sleeve position) it is only necessary to lower the screwed bush on the valve spindle. The first result of this is to lower the pilot valve, admit more oil to the underside of the relay piston and so increase the opening of the throttle. Conversely, to lower the speed of the turbine, the handwheel H is turned so as to raise the bush relative to the valve spindle. This process must always be carried out when paralleling alternators.

The special case of turbo-alternators which are electrically connected to the same network may now be very briefly considered. The turbines will, of course, run at the same fraction of their normal speed since they are generating at the same frequency. If the normal speeds of the turbines are identical, e.g. 3,000 r.p.m. then their speeds will also be identical for any frequency. The proportion of the load taken by the individual machines will, however, depend entirely on the adjustment of the respective speeder gears. For example, a machine which has just been paralleled will not be supplying any energy and the turbine throttle valve will be open very little. In order to increase the load on such a machine, it is only necessary to lower the bush H relative to the valve spindle. Since the turbine speed is practically constant, the position of the governor sleeve is also fixed, and any movement of the speeder gear handwheel will displace the pilot valve from its central position and so open or close the throttle valve. It will be seen, therefore, that once a turbo-alternator is connected to a network, the speeder gear may be used to vary the load on any individual turbine within its full range and to vary the distribution of load amongst the various turbines in the system. The speeder gears may also be used to alter the frequency of the alternating current generated by all the sets.

264. Alternative Form of Oil Relay. An alternative form of relay gear is shown in Fig. 365. In this case oil is admitted by the pilot valve E to the underside of the relay piston B, the upper side of the piston being loaded by a stiff spring. The lower piston only of the valve E is effective and controls the admission and release of oil to and from the relay cylinder. The upper piston of the pilot valve serves to balance the forces on the valve due to oil pressure.

A small hole is usually drilled in the relay piston to allow a slight constant leakage of oil. This prevents the formation of air pockets under the piston and merely means that the pilot valve will

normally be a little below its central position when the conditions of operation are steady.

The connection of the floating lever to the extension of the relay piston rod is made rigid and the control of speed is effected by varying the tension in the auxiliary spring H. This may be done by turning the handwheel K. The spring H provides part of the controlling force of the governor. By increasing the tension in this

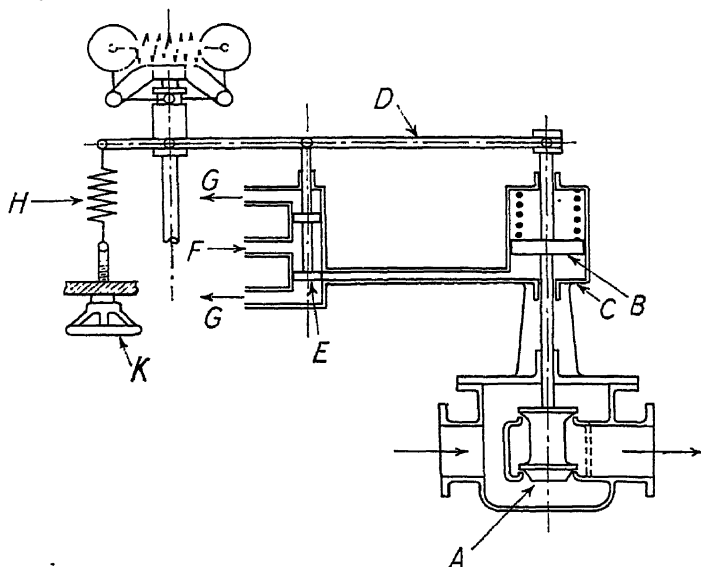


FIG. 365.—Diagrammatic Arrangement of Alternative Form of Relay Gear

spring the speed of the turbine may be raised, and by reducing the tension it may be lowered.

The operation of this type of relay gear is somewhat similar to that described in Art. 364 and needs no further explanation.

EXAMPLE 37. A power station contains two steam turbines Nos. 1 and 2. No. 1 is of 20,000 kW and has a governor regulation of 3 per cent, while No. 2 is of 50,000 kW and has a governor regulation of 4 per cent. At a given instant the total load on the station is 35,000 kW, borne as follows: Turbine No. 1, 12,000 kW and Turbine No. 2, 23,000 kW. If the total load is increased to 55,000 kW, what will be the loads on turbines 1 and 2, assuming that no alteration has been made to the governor gears of the turbines and that there is "straight-line" regulation? If the frequency at 35,000 kW is 50 per sec., what will it be when the load is 55,000 kW and the conditions are steady? The alternators are connected electrically.

Let l_1 = additional load taken by turbine No. 1.

l_2 = " " " " " " 2.

Then $l_1 + l_2 = 20,000 \text{ kW.}$

Percentage decrease of speed—

No. 1. $3l_1 \div 20,000$

No. 2. $4l_2 \div 50,000.$

Since these are equal, $l_2 = \frac{15l_1}{8}$

$\therefore \frac{23l_1}{8} = 20,000$ or $l_1 = 6,960 \text{ kW.}$
 $l_2 = 13,040 \text{ kW.}$

$\therefore$ Load on Turbine No. 1 = $12,000 + 6,960 = 18,960 \text{ kW.}$

“ “ “ 2 = $23,000 + 13,040 = 36,040 \text{ kW.}$

Percentage reduction of speed = $\frac{6,960 \times 3}{20,000} = 1.042 \text{ per cent.}$

$\therefore$ New frequency = 50×0.98958
 $= 49.479 \text{ cycles per second.}$

An alternative graphical solution is illustrated by Fig. 366. The frequency of the alternating current is plotted against power in megawatts.

Points A and B show the outputs of turbines No. 1 and No. 2, respectively, at the frequency 50 cycles per second. Since for turbine No. 1 the regulation is 3 per cent, the change of frequency for an

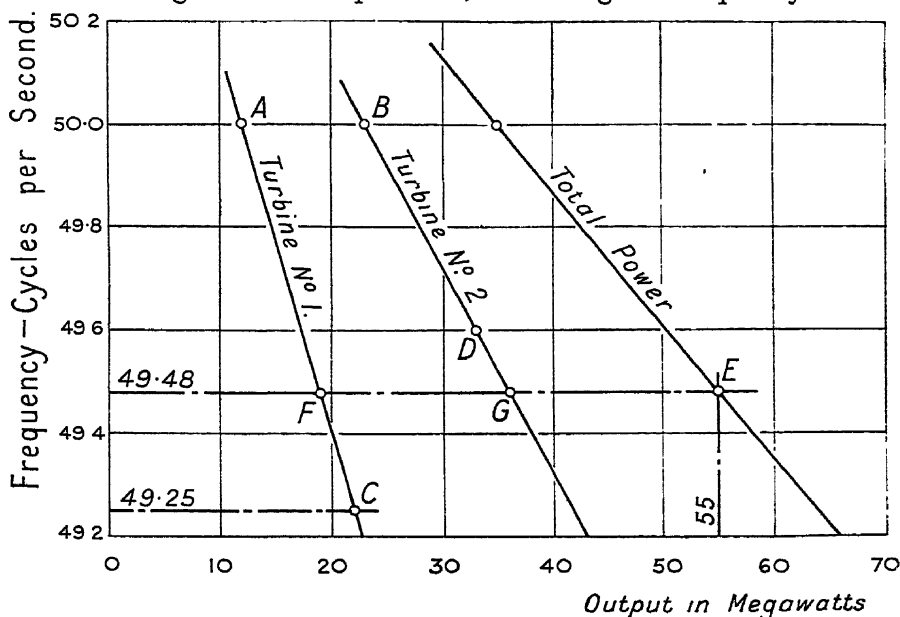


FIG. 366.—Load Distribution with Turbo-alternators Operating in Parallel

increase of output of 10,000 kW is 1.5 per cent and the frequency would be 49.25. This condition, load being 22,000 kW, is represented by the point C. Since there is assumed to be straight-line regulation, the relationship between frequency and load will be given by the straight line AC.

Similarly, an increase of 10,000 kW on turbine No. 2 reduces the frequency from 50 to 49.6, point D. The total power line is found

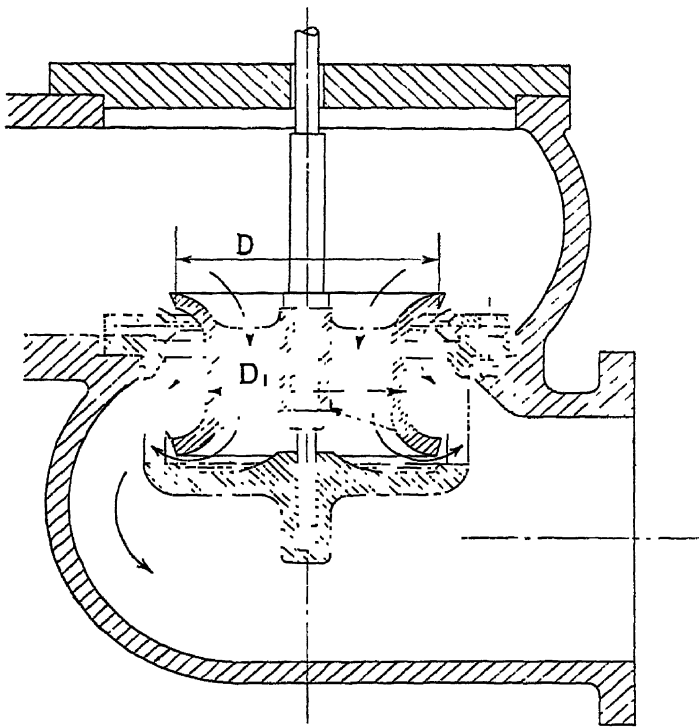


FIG. 367.—Simple Double-seated Throttle Valve

by adding the powers of turbines 1 and 2. Taking point E on this line for 55 megawatts and drawing a horizontal straight line, the outputs of turbines 1 and 2 are given (points F and G) as 19 and 36 megawatts, respectively, and the frequency as 49.48.

265. The Throttle Valve. Various forms of throttle valve are used to control the admission of steam to a steam turbine. The commonest form is the double-seated equilibrium valve in which the forces due to steam pressure are either completely or partially balanced. There are, however, certain dynamic forces on the valve caused by the flow of steam through it.

Fig. 367 shows a simple double-beat throttle valve in which the steam flows through the valve seat in a direction perpendicular to

the axis of the valve. The outer portion of the valve is attached to the central boss by ribs or wings, the number of such ribs being about three or four. The steam which passes through the lower seat flows through the inner part of the valve.

In another form of valve, shown in Fig. 368, the part of the valve which controls the actual throttling of the steam is given a special shape. The direction of steam flow in passing through the

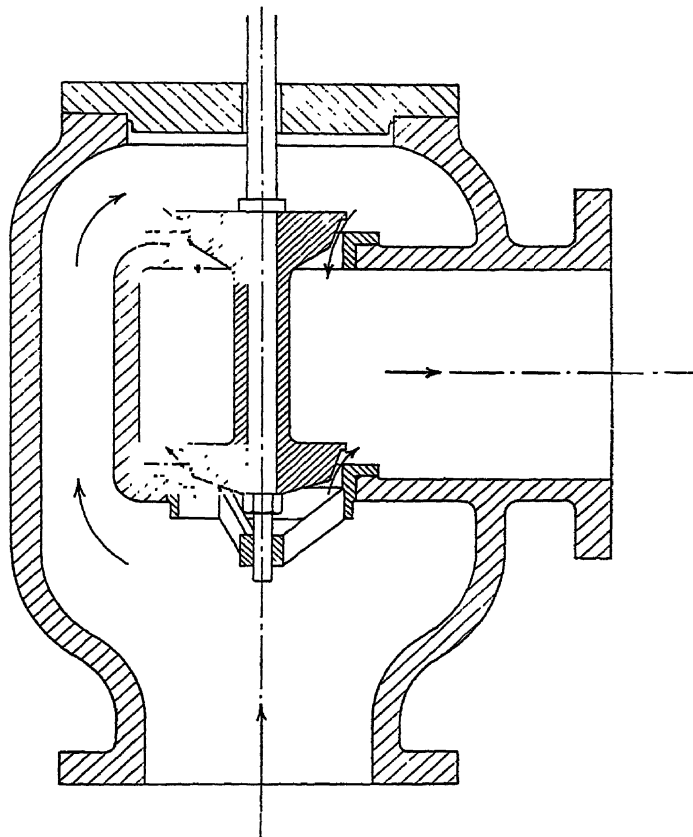


FIG. 368.—Throttle Valve with Shaped Profile

valve is more or less parallel to the valve axis. With a valve body of the type shown in Fig. 368 the valve may be made solid, but in certain cases the valve body is modified in form and part of the steam passes through the hollow valve to the upper or lower seat, as the case may be.

In the simple type of throttle valve the seat diameter is sometimes made equal to the diameter of the pipe. The larger the diameter of the valve, the smaller the required lift at full load and

the less sensitive the governor gear as a whole. Consequently, a somewhat smaller valve and a larger lift would appear to provide more stable governing. On the other hand, too small a valve leads to higher steam velocities through the centre of the valve and through the passages close to the valve seat. In shaped valves

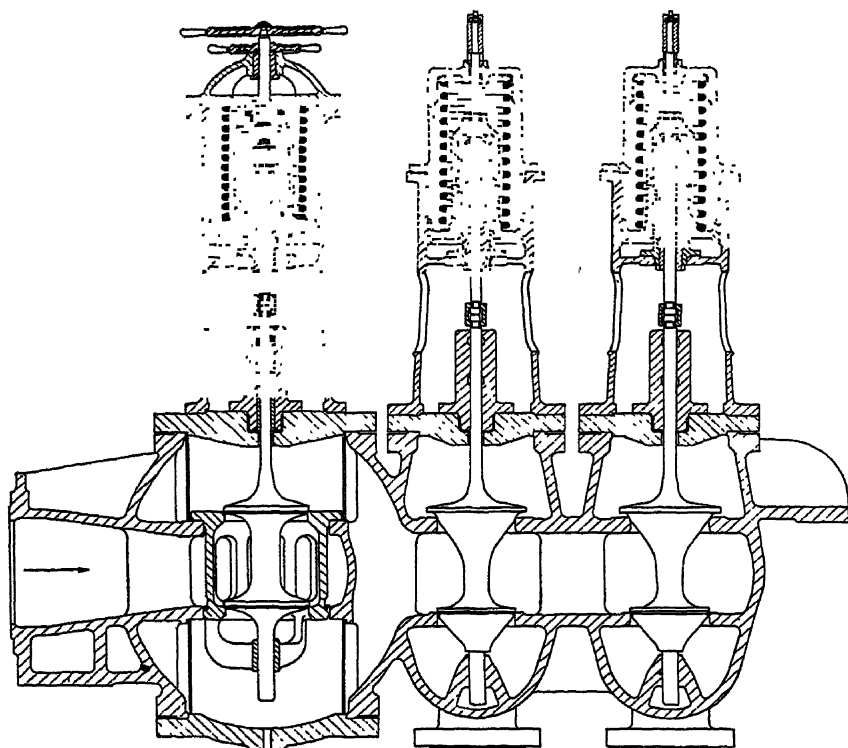


FIG. 369.—Section through High-pressure Steam Chest

of the second type, the seat diameter is normally about 0.7 times the pipe diameter.

Fig. 369 shows the section through a 3-pot steam chest for a Parsons turbine. This is bolted to the foundation alongside the turbine and connected to the casing by flexible pipes. The chest is normally made of cast steel having a chemical composition similar to that of the high-pressure part of the cylinder. The valve on the left is the emergency stop valve which shuts off automatically if the speed of the turbine should exceed a predetermined limit, or if the oil pressure should fail. The valve is usually of the double-beat type as shown, the seats being formed in a separate cage of forged steel bolted into the steam chest casting.

There are two governor valves, also double-seated. The valve

nearer the stop valve supplies steam to the high-pressure inlet branch while the second valve supplies steam to the bypass belt when the load exceeds the economic load. For high steam pressures the governor valve seats are often formed in forged steel cages similar to that used for the emergency stop valve. The muff coupling

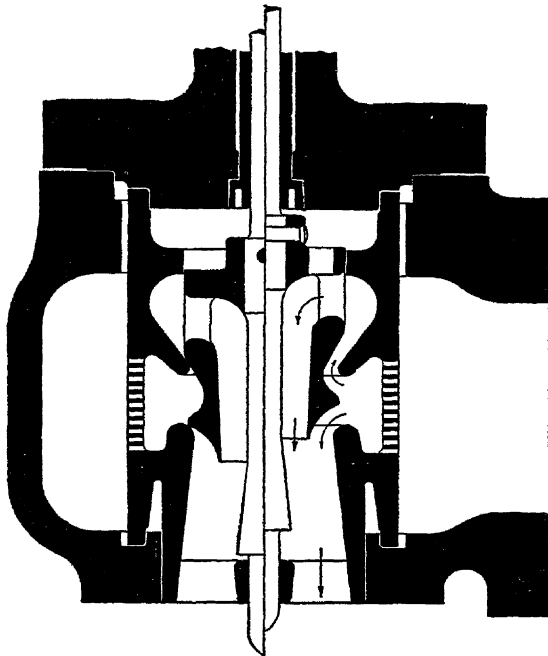


FIG. 370.—Combined Section of Double-beat Governor Valve. Left Hand Closed, Right Hand Open

between the relay piston rods and the valve stems allows for freedom of self-alignment. A steam strainer is provided between the stop valve and the governor valves to trap any foreign matter.

For high steam conditions, the type of governor valve illustrated in Fig. 370 is employed by the Metropolitan-Vickers Electrical Co., Ltd. The left-hand half of the section shows the valve closed and the right-hand half shows it in the normal open position. It will be seen that when the valve is in the open position, curved passages with gradual changes of area are provided, thus enabling the use of high steam velocities, reduced valve diameter, and less likelihood of valve seat leakage when the valve is closed. The symmetrical formation of the valve and its seatings ensure that the seal at the valve seats will not be affected by thermal expansion.

266. Flow of Superheated Steam Through Throttle Valves. In practically all cases, the steam passing through a throttle valve

is in the superheated condition. Assume that the steam pressure before the valve is P_1 and after the valve P_2 lb. per sq. ft., and that the efficiency of the valve, considered as a nozzle, is η . Then assuming that the velocity of the steam before expansion through the valve is negligibly small, the velocity V_2 through the valve is given by the expression -

$$\frac{V_2^2}{2g} = \eta \int_{P_2}^{P_1} v \cdot dP$$

Let n be the adiabatic index (1.30) for superheated steam and let m be the actual index for the frictionally resisted expansion. Then, as shown in Art. 93,

$$m = \frac{n}{n - \eta(n - 1)}$$

$$\begin{aligned} \text{Then } \frac{V_2^2}{2g} &= \eta_n \cdot \frac{m}{m - 1} \left\{ P_1 v_1 - P_2 v_2 \right\} \\ &= P_1 v_1 \left[\frac{\eta_n \cdot m}{m - 1} \left\{ 1 - r^{\frac{m-1}{m}} \right\} \right] \end{aligned}$$

$$\text{where } r = \frac{P_2}{P_1}.$$

$$\text{from which } V_2 = \sqrt{P_1 v_1 \left[\frac{2g \cdot \eta_n \cdot m}{m - 1} \left\{ 1 - r^{\frac{m-1}{m}} \right\} \right]} \quad (230)$$

Let w = steam flow through valve in lb. per second.

A_1 = total effective area required, sq. ft.

A = " " " " sq. in.

$$\begin{aligned} \text{Then } \frac{w}{A_1} &= \frac{V_2}{v_2} \\ &= \sqrt{\frac{P_1}{v_1} \left[\frac{2g \cdot \eta_n \cdot m}{m - 1} \left\{ r^{\frac{2}{m}} - r^{\frac{m+1}{m}} \right\} \right]} \\ &= C_1 \sqrt{\frac{P_1}{v_1}} \quad (231) \end{aligned}$$

$$\text{where } C_1 = \sqrt{\frac{2g \cdot \eta_n \cdot m}{m - 1} \left\{ r^{\frac{2}{m}} - r^{\frac{m+1}{m}} \right\}} \quad (232)$$

If p_1 = initial pressure in lb. per sq. in. abs.

$$\text{Then } \frac{w}{A} = C \sqrt{\frac{p_1}{v_1}} \quad (233)$$

$$\text{where } C = \frac{C_1}{12}.$$

Values of C are given in the adjoining Table and in Fig. 371.

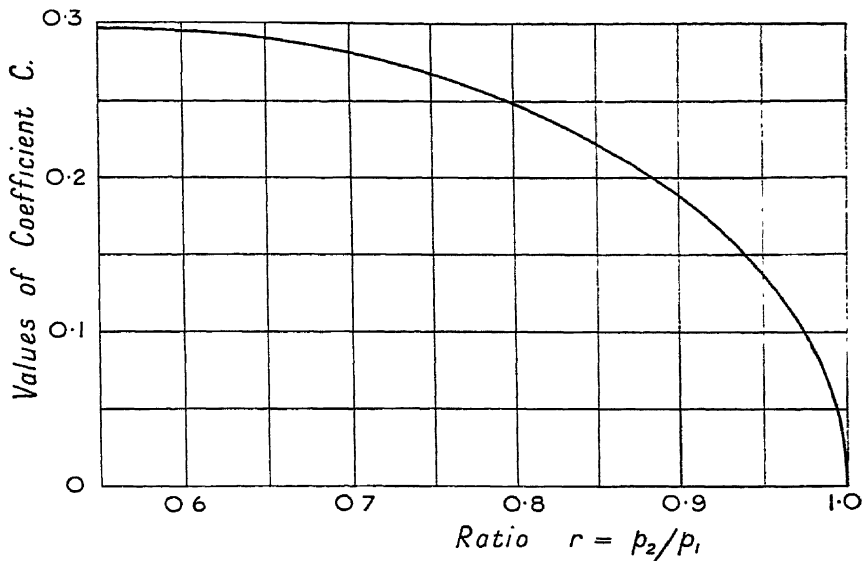


FIG. 371.—Coefficient for Discharge of Superheated Steam through Throttle Valve

TABLE XXXIV
VALUES OF FACTOR C

Assumed efficiency $\eta_n = 0.9$.

„ adiabatic index $n = 1.3$.

Actual index $m = 1.262$.

r	C	r	C
0.55	0.2963	0.93	0.1613
0.60	0.2947	0.94	0.1502
0.65	0.2901	0.95	0.1382
0.70	0.2806	0.96	0.1241
0.75	0.2674	0.97	0.1082
0.80	0.2479	0.98	0.0891
0.85	0.2236	0.99	0.0650
0.90	0.1894	0.995	0.0451
0.91	0.1799	1.000	0.0
0.92	0.1712		

267. Design of Simple Throttle Valve. We shall now consider the characteristics of the type of throttle valve shown in Fig. 367. This is best done by taking a specific example.

EXAMPLE 38. Calculate the throttle valve areas necessary for the 3,000 kW turbine of example 18, page 244, at different loads.

Hence, having selected a suitable diameter for the valve, calculate the variation of lift with turbine output and hence the variation of turbine speed with varying outputs.

The pressure in the nozzle box is 200 lb. per sq. in. abs. The area through the valve at full load will depend entirely on the steam pressure before the valve, i.e. the steam pressure after the stop valve. The greater the pressure difference across the throttle valve the greater the steam velocity through the valve and the smaller the required lift. In this case we shall assume that the steam pressure just before the throttle valve is 204 lb. per sq. in. abs., i.e. that the pressure drop across the valve is 4 lb. per sq. in., or 2 per cent of the initial pressure.

As the enthalpy after throttling, corresponding to $p = 200$ lb. per sq. in. and 150° F. superheat, is 1288.8 and as the pressure before throttling is 204 lb. per sq. in. abs. then using the original Callendar equation, we obtain the specific volume

$$v = \frac{1.2464 (1288.8 - 835.2)}{204} = 2.765 \text{ cu. ft./lb.}$$

Assume a mean steam velocity through the main steam inlet pipe = 100 ft. per second; then since the steam flow per second at full load = 10.12 lb. per second, the pipe area

$$A = \frac{10.12 \times 2.765}{100} \times 144 = 40.4 \text{ sq. in.}$$

Thus the required pipe diameter = 7.17 in., say 7 in.

Consider the area through the valve at full load.

Let w = total steam flow through the valve, lb. per sec.

A = effective area through valve, sq. in.

D = valve diameter, in inches.

l = valve lift, inches.

Then, since the valve is double-beat, the effective area is $2\pi D \cdot C_c \cdot l$, where C_c is the coefficient of contraction. The latter quantity can be determined only by experiment and its value is very uncertain. It would depend on the shape of the valve and its seat. For the purpose of this example, we shall assume that $C_c = 0.8$.

Then $A = 5.03 D l$.

Then, by eq. 233 of Art. 266,

$$A = \frac{w}{C \sqrt{\frac{204}{2.765}}} = \frac{w}{8.59 C} \text{ sq. in.}$$

Let p_0 = nozzle box pressure

p_{sv} = stop valve pressure.

Then at full load $\frac{p_0}{p_{sv}} = \frac{200}{204} = 0.980$, for which $C = 0.089$.

$$\text{Hence } A = \frac{10.12}{8.59 \times 0.089} = 13.25 \text{ sq. in.}$$

$$\text{i.e. } 5.03 D l = 13.25$$

$$\text{or } D l = 2.64.$$

Thus the valve diameter depends on the selected ratio of lift to diameter. The permissible value of this ratio depends upon the

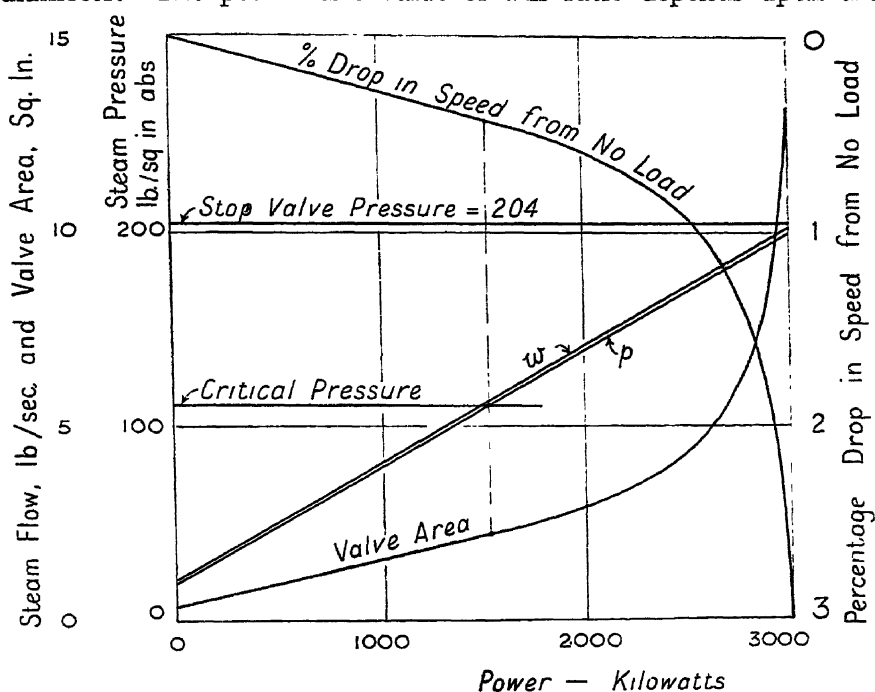


FIG. 372

ratio of the diameters D_1 to D , Fig. 367. In order to limit the velocity of the steam through the centre of the valve, this ratio is usually made about 0.76 to 0.80. Hence if the control of steam flow is to take place at the valve seat, the lift of the valve should not exceed about 0.1 of the diameter. Hence, making $l = 0.1 D$, the valve diameter $D = 5.14$, say 5 in.

$$\text{Hence area through valve} = 2\pi \times 5 \times 0.8 l$$

$$= 25.1 l \text{ sq. in.}$$

The calculation of areas is given in Table XXXV for different loads. The values of the nozzle box pressure p_0 are taken from

Fig. 372 and thus values of the ratio $\frac{p_0}{p_{sv}}$ calculated. The corresponding values of the coefficient C are then taken from Fig. 371. In the next column, values of the steam flow w are set down from Fig. 372. The required effective area is then calculated by eq. (233) and from this the valve lift is calculated. In the next column of the table, the valve lift from the no-load position is set down by deducting the lift at no-load from all values of the lift. In a simple linkage between the governor sleeve and the throttle valve spindle, such as that shown in Fig. 364, the travel of the governor sleeve is directly proportional to the valve lift, and if the relationship between sleeve travel and governor speed is taken to be linear, then the percentage drop in speed from the no-load speed in the given case is—

$$\frac{l_1}{0.512} \times \text{Governor Regulation,}$$

l_1 being the valve lift from the no-load position. Assuming the governor regulation to be 3 per cent in this case, we obtain the speed changes given in the last column of the table.

TABLE XXXV
CALCULATION OF THROTTLE VALVE LIFTS AND TURBINE SPEED CHANGES
WITH VARIATION OF LOAD IN A 3,000 KW STEAM TURBINE

Load kW	p_0	$\frac{p_0}{p_{sv}}$	C	w	Effective Area A sq. in.	Valve Lift l inches	Lift of Valve from No-load Position	% Drop in Speed
0	20	0.098	0.2963	1.012	0.398	0.016	0	0
500	50	0.245	"	2.532	0.995	0.040	0.024	0.140
1,000	80	0.392	"	4.052	1.592	0.063	0.047	0.275
1,500	110	0.540	"	5.562	2.182	0.087	0.071	0.415
1,600	116	0.569	0.296	5.872	2.306	0.092	0.076	0.445
1,800	128	0.628	0.292	6.479	2.58	0.103	0.087	0.509
2,000	140	0.686	0.283	7.086	2.92	0.116	0.100	0.585
2,200	152	0.745	0.269	7.693	3.33	0.133	0.117	0.684
2,400	164	0.805	0.246	8.305	3.93	0.157	0.141	0.825
2,600	176	0.863	0.215	8.907	4.82	0.192	0.176	1.030
2,800	188	0.922	0.170	9.513	6.51	0.260	0.244	1.430
2,900	194	0.951	0.137	9.816	8.35	0.333	0.317	1.850
3,000	200	0.980	0.089	10.12	13.25	0.528	0.512	3.000

On examining Table XXXV and Fig. 372 it will be seen that up to about half load, the drop in pressure across the throttle valve exceeds the critical, and the required area through the valve is directly proportional to the steam flow. Moreover, the percentage

drop in speed is uniform. At loads greater than half load, however, the pressure drop across the valve is less than the critical and, as full load is approached, the velocity of flow through the valve falls to such an extent that the required valve area, and hence the valve lift, increases rapidly. In association with this, the speed changes quite rapidly as full load is approached, unless some adjustments are made by means of the speeder gear.

268 Modified Linkages to give a Closer Approach to Straight-line Regulation. In order to obtain a more uniform change of speed

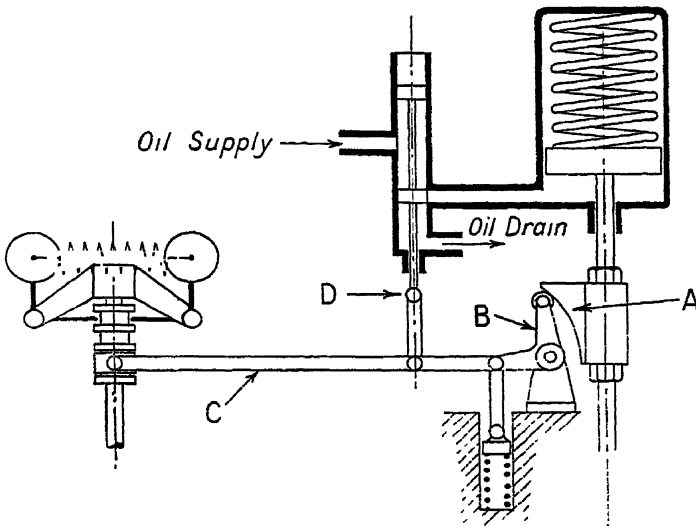


FIG. 373.—Diagrammatic Arrangement of Relay Gear Modified to give Approximately Straight-line Regulation

with change of load, the linkage between the governor and the throttle valve spindle is sometimes modified. Such a modification is shown in Fig. 373. Here a plate cam A is fixed to the throttle valve spindle; this actuates a bell crank lever B to which the floating lever C is attached. The curvature of the cam is greatest at the top and least at the bottom. Bearing in mind that, for a steady condition, the point D must be fixed, it will be clear that as the lift of the throttle valve is increased towards the full-load lift, the

$$\frac{\text{Governor sleeve travel}}{\text{Throttle valve lift}}$$

ratio diminishes, thus reducing the change of speed which would otherwise take place above half load.

Another form of gear, described in a paper by Mr. C. D. (now Sir Claude) Gibb, is shown in Fig. 374. Here a toggle joint A causes

the upper end of the link B to move in a slot machined in the floating lever C. The increased lift of the throttle valve above half load again causes the ratio of governor sleeve travel to throttle valve lift to fall off as the full-load lift is approached.

The satisfactory operation of such gears would appear to be dependent on the maintenance of a steady steam pressure at the

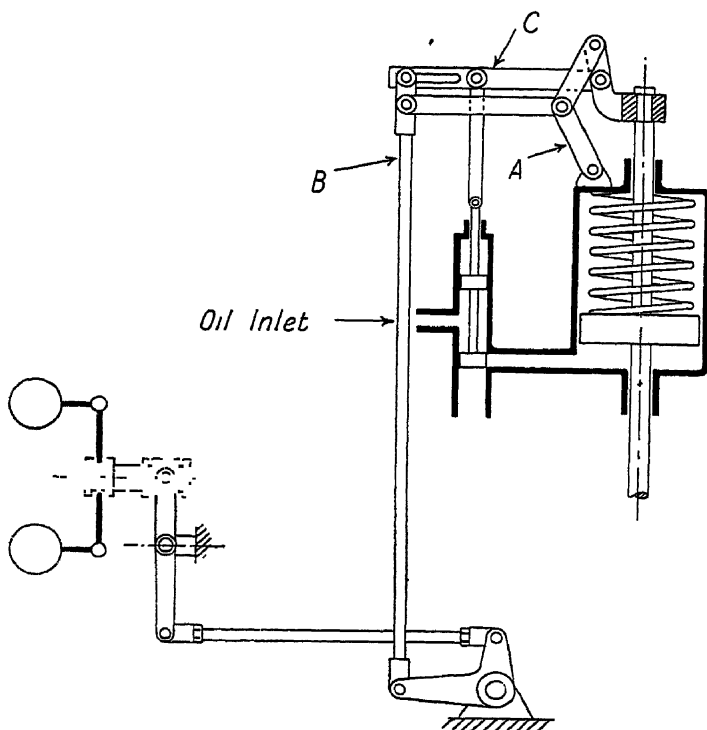


FIG. 374.—Diagram of Relay Gear with Toggle Joint Mechanism to give Closer Approximation to Straight-line Regulation

stop valve; for if, at full load, the stop valve pressure is higher than the normal value, the greater pressure drop across the throttle will enable the necessary steam flow to be maintained with a smaller lift than normal and hence with a smaller variation of speed from the no-load value.

269. Design of Shaped Throttle Valve. The area through the valve shown in Fig. 368, corresponding to any value of the lift, depends on the shape of the tapered part of the valve. By selecting a suitable shape, an arbitrary relationship between load and turbine speed may, to a certain extent, be obtained. But there are limits imposed by the conditions of operation as will be shown by means of an example.

The diameter of the valve is generally made from 0.7 to 0.75 times the diameter of the steam pipe, but the ratio is, of course, quite arbitrary. The lift is also arbitrary but may be made from 0.12 to 0.2 times the diameter of the valve. The coefficient of contraction is probably not the same for the two seats owing to the different conditions of approach.

EXAMPLE 39. Derive the shape of a throttle valve suitable for the 3,000 kW turbine of example 18, page 244.

The pipe diameter will be made the same as in the previous example, viz. 7 in. Taking the valve diameter as 0.7 times the pipe diameter, we have a diameter of 4.9 in., say, 5 in. We shall assume that the lift of the valve from the no-load position is equal to $0.2D$, i.e. 1 in.

We shall first attempt to design the valve so as to obtain straight-line regulation, i.e. the change in speed, plotted against load, is represented by a straight line. Since the relationship between governor travel and speed is very nearly linear, we may assume with very small error, that the relationship between valve lift and load is also linear. On this basis the data in Table XXXVI were obtained for two values of the stop-valve pressure, viz. 205 and 210 lb. per sq. in. absolute. The method of calculating the effective area is the same as that used in the preceding example and need not be given in detail. In calculating the "opening" x of the valve, Fig. 375, it is assumed that the coefficient of contraction is 0.8 for the upper seat and 0.9 for the lower seat. Hence the total effective area—

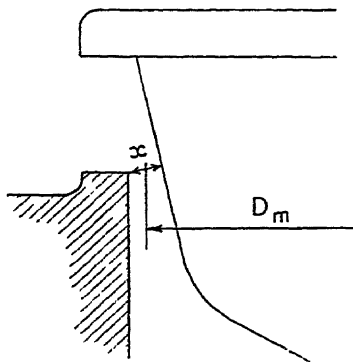


FIG. 375

$$\begin{aligned} A &= \pi D_m \cdot x (0.8 + 0.9) \\ &= 5.34 D_m \cdot x. \end{aligned}$$

x is calculated by successive approximations.

In obtaining the shape of the valve, the lift is set down to an enlarged scale, as shown in Fig. 376, as though the valve remained stationary and the valve seat moved downwards as the valve opened. The various "openings" are then set out as arcs of circles and a curve drawn tangential to them. It will be seen that this method is successful up to a lift of 0.825 when $p_{sv} = 205$, and 0.875 when $p_{sv} = 210$. Above these lifts, the openings increase so rapidly that there is interference. It will be seen that the interference is smaller at the higher pressure, and it is quite possible to obtain a valve shape which will result in straight-line regulation provided the pressure drop across the throttle valve is made sufficiently high.

The resulting change in the upper part of the valve is relatively small.

We shall now set out a practical shape of valve for the second case, i.e. when the pressure before the throttle is 210 lb. per sq. in. absolute. First set down the full lift of one inch from the no-load position, Fig. 377. To the same scale draw an arc of radius 0.344 in.

TABLE XXXVI
CALCULATION OF "OPENING" OF THROTTLE VALVE REQUIRED TO GIVE
STRAIGHT-LINE REGULATION

Lift of Valve above No-load Position (Inches)	Load kW	$p_{sv} = 205$		$p_{sv} = 210$	
		Effective Area sq. in.	"Opening" $\times$ Inch	Effective Area sq. in.	"Opening" $\times$ Inch
0	0	0.396	0.0149	0.386	0.0145
0.1	300	0.751	0.0283	0.733	0.0276
0.2	600	1.107	0.0418	1.080	0.0408
0.3	900	1.460	0.0554	1.426	0.0541
0.4	1,200	1.820	0.0692	1.774	0.0675
0.5	1,500	2.175	0.0829	2.122	0.0810
0.6	1,800	2.560	0.0980	2.480	0.0950
0.7	2,100	3.075	0.118	2.962	0.113
0.8	2,400	3.87	0.149	3.66	0.141
0.825	2,475	4.16	0.161	3.91	0.151
0.85	2,550	4.48	0.174	4.18	0.162
0.875	2,625	4.88	0.190	4.49	0.174
0.9	2,700	5.40	0.211	4.90	0.191
0.925	2,775	6.06	0.238	5.40	0.212
0.95	2,850	6.99	0.277	6.04	0.237
0.975	2,925	8.68	0.350	6.97	0.276
1.0	3,000	11.98	0.500	8.55	0.344

as shown. Reference to Table XXXVI will show that the "opening" for straight-line regulation varies uniformly from no-load up to a load of about 1,800 kW when the lift is 0.6 in. and the opening 0.095 in.

Draw arcs of circles from centres A and B with radii of 0.0145 in. and 0.095 in., respectively for lifts of 0 and 0.6 in. from the no-load position. Draw a straight line CD tangential to these arcs, and draw BD perpendicular to CD. The required profile of the valve below point D may be any curve to which CD is a tangent and which touches the circle of 0.344 in. radius. Assume that the curve is a circular arc. The centre of this lies on BD produced and the radius, found by trial, is 0.15 in. By tapering the valve uniformly up to the seat, a lift of 0.075 in. is obtained at no-load for a radial clearance of 0.005 in.

By setting off the various openings required for different loads we obtain the corresponding valve lifts. For example, setting off the distance 0.237 in. for 2,850 kW we obtain the lift 0.871 in. Assuming the change in speed to be approximately proportional to the movement of the governor sleeve the speed at any load may

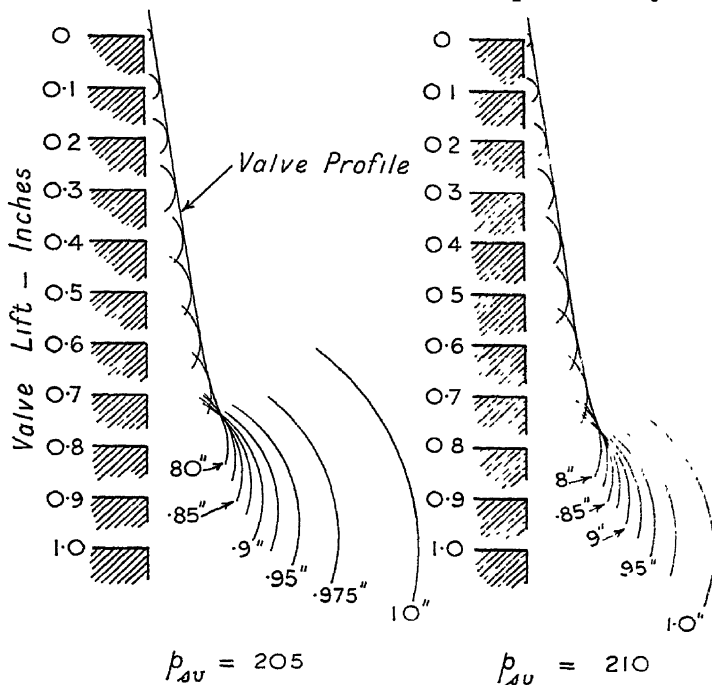


FIG. 376.—Derivation of Valve Profile

be calculated. These figures are given in Table XXXVII and Fig. 378. It will be seen that this valve profile gives a close approximation to straight-line regulation.

270. Nozzle Control Governor Gear. Automatic control of a number of separate nozzle valves may be effected in several ways. Each valve may have its own relay cylinder and pilot valve, the linkage being arranged so that with increasing load the valves are opened in a certain order, and with a decreasing load they close one after another in the reverse order. Alternatively, the valves may be opened mechanically by a single relay cylinder and a system of levers and links in which there is a certain amount of lost motion. In another method, the nozzle valves are arranged in a row, e.g. on top of the turbine, and opened, in order, by levers actuated by cams.

Fig. 379 shows a B.T.H. turbine employing this form of governor gear, Fig. 380 shows a diagrammatic arrangement, while Fig. 381 gives a section through a nozzle valve.

Each cam is positive in its action and consists of a pair of discs carrying a roller in the space between them. This engages with a horse-shoe shaped lever which, as the cam rotates, either opens or closes a nozzle valve. The cam shaft is rotated by a servo motor of the moving blade type, the operating fluid being lubricating oil

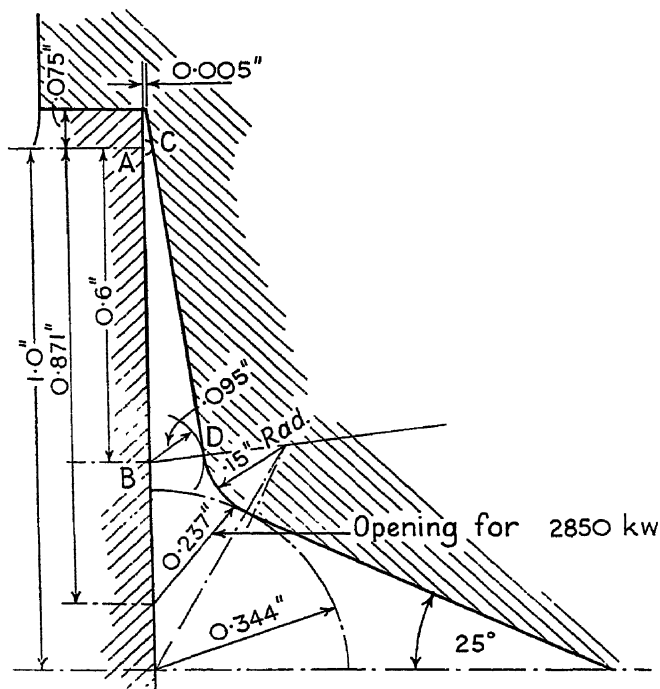


FIG. 377

under a pressure of about 50 lb. per sq. in. A small pinion on the end of the cam shaft engages with a rack which is connected to one end of the floating lever and moves the pilot valve back to its mid-position when any rotation of the cam shaft has taken place. Oil exhausted by the servo motor and also oil passing through the high-pressure by-pass valve is supplied to the bearings at a pressure of 5 to 10 lb. per sq. in.

The action of the governor may now be briefly described. If, owing to an increase of load on the turbine, the speed should fall slightly, the governor sleeve will move downwards, so raising the pilot valve above its central position, in which position it covers both ports. A certain amount of high-pressure oil is thus admitted to one side of the moving blade of the servo motor and an equal amount is expelled from the other side. The rotation of the cam shaft will either open another valve or increase the lift of the one

last opened. As the cam shaft rotates, the pinion and rack tend to move the pilot valve back to its central position. Equilibrium of the

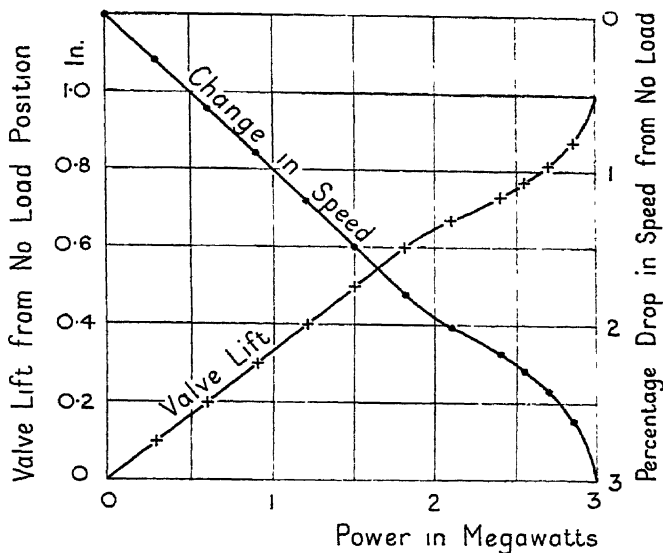


FIG. 378.—Characteristics of Throttle Valve Shown in Fig. 377

TABLE XXXVII

THROTTLE VALVE LIFT AND SPEED CHANGES FOR A 3,000 kW STEAM TURBINE

Note. Governor Regulation assumed to be 3 per cent.

Load kW	Valve Lift from No-load Position Inches	% Drop in speed from No-load	Load kW	Valve Lift from No-load Position Inches	% Drop in speed from No-load
0	0	0	2,100	0.671	2.013
300	0.100	0.3	2,400	0.730	2.190
600	0.200	0.6	2,550	0.767	2.301
900	0.300	0.9	2,700	0.807	2.421
1,200	0.400	1.2	2,850	0.871	2.613
1,500	0.500	1.5	3,000	1.000	3.000
1,800	0.600	1.8			

gear is only possible when the pilot valve is in its central position and thus in order to meet the steam demand due to an increase in load there will be a definite reduction in turbine speed, and vice versa.

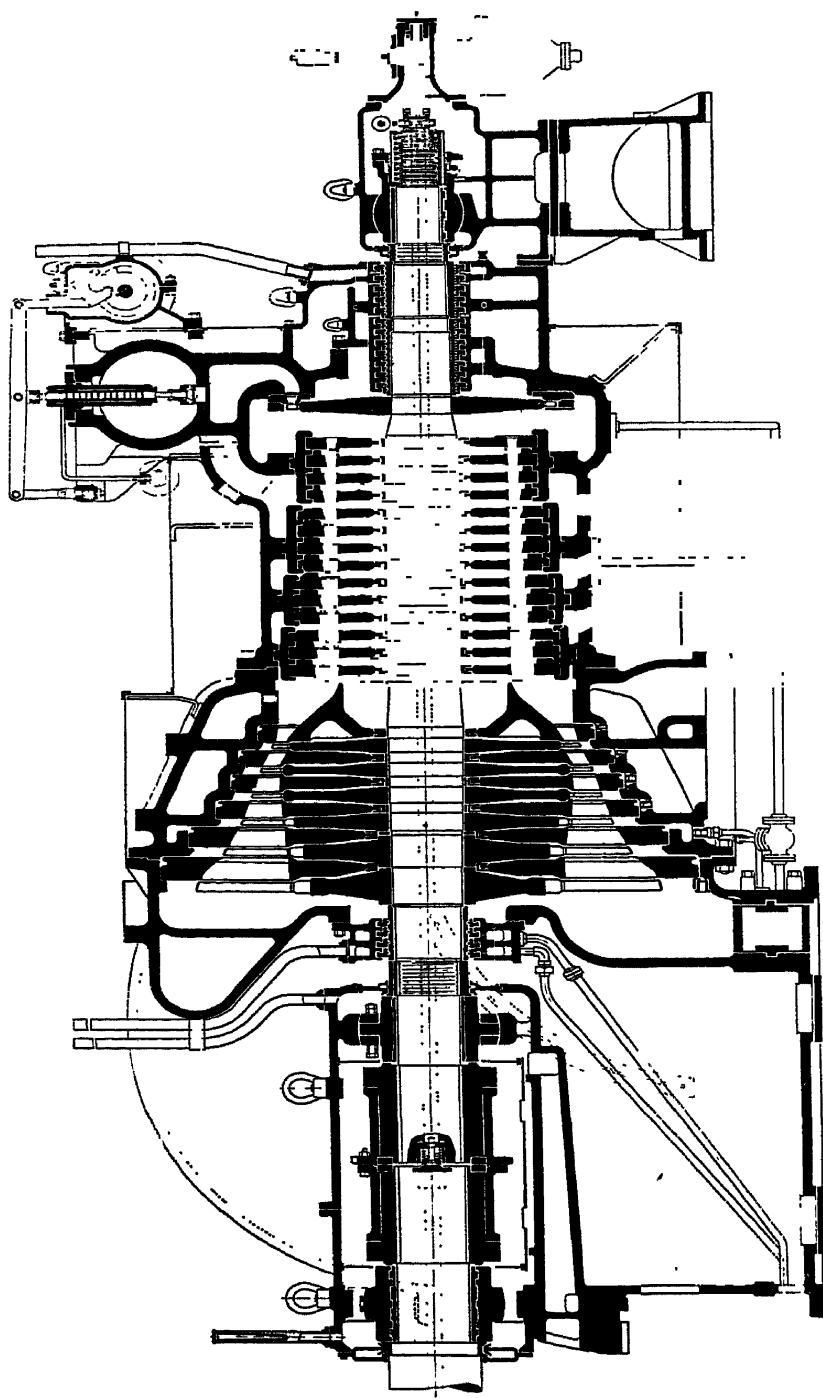


FIG. 379.—Sectional Elevation of B.T.H. Turbine, with Nozzle Control Governor Gear

271. Oil Pressure Governing without Linkage. A relatively simple and easily adaptable form of governor gear is shown diagrammatically in Fig. 382. The governor actuates a small piston valve A, balanced by a similar piston B. Oil under pressure is supplied to the pipe C, and through the port D to the space between A and B. A further port E, when uncovered by the valve A, permits oil to

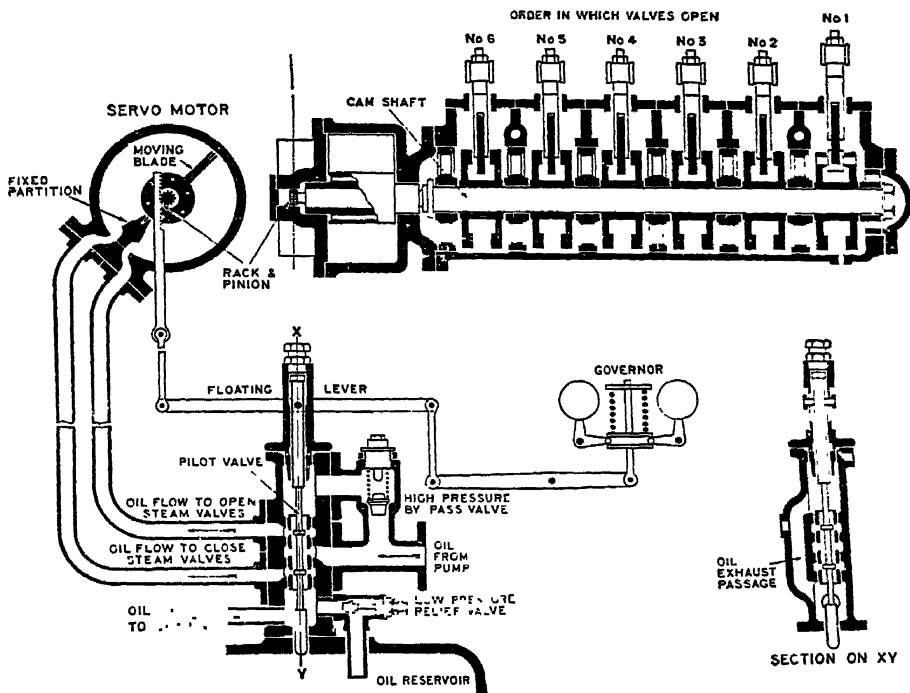


FIG. 380.—Diagrammatic Arrangement of Nozzle Control Governor Gear for B.T.-H. Turbine

escape. The pipe C is also coupled to the underside of the relay cylinder, the relay piston being spring-loaded on its upper side.

The volume of oil delivered by the oil pump will be nearly constant at all loads, and the pressure of the oil in the pipe C will depend on the port opening l ; the larger the opening the lower the pressure, and vice versa. But the oil pressure also depends on the lift of the throttle valve, for if this be increased the compression of the spring above the relay piston is greater and the oil pressure under the piston must also be greater. Consequently, there is a correspondence between the lift of the throttle valve and the position of the piston valve A, which, in turn, depends on the speed of the turbine.

It will be seen from a study of Fig. 382 that, just as in the ordinary type of relay with links, an increase of load is accompanied by a

reduction in speed, and vice versa. A form of speeder gear is shown in Fig. 382. The sleeve in which the ports are cut may be moved inwards or outwards by means of the handwheel F, rotation of the sleeve being prevented by the feather G. To raise the speed, the sleeve is moved inwards. This reduces the opening *l*, raises the oil pressure, and increases the lift of the throttle valve. Conversely, the speed is lowered by moving the sleeve outwards.

The same type of gear may be used with nozzle-control valves.

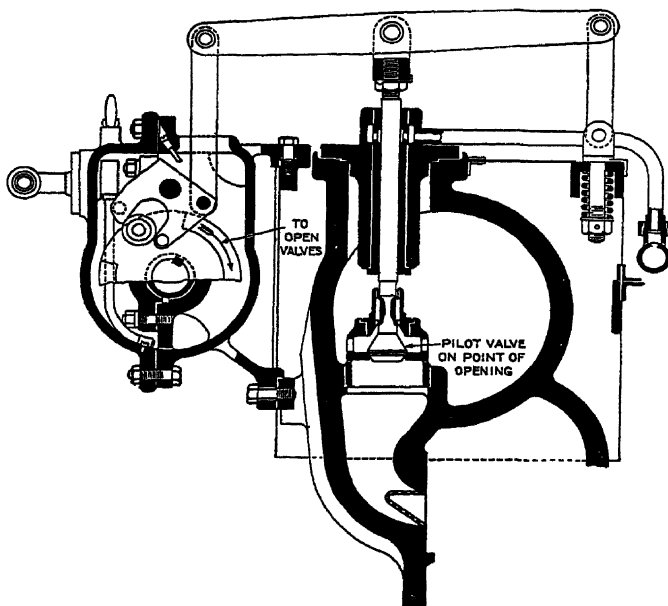
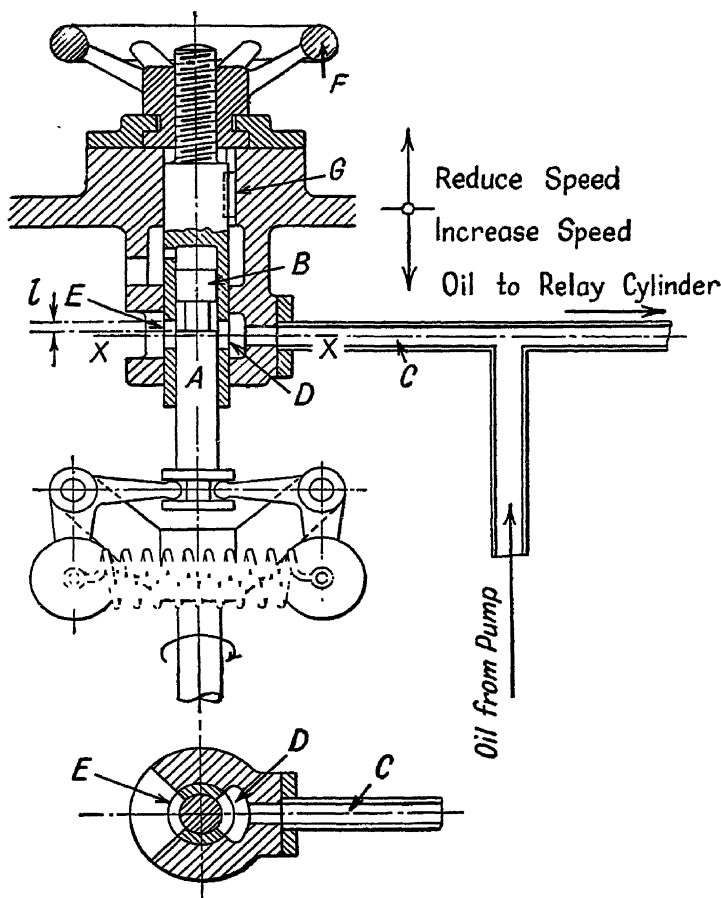


FIG. 381.—Sectional View of Nozzle Control Valve with Cam-operated Gear

Here each valve has its relay piston, but the spring loads are made progressively greater so that as the oil pressure is increased, due to a drop in speed, the valves open in turn.

272. Compound Oil Relay. In turbines with large throttle valves, or those operating with high-pressure steam, the dynamic forces on the throttle valve may be appreciable, and are sometimes sufficiently large to interfere with the satisfactory operation of the relay gear described in the previous article. A compound relay is sometimes employed in such cases. One type is shown in Fig. 383. There are two oil systems operating at different pressures. The first, operating at a variable pressure, is controlled by the governor and is coupled to A, Fig. 383. The second, which is kept at a constant and relatively high pressure, is connected to B. Thus the governor regulates the oil pressure under the spring-loaded piston

C, and hence controls the position of the pilot valve D. The duty of the governor is, therefore, light and the sensitivity of the gear is not affected by dynamic forces on the throttle valve. The valve D works in the hollow piston rod of the main relay piston E, and



SECTION THROUGH X-X

FIG. 382.—Diagrammatic Arrangement of Oil Relay Gear without Links

oil admitted at B has free access to the space under D. If, due to an increase of load, the speed falls, the increase of oil pressure under C raises D and admits high-pressure oil to the underside of E. This is forced upwards until the pilot valve D again covers its port. Similarly, a drop in load causes an increase in speed, the oil pressure under C is reduced, C and D move downwards, and some of the oil under E is allowed to escape into the space above E. The throttle

is thus partially closed until there is equilibrium. Provision is made for continuously draining the spaces above the pistons C and E.

An alternative form of relay operating on a similar principle and

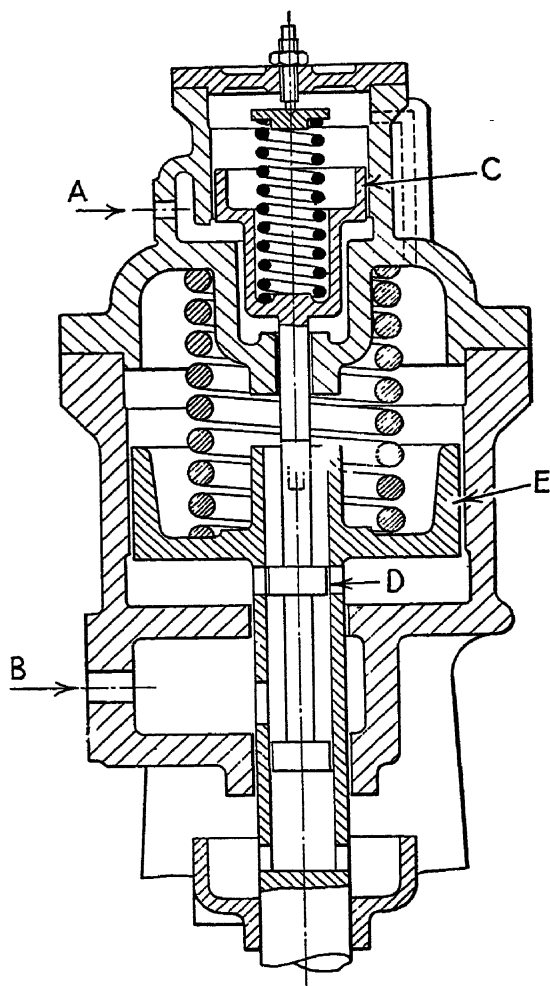


FIG. 383.—Compound Relay Gear

employed on turbines constructed by Messrs. C. A. Parsons & Co., Ltd., is shown diagrammatically in Fig. 384.

At the high-pressure end of the turbine there is a cross shaft driven by worm gearing from the turbine spindle and carrying at one end the speed governor and at the other end two oil pumps. One of these is a low-pressure pump supplying lubricating oil and the other a high-pressure pump A supplying oil for governing. The

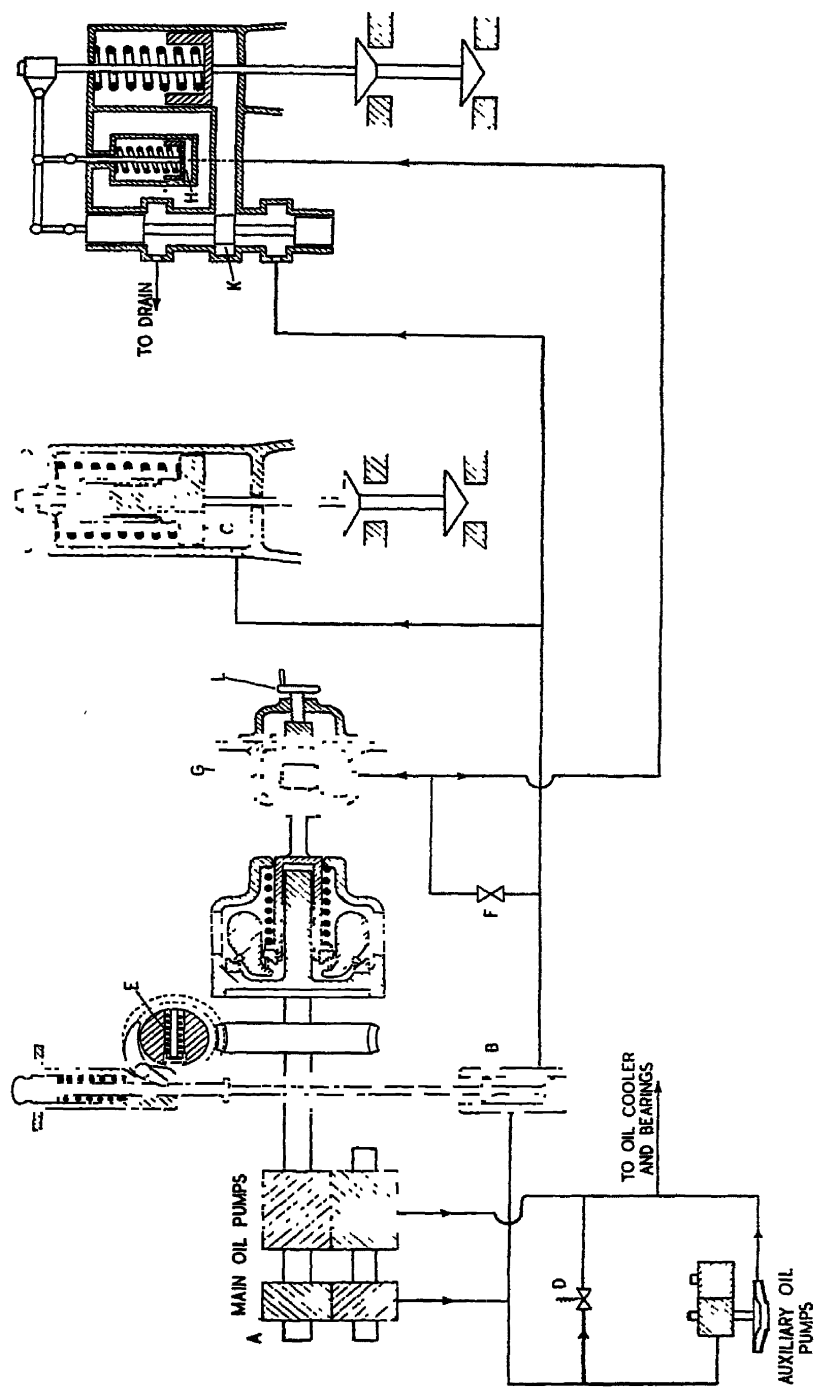


Fig. 384.—Governing System for Parsons Turbine

pressure of the oil supplied for governing is kept sensibly constant by a relief valve D which discharges to the lubricating oil supply system.

The governing oil passes through the trip gear B to the piston C of the combined emergency stop valve. A branch from the pressure oil pipe passes through the control valve F to a space surrounding the ported governor bush G. A piston valve sliding in G and operated by the speed governor regulates the opening of the port, the rate of oil leakage, and hence the oil pressure in the pipe after the control valve F. The oil pressure, thus controlled by the speed governor, does not act direct on the main relay piston, but on an auxiliary piston H which actuates the pilot valve K. It will be seen that for equilibrium, since the pilot valve must occupy its central position covering the oil port, the throttle valve will follow the movement of the piston H, the ratio of their displacements depending only on the dimensions of the linkage.

It will be clear that a small displacement of H, producing a displacement of the pilot valve K will admit the high-pressure relay oil to the relay piston and so provide a large restoring force. The control valve F acts as a choke and prevents the flow of oil, through the ports in the sleeve G and to the cylinder H, from affecting the oil pressure in the relay pipe.

If the turbine speed should rise due to a reduction of load, the governor will move the piston in G to the right, thus increasing the area of the open port, reducing the pressure of the oil under the piston H and partially closing the throttle valve. A fall in turbine speed will cause the reverse effects. The speed of the turbine may be changed by rotating the wheel L which moves the sleeve G inwards or outwards according to whether the speed is to be reduced or increased.

If the emergency governor E should come into operation, it trips the vertical plunger and lifts the piston valve in B. This releases the oil pressure from under C and H, and causes the emergency valve and main throttle to close.

The combined emergency stop valve is shown in greater detail in Fig. 385. The upper part of the valve spindle carries a piston J, to the underside of which oil from the power supply is admitted. The piston is held down against the pressure of this oil by a hand-controlled sleeve, the end of which abuts against a shoulder H on the piston. In order to open the valve, the sleeve is slowly raised by turning the handwheel, when the piston will rise under the action of the oil pressure, the shoulder H being kept all the time in contact with the end of the sleeve.

Should the oil pressure fail at any time, due to the operation of the trip gear or for any other reason, the valve will at once shut automatically. The piston will be forced downwards by the strong spring above it, returning the valve rapidly to its seating. As soon as the piston begins to move, the shoulder H will lose contact with

the end of the sleeve, so that any oil remaining under the piston will be free to escape through ports in the piston which are normally kept closed by the end of the sleeve. To limit the flow of oil when

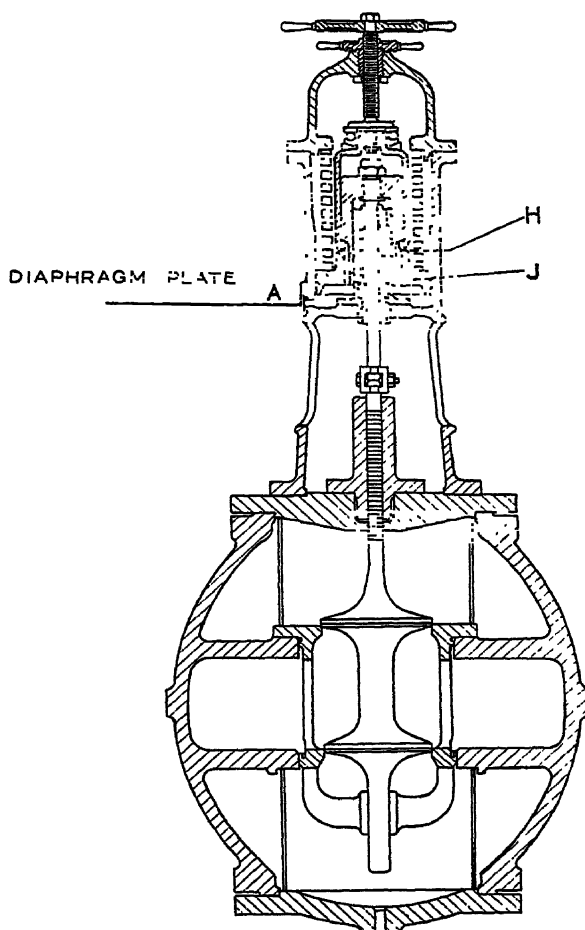


FIG. 385.—Combined Emergency Stop Valve for Parsons Turbine

the ports are uncovered, a diaphragm, pierced only by a small hole, is fitted at A in the supply pipe.

273. Governor Gear for Mixed-pressure Turbine. The function and operation of mixed-pressure turbines has already been explained in Chapter XIV. A typical governor gear for such a turbine is shown diagrammatically in Fig. 386. The gear is designed to maintain a sensibly constant speed of rotation and to utilize only low-pressure steam so long as a supply of such steam is available. It will

be clear that if the supply of low-pressure steam should fail then its pressure would rapidly fall. Thus the governor gear comprises two main elements, a speed-responsive element, i.e. the ordinary centrifugal governor A, and a pressure-responsive element, or "pressure governor" B.

The speed governor controls the relay piston C in the usual manner and this, in turn, controls the position of the point D on the

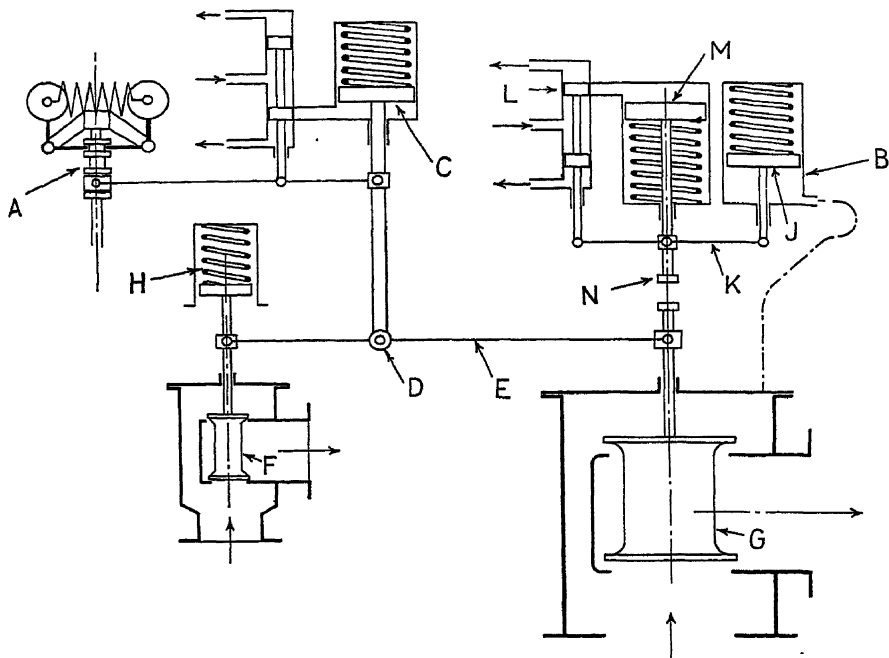


FIG. 386.—Diagrammatic Arrangement of Governor Gear for Mixed-pressure Turbine

link E, which is connected at one end to the high-pressure throttle valve F and at the other end to the low-pressure throttle valve G. A spring H is compressed to such an extent that, taking into account the weight of the valves F and G and of their spindles, etc., the moment of the forces on the left of D is greater than that of the forces on the right of D. Consequently, provided the valve G is free to lift, any upward movement of the point D will cause the l.p. throttle valve to open to a greater extent, whereas any downward movement of D would partially close the valve. Meanwhile the h.p. throttle valve is held shut. Under these conditions, the governor gear operates in the same manner as any normal relay gear, a fall in turbine speed due to increase of load causing the lift of the l.p. throttle valve to increase, and vice versa. Moreover, only l.p. steam is being used.

The pressure governor B comprises a spring-loaded piston J subjected on its underside to the pressure of the available l.p. steam. The piston rod is connected through the floating lever K to the pilot valve L which controls the supply of oil to the relay piston M. The piston rod N is not fastened in any way to the spindle of the throttle valve G, but downward movement of M and N may cause the valve G to be entirely or partially closed. It will be clear

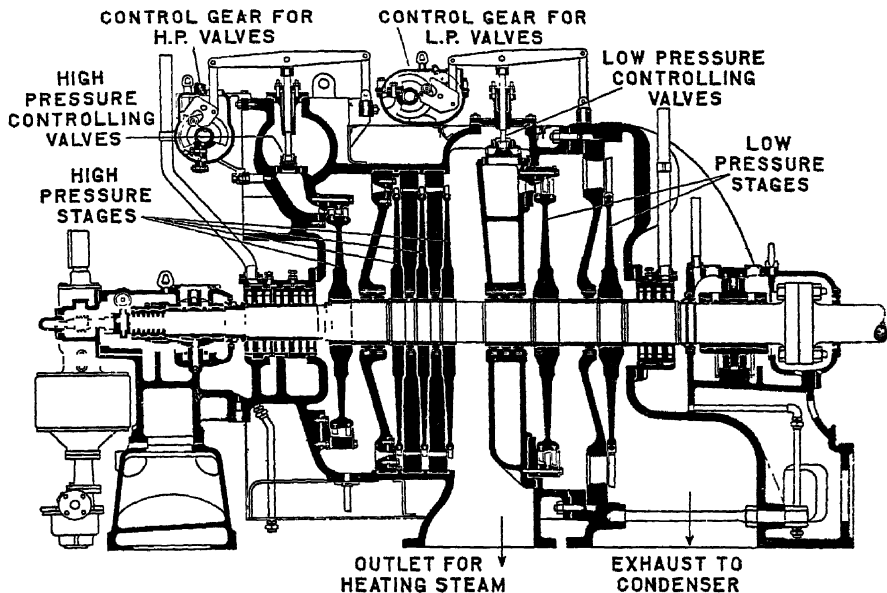


FIG. 387.—Sectional Elevation of B.T.-H. Pass-out Turbine with Cam-operated H.P. and L.P. Valve Gear

without further explanation that as the pressure of the l.p. steam falls, the relay piston M moves downwards, partially closing the valve G. Any such closure of the valve G causes the h.p. throttle valve F to open, thus supplying live steam to the h.p. stages of the turbine. These are mixed-pressure conditions of operation. If the supply of l.p. steam fails completely, then the valve G is forcibly closed and the turbine runs as a high-pressure unit.

274. Governor Gear for Pass-out Turbine. The general principles of the pass-out turbine and its governor gear have been outlined in Chapter XV. A sectional elevation of a British Thomson-Houston pass-out turbine is given in Fig. 387. There are four impulse stages in the high-pressure section and two in the low-pressure section. A two-row velocity wheel forms the first stage in each section. Nozzle control governing similar to that described in Art. 270 is employed for both the h.p. and l.p. controlling valves, the valves being cam-operated in each case.

The advantages of interconnecting the speed governor and pressure regulator have already been explained in Art. 127. The governors for the B.T.-H. turbine, shown diagrammatically in Fig. 388, are interconnected in the same way. A point on the h.p. rack forms a fulcrum for the floating lever which is connected to each of the pilot valves.

If the action of the mechanism is traced out, it will be seen that if the power load should be reduced while the heating load remains

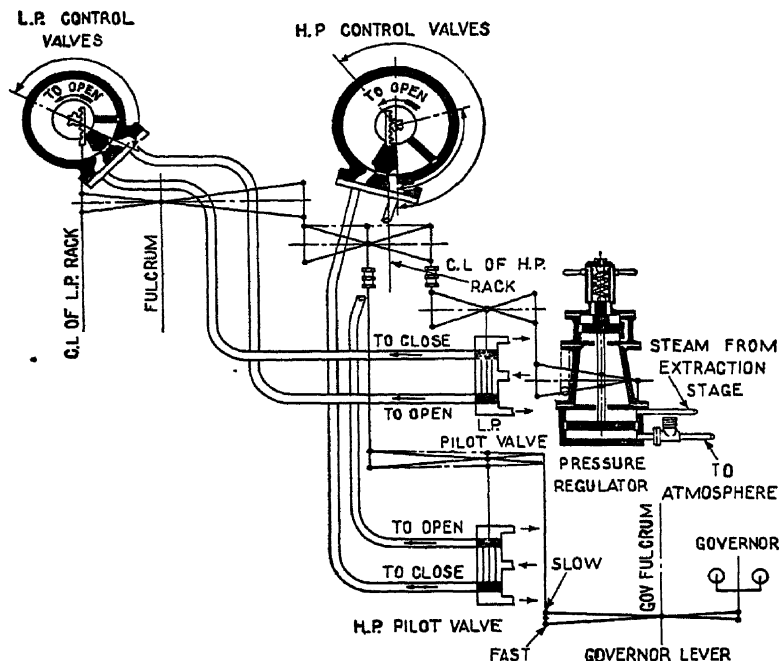


FIG. 388.—Diagram showing the Action of Cam-operated H.P. and L.P. Valve Gear for B.T.-H. Pass-out Turbine

constant, the increase of speed causes the h.p. pilot valve to be lowered, thus partially closing the h.p. control valves. The consequent upward movement of the h.p. rack tends to restore the h.p. pilot valve to its mid-position and simultaneously raises the l.p. pilot valve, thus admitting oil to the l.p. servo-motor and partially closing the l.p. control valves. The clockwise movement of the l.p. servo-motor lowers the l.p. rack and tends to restore the l.p. pilot valve to its central position. It will be seen, therefore, that this corresponds to the action of the governor gear shown in Fig. 151.

Suppose now that the heating load is reduced while the power load remains constant. Then the heating steam pressure will rise and cause the l.p. pilot valve to be lowered. This first increases the

opening of the l.p. control valves, but as soon as the servo-motor begins to move, the upward movement of the l.p. rack causes the h.p. pilot valve to move downwards as well as moving the l.p. pilot valve towards its central position. Thus the h.p. control valves are partially closed.

It will be seen, therefore, that a change in turbine speed due to a change in load causes both h.p. and l.p. control valves to move the same way, i.e. both to close or both to open, whereas a change in heating load causes these valves to move in opposite directions. This, also, is in agreement with Fig. 151.

The heating steam pressure may be regulated, within limits, by varying the tension in the spring in the pressure regulator.

In certain cases, the low-pressure control valves are replaced by a circular grid-iron valve covering the inlet to the first stage of the l.p. section of the turbine. The partial rotation of the grid-iron valve causes a change in the area through the nozzles and is effected by an external hydraulic cylinder.

275. Fluid-pressure Governors. As already mentioned in Art. 259, fluid-pressure governors are sometimes used to control the speed of steam turbines in place of the more conventional fly-ball type. An impeller of some form is attached to the turbine shaft or is driven from it through gearing. The operating medium may be either air or oil. The fluid pressure produced by the rotation of the impeller is caused to act either directly on a piston operating the throttle valve or, more usually, on a relay actuating such a piston. The fluid pressure generated varies as the square of the speed and the change of pressure produced by a change of speed is utilized to operate the throttle valve.

276. Early Parsons Air Governor. The fluid-pressure governor was first used on Parsons' turbine and is shown in Fig. 389. A collar A is provided on the shaft and has a series of communicating radial and axial holes which make it, in effect, a small centrifugal impeller. On the side having the axial holes, the impeller runs in close proximity to the bearing housing B, the upper part of which has a cast pipe C which makes the air pressure behind the bellows D and in the chamber E above the oil reservoir equal to that in the space F. The diaphragm D is connected by a rod to a lever actuating the steam throttle valve.

When the impeller is rotating at speed, since the air pressure at the periphery is atmospheric, a sub-atmospheric pressure is produced in the space behind the diaphragm D. This tends to pull the diaphragm inwards against the tension of a spring and so partially close the throttle valve. This part of the governor gear by itself would keep the turbine speed sensibly constant.

These turbines were, however, used to drive direct-current generators in which there was a considerable voltage drop with increase of load. This was corrected by an auxiliary electrical

governor which, as the current in the field winding decreased, caused air to enter the governor at G and so open the throttle and raise the speed sufficiently to bring back the voltage to the normal value.

The partial vacuum produced by the impeller was also used for priming the lubricating system. When the turbine was brought up

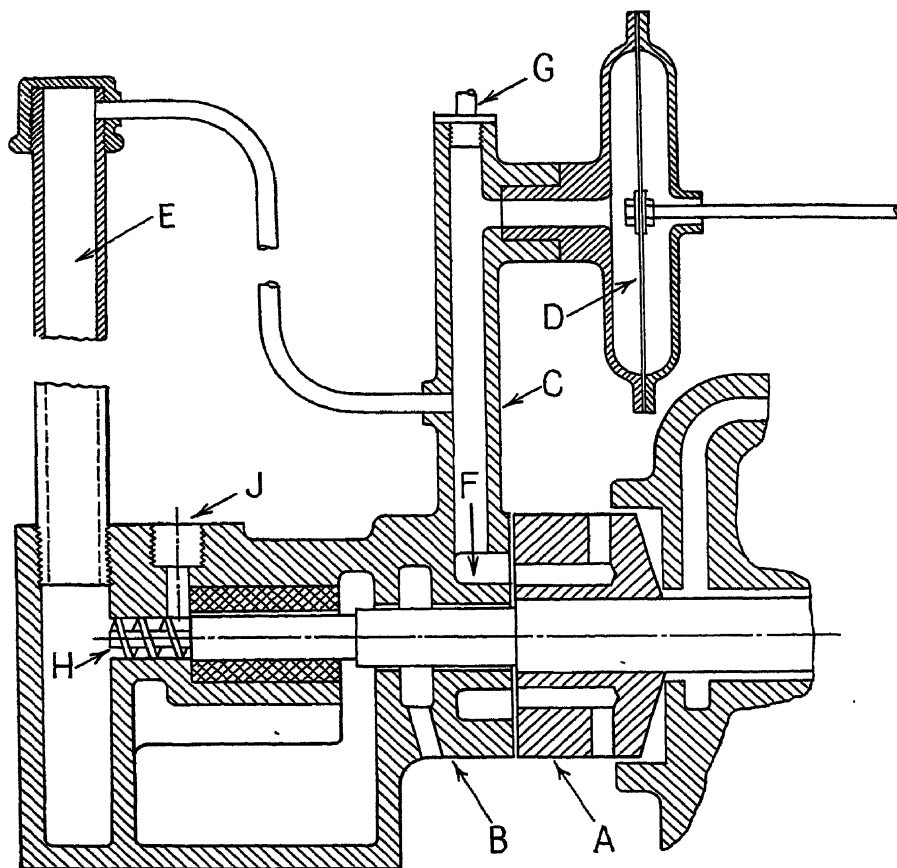


FIG. 389.—Air Governor on Early Parsons Turbine

to speed, the partial vacuum maintained in the chamber above the oil reservoir caused the oil to be forced into this chamber by atmospheric pressure. The end of the turbine spindle was provided with a worm H running in a closely fitting housing opening into the passage between the air chamber and the oil reservoir, so that as the oil rose in the passage it flowed into the worm pump which delivered it to the turbine bearings via the outlet J.

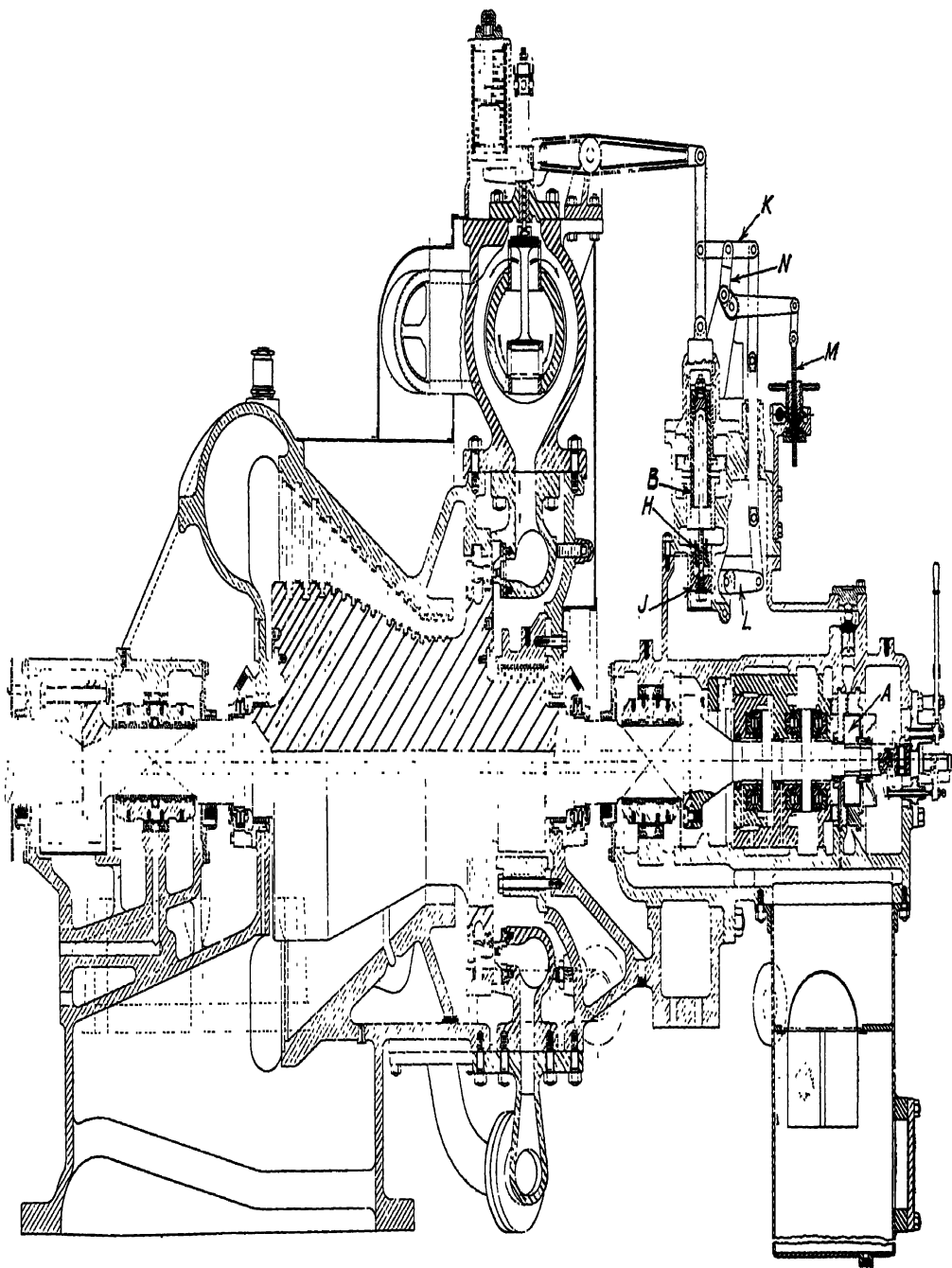


Fig. 390. 2,500 kW Westinghouse Turbine, with Oil Governor

There have been many other applications of the fluid-pressure governor. Particular mention may be made of that developed by Sulzer Brothers about 1911.

277. Westinghouse Fluid-pressure Governor. The Westinghouse Electric and Manufacturing Company, Philadelphia, now use fluid-pressure governors on steam turbines of all sizes.

Fig. 390 shows a cross-section through a 2,500 kW Westinghouse impulse-reaction turbine, designed for an operating speed of 3,600 r.p.m.; in which the construction of the type of oil governor employed on the smaller turbines can be seen. The governor oil impeller A on the end of the spindle supplies oil to the sensitive control piston B, an enlarged detail of which is shown in Fig. 391. The control piston B works within the operating or relay piston C, which has ports D and E communicating with the spaces above and below the piston, respectively. The pressure oil is admitted to the inside of the control piston, and so has access to the recess F. The unrecessed portion G of the control piston covers the ports D with a very small amount of lap when the gear is in the neutral position, and controls the admission and release of the pressure oil to and from the upper side of the relay piston. The space beneath the operating piston is connected by a drain pipe to the oil tank. As may be seen from Fig. 390, the spring H is connected at its lower end to the sliding block J. The normal operating oil pressure is 60 lb. per sq. in.

An increase in turbine speed brings about an increase of oil pressure which, acting on the control piston against the tension of the spring, causes it to move upwards, and so uncover the ports G. Oil under pressure is thus allowed to escape from the upper side of the relay piston, and the latter thus moves upward slightly and partially closes the throttle valve. These movements are brought about by the compression in the spring which tends to close the throttle valve.

Stability is obtained in the following manner. The levers K and L are so arranged that as the relay or operating piston is moving upwards, the sliding block J moves upwards at a slower rate, and the tension of the spring gradually increases. Thus, for every turbine speed and, consequently, every oil pressure, there is a definite position of the operating piston with respect to its cylinder and, hence, a definite throttle opening and corresponding power. This differential follow-up motion is not absolutely essential, but without its use, the design of a satisfactory spring is a matter of some difficulty.

Governors of this type are ordinarily given a speed variation of from 4 to 6 per cent. For large central station turbines, the speed variation is usually set at 4 per cent. The speed changer is the hand- or motor-operated screw M, which actuates the fulcrum link N, and so varies the tension in the spring H.

The oil from the main tank is forced up to the impeller suction

by a hydraulic ejector. The ejector is, in turn, supplied with high-pressure oil from the discharge of the governing impeller. The cycle is started by the usual steam-driven auxiliary pump.

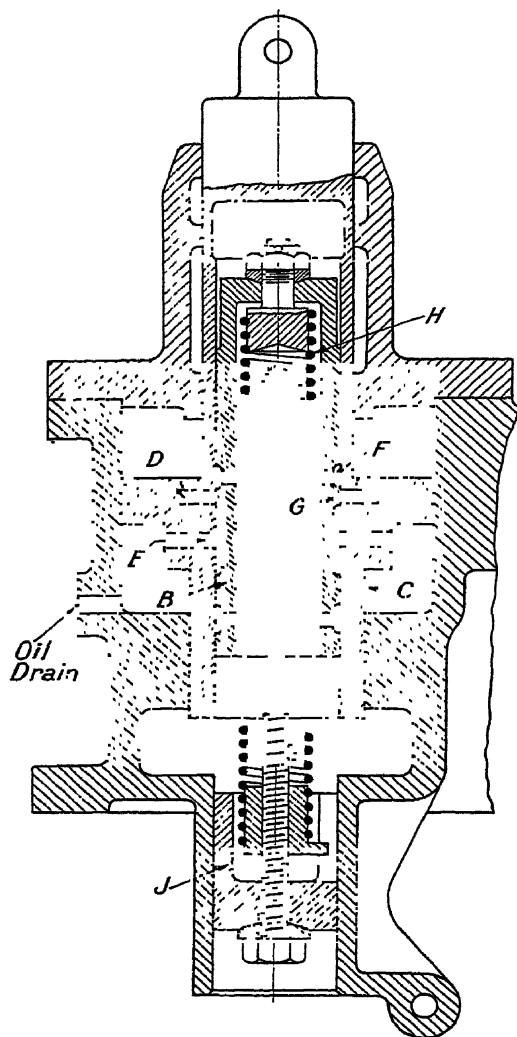


FIG. 391.—Details of Control Piston

A diagrammatic arrangement of the oil governor mechanism for a 40,000 kW turbine is shown in Fig. 392, which is largely self-explanatory. The oil pump impeller is formed on an extension of the turbine shaft, and delivers oil at a pressure of 80 lb. per sq. in. to the relay. It will be seen that the relay is in equilibrium under

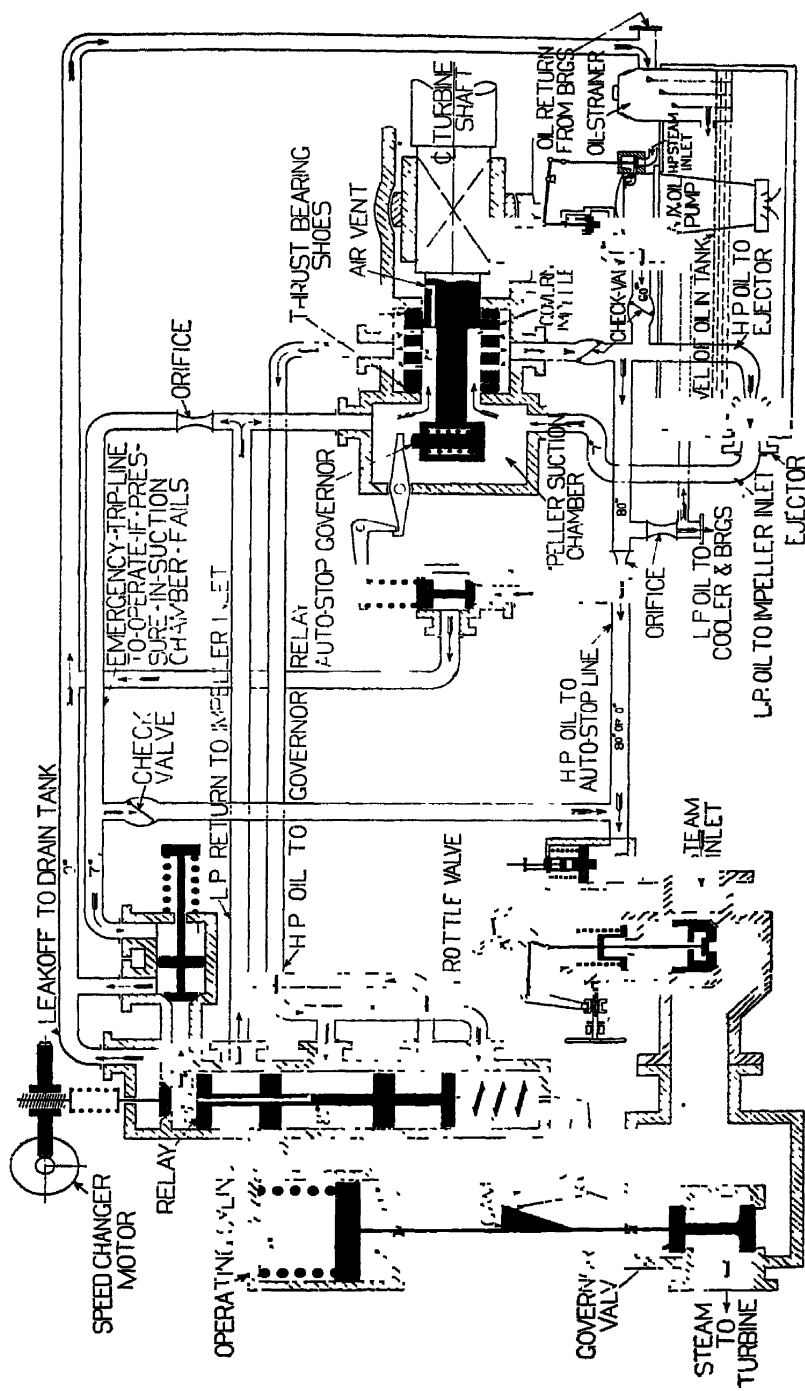


FIG. 392.—Diagrammatic Arrangement of Oil Pressure Governor for 40,000 kW Westinghouse Turbine

the action of forces produced by the full oil pressure, a spring, and a variable oil pressure. The force exerted by the spring balances the difference between the forces produced by the operating oil pressure and the leak-off oil pressure. Stability is secured by a cam-operated bell crank which restores the relay to its neutral position after a change in speed has taken place, by altering the spring tension to balance the changed oil pressure.

Speed change, for paralleling, etc., is effected by varying the back pressure on the upper side of the sensitive piston, there being a small flow through the relief valve at all times, by way of the orifice shown.

While the oil ejectors are extremely reliable for the purpose of supplying oil to the impeller suction, a low-pressure closing device has been provided, so that in the event of the supply of suction oil to the main impeller being lost, the low-pressure closing device will operate and shut the main valve.

Details of the actual governor mechanism are shown in Fig. 393, opposite.

The impellers for a 60,000 kW, 1,800 r.p.m. turbine are shown in Fig. 394. In order to give a flat characteristic curve, radial vanes are used. The impeller does not resemble a centrifugal pump of the ordinary type, as the overhanging rim that forms the pumping part of the impeller is simply a part of the forged steel shaft end, with radial holes drilled for the flow of oil. Actually, the collar on the shaft functions, in addition, as the Michell thrust bearing. The adjusting gear for the latter is clearly shown in the illustration. The Westinghouse Company have attempted in this design to obtain an impeller which would hold a large volume of oil, passing through the impeller at a low velocity, thus permitting the air to be separated and forced to the centre of the impeller, from which it passes out through an air vent drilled in the shaft. The smaller impeller, which is of higher capacity, furnishes the oil for the bearings.

A number of points arise for consideration in connection with oil pressure governors. It might be thought that the constant churning of the oil would cause oxidation and sludging. Actual experience has shown that such is not the case, probably owing to the continual de-aeration of the oil. The reduction in density of the oil as its temperature rises causes a reduction in the pressure generated at any given speed, and thus tends to bring about a change of speed. This is compensated in two ways. Firstly, by immersing the governing springs in the hot oil, which, it will be noted, is done in every case. There is thus a partial compensation for the reduction in density, as the lowering of the elastic modulus gives a corresponding reduction in load due to the spring. The remainder of the compensation comes about by reason of the fact that with hotter and less viscous oil, the air separation in the impeller is more complete.

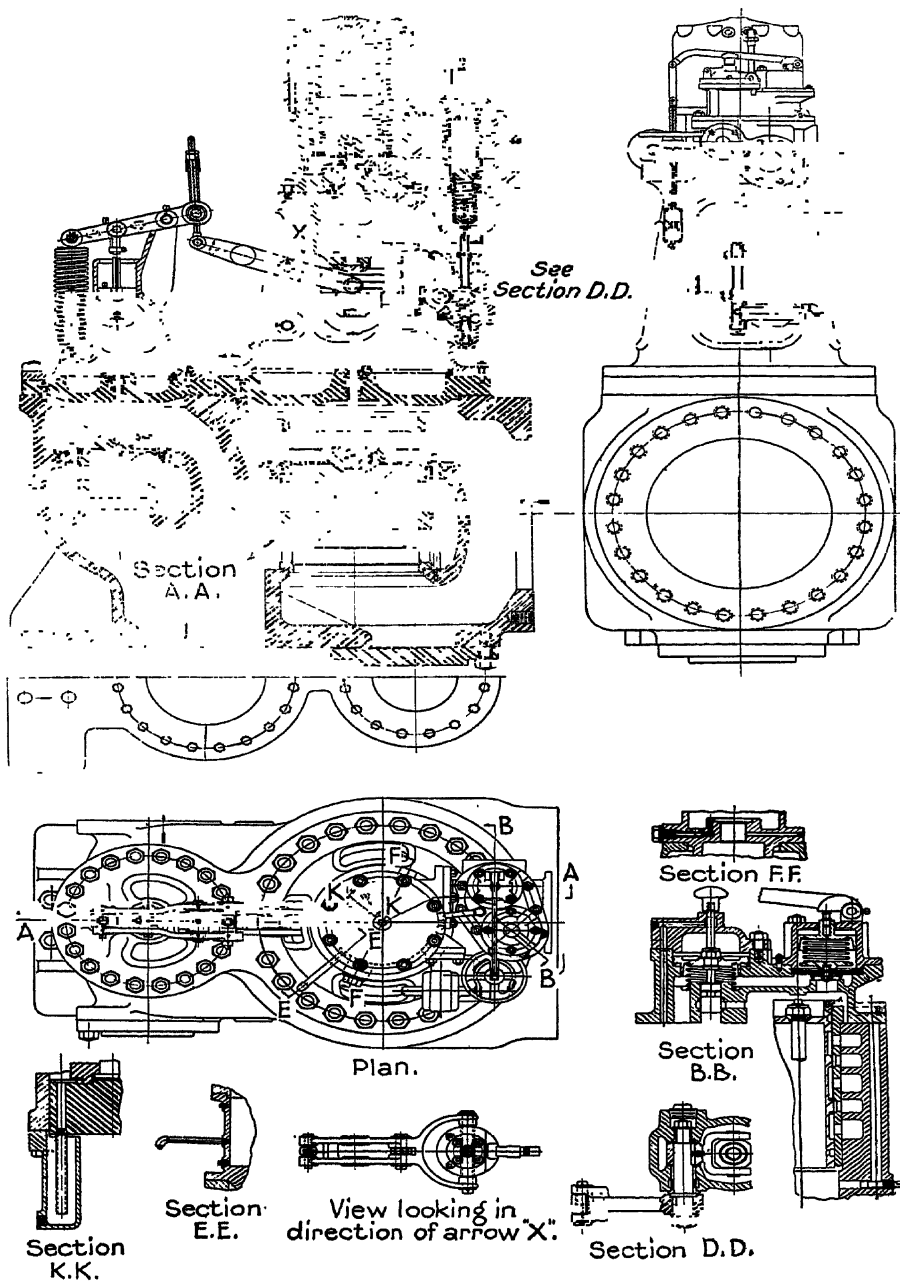


Fig. 393. Details of Throttle Valves and Governor Gear for 40,000 kW Westinghouse Turbine

Section B.B. Through speed changer leak-off valve, also low-pressure closing device. Note the leak-off passage to the right, shown diagrammatically in Fig. 392 by a central hole in the relay valve.

Section A.A. Through main throttle and relay valves and relay gear.

Section K.K. Through drip collector, for the leak-off from the valve spindle guide.

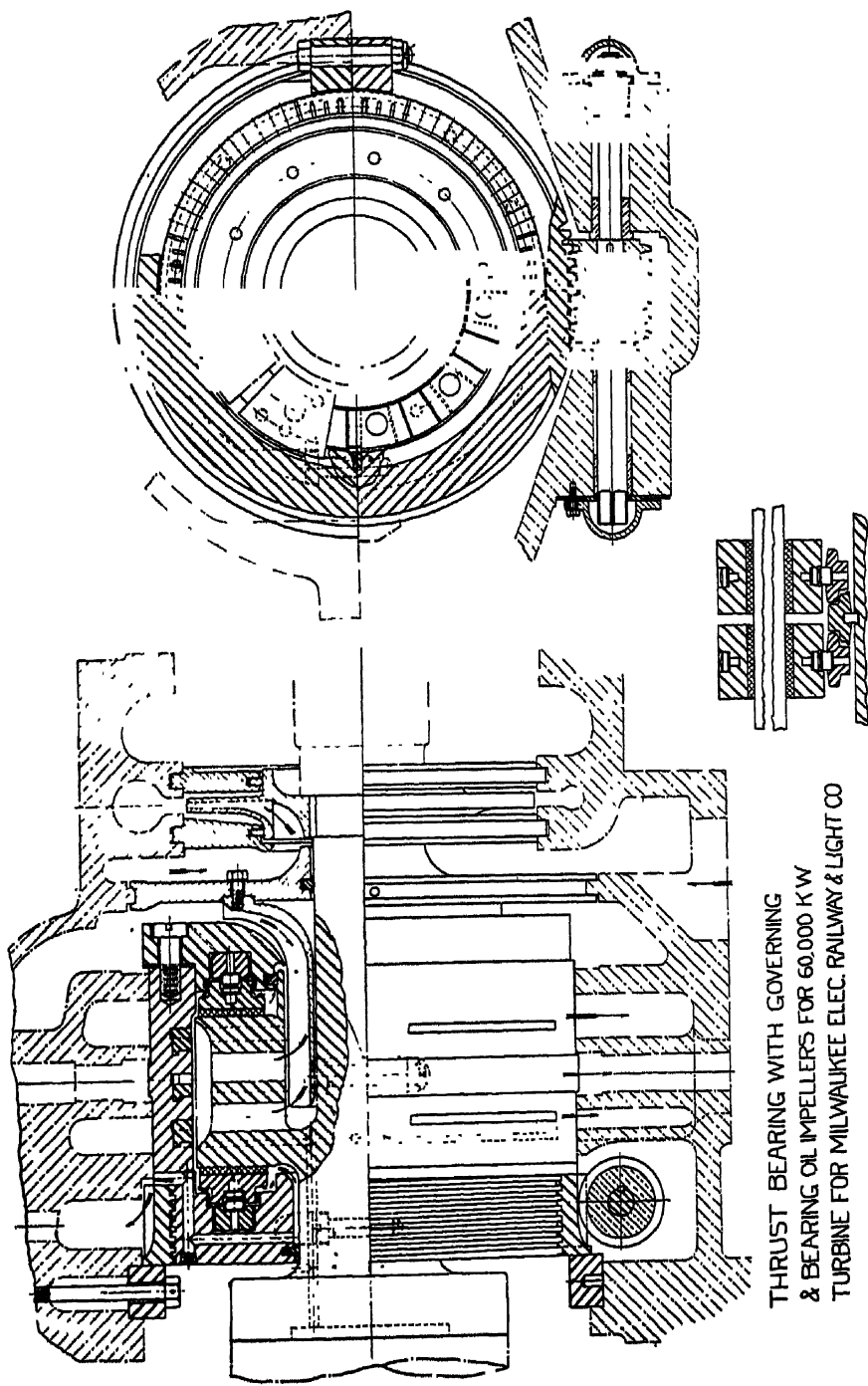


FIG. 394.—Impellers for 60,000 kW Turbine

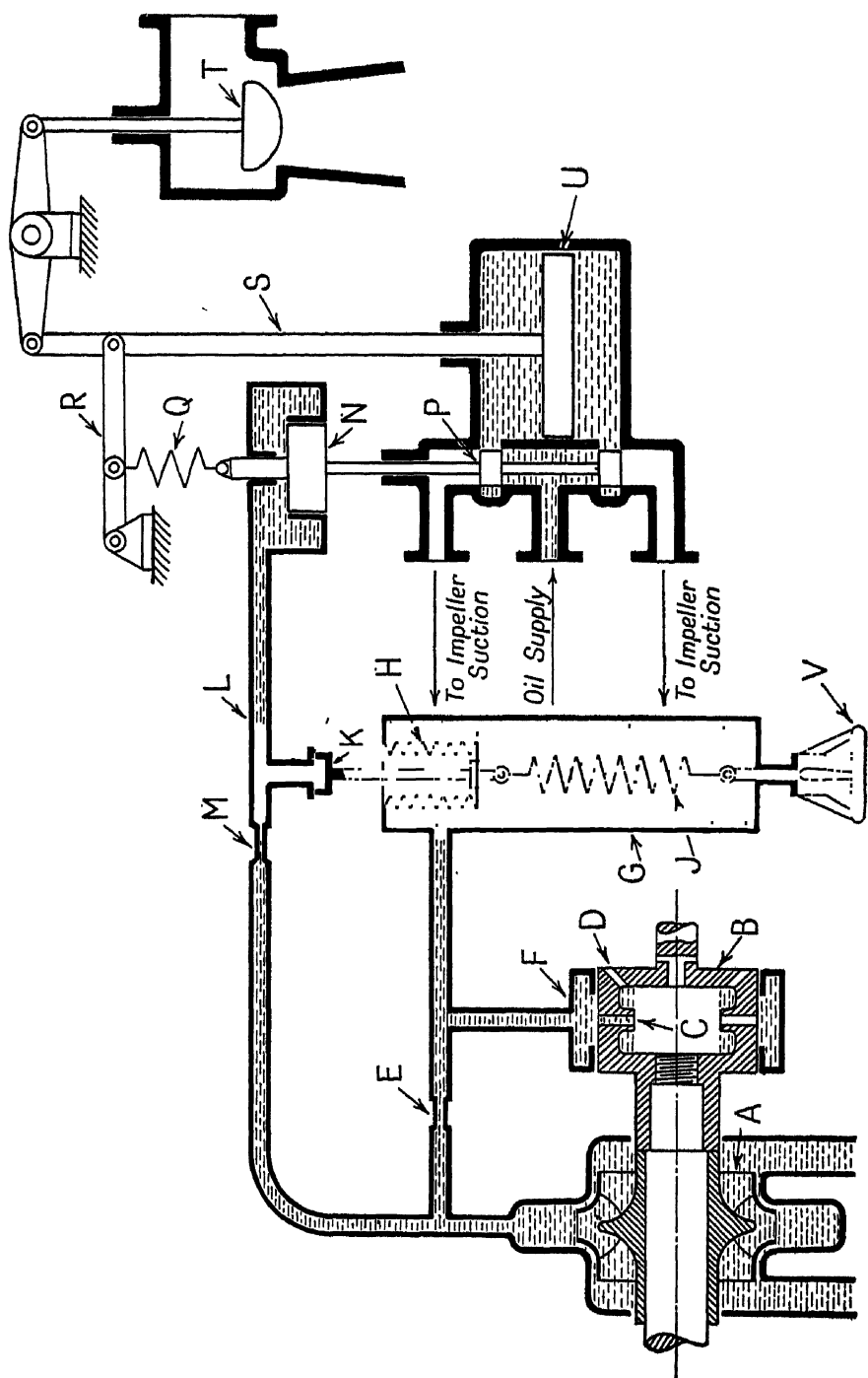


FIG. 395.—Diagrammatic Arrangement of Fluid Pressure Governor Gear on Westinghouse Steam Turbine

278. Westinghouse Fluid-pressure Governor with Governor Transformer. A more recent form of the Westinghouse governor is illustrated in diagrammatic form in Fig. 395. This type of governor is used on central station machines and the modifications which have been made in the earlier types have for their object to make the governor gear more sensitive to small changes of speed.

The impellers A and B are mounted on the turbine shaft. A is the main oil pump and supplies all the oil required while the turbine is operating at normal speed. It is a centrifugal pump and discharges at a pressure of about 120 to 150 lb. per sq. in. gauge at the normal operating speed with 10 to 30 lb. per sq. in. g. suction pressure. During the starting period oil is supplied to the impeller suction by the auxiliary oil pump, as the impeller is not self-priming. Under normal operating conditions the suction is supplied by an ejector which utilizes a part of the high-pressure oil from the impeller discharge as the operating medium.

The ejector is designed to handle all the oil required for lubrication and for normal leakage. The relay piston discharges directly into the impeller suction line and therefore does not add load to the ejector.

The governor impeller B is made integrally with the overspeed trip body and consists of a hollow cylindrical body with a series of radial tubes C. It is also connected to the drain chamber of the bearing pedestal by a number of holes D. An air vent is provided at the centre and communicates with two radial holes in the overspeed trip body.

Oil from the main pump A is admitted through a variable orifice E into the annular chamber F and maintains a small flow through the impeller holes into the drain chamber. With the impeller B rotating the centrifugal force of the oil in the radial tubes opposes the flow of oil through the holes and maintains in the chamber F a pressure which varies as the square of the turbine speed, being about 30 lb. per sq. in. g. at 3,600 r.p.m. The variable orifice E must be so adjusted that the oil supply to the impeller is slightly in excess of the normal leakage around the governor impeller B. This ensures that the impeller tubes will be full of oil and will thus function correctly.

The space F is connected to a chamber G which forms part of the governor transformer and contains a flexible metal bellows H and a load spring J. Connected to the bellows is a small "cup" valve K which controls the flow of oil from a space L which is supplied with high-pressure oil through a choke M. The pressure of the oil in the chamber L will depend on the opening of the cup valve. Furthermore, the force on the cup valve depends on the pressure of this oil, and under all conditions the cup valve is in equilibrium under the action of the forces due to (1) the oil pressure on the bellows H, (2) the tension of the spring J, and (3) the pressure of the oil in L. It will be apparent, therefore, that an increase in oil pressure in G

caused by an increase of turbine speed will be followed by an increase in the secondary control oil pressure in the chamber L. Such pressure increase is transmitted to the upper side of the relay piston N which is, in effect, coupled to the relay piston valve P. A tension spring Q connects the upper end of N to the follow-up lever R which is attached to the relay piston rod S. As shown in the diagram, the relay piston controls the opening of the steam valve T.

The piston N is held in equilibrium by the oil pressure above it and the tension of the spring Q and this latter depends on the position of the main relay piston U. We have seen that the pressures in G and in L depend on the turbine speed and, therefore, for every turbine speed there is a corresponding equilibrium position of the throttle valve T.

The combination of the chamber G with its bellows H, the cup valve K and the piston N operating the pilot valve P is termed the governor transformer and its purpose is to magnify the relatively small pressure changes delivered by the governing impeller into larger pressure changes which are utilized to actuate the pilot valve of the servomotor. Assuming that the mechanism is in operation, the following describes a complete cycle of control.

If the load increases, the turbine speed decreases and the governing oil pressure below the transformer bellows H decreases, thus allowing the tension spring J to move the transformer cup valve K downwards. Downward movement of this valve passes more oil to the drain chamber, thus decreasing the pressure of the governing secondary control oil in L. With the reduction of pressure in L, the existing force of the oil pressure below the bellows H again balances the pressure above the cup valve.

The decreased governing secondary control pressure thus established by the transformer cup valve has at the same time decreased the downward force on the piston N. This causes an upward movement of the pilot valve P with the result that high-pressure oil is admitted above the main operating piston U and the space beneath U is connected to the impeller suction. The main relay piston U is therefore moved downward thus increasing the opening of the steam throttle valve. The downward movement of the main relay piston decreases the tension in the spring Q until the diminished spring tension balances the decreased secondary control pressure and thus returns the relay to its neutral position.

If the load decreases, the turbine speed increases and generally things happen in the reverse order to that described above.

279. Speeder Gear. If the turbine is running alone and is controlling frequency then the speed of rotation may be changed by changing the tension of the spring J. This may be effected by turning the handwheel V, usually by remote control. If the spring tension is increased, then since the turbine speed is, for the time being, constant, the oil pressure under the bellows H remains unchanged

and the opening of the cup valve K will increase, with a consequent reduction of oil pressure in L. This causes the piston N and the pilot valve P to be displaced upwards with the result that the main relay piston U is lowered and the opening of the throttle valve T increased. As already explained above, downward movement of U reduces the tension in the spring Q which balances the reduced pressure in L. The increased opening of the throttle T will raise the turbine speed and the oil pressure in G. This increase in oil pressure will partially balance the increased tension in the spring J. The oil pressure in L will still be lower than its former value and will correspond to the reduced tension in the spring Q. Thus, for every setting of the speeder gear there will be a corresponding equilibrium speed at a given load.

A reduction of the tension in J would likewise cause the oil pressure in L to rise, with the result that the pilot valve P would be lowered, the piston U raised, and the throttle valve T partially closed.

If the turbine is driving an alternator which is electrically connected to other sets and is, therefore, running at a more or less fixed speed, then the speeder gear may be used for increasing or reducing load on the turbine in question. For example, if the tension in the spring J is reduced, the pressure in L will be increased, and the resultant downwards movement of N and P will cause the throttle valve T to close partially. Equilibrium will be restored by the increased tension in Q, and the amount of movement of T will depend entirely on the amount by which the tension in J is reduced. Similarly, load may be added to the turbo-alternator, within the limits of the set, by increasing the tension in the speed changer spring.

ANSWERS TO EXAMPLES

II (page 35)

1. (a) 368; (b) 374.096; (c) 372.511 B.Th.U. per lb.
2. 298.469 B.Th.U. per lb.
3. (a) 81.68; (b) 810.52 B.Th.U. per lb.
4. 0.8626.
5. 1,126.2; 448.3 B.Th.U. per lb.
6. 900.1 B.Th.U., 30 lb.
7. 0.8397.

III (page 48)

2. 0.1353.
3. (a) 0.4714; (b) 1.1346; (c) 0.1274.
4. 0.73713; 0.7144.
5. (a) 190 lb.; (b) 2,029,600 B.Th.U.; (c) 2,289 units of entropy.

IV (page 64)

1. (a) 1,124.6 B.Th.U. per lb.; (b) 1.4377; (c) 807.1 B.Th.U. per lb.;
(d) 28.2 per cent.
2. (a) 951.0 B.Th.U. per lb.; (b) 1.2255; (c) 687.9 B.Th.U. per lb.;
(d) 27.67 per cent.
3. (a) 1252.4; (b) 854.9; (c) 397.5; (d) 398.2 B.Th.U. per lb.;
(e) 31.8 per cent; (f) 8.572 lb. per kW-hr.; 76.4 per cent.
4. 20.48 per cent.

	Turbine A	Turbine B
5. (a)	9.72	8.40
(b)		13.60
(c)	0.290	0.326
(d)		12.4

6. 7.41 B.Th.U. per lb.
7. 22.52 per cent; 71.45 per cent.
8. Turbine A: 34.46 per cent; 0.866. Turbine B: 40.80 per cent; 0.764.
9. 0.115 penny.

	Turbine A	Turbine B
10. (a)	9.298	8.204
(b)	31.97 per cent	32.92 per cent
(c)		11.8 per cent
(d)		2.97 per cent

11. Initial superheat = 286.3° F.
12. 38.2 lb. per sq. in. abs.; 0.8428; 385.8 B.Th.U. per lb.
13. (a) 337.3; (b) 349.8; (c) 365.6; (d) 387.4; (e) 421.8 B.Th.U. per lb.
(a) 0; (b) 3.7; (c) 8.4; (d) 14.9; (e) 25 per cent.

V (page 101)

1. 1,416 ft. per sec.
2. 1,343 ft. per sec.
3. 0.964.

4. (a) 1,467 ft. per sec.; 0.494 sq. in.
5. 351.1° F.; 1,431 B.Th.U. per lb.; 74.6 B.Th.U. per lb.; 1,932 ft. per sec.
7. 0.374 sq. in.
8. (a) 0.366; 0.488 sq. in. (b) 0.394; 0.522 sq. in.
9. 0.0154.
10. 2.956; 70.6° F.; 0.0024; 1.85 B.Th.U. per lb.; 2.51 per cent.

VII (page 152)

1. (a) 29° 48'; (b) 2,405 ft. per sec.; (c) 58.4 lb.; (d) 127.2; (e) 73.8 per cent
2. (a) 642; (b) 640; (c) 47.5 lb.; (d) 640; (e) 82.8 per cent.
3. 375 h.p.
4. 646.8; 2,660 ft.-lb. per lb.
5. 0.4851; 26.5°; 89.4 per cent.
6. 56.3 lb.
7. 80.8 per cent; (1) 0.3208, $\beta_1 = 29^\circ$; (2) 0.6188, $\beta_1 = 47^\circ$.
8. 7.4 lb.; 252 h.p.
9. 690 ft. per sec.; 34°; 84.8 per cent.
10. 408 and 353 ft. per sec.; 82.9 per cent.

	First Moving Row	Second Moving Row
11. (a)	2,678	663
(b)	441	109
(c)	20.3	11.5
(d)	305 h.p.	75.5 h.p.
	70.3 per cent.	

	First Row	Fixed Row	Second Row
12. (a)	19.9°	25°	38°
(b)	605 h.p.	—	171 h.p.
(c)	74 per cent.		

VIII (page 167)

1. (a) 49° 32'; (b) 311 lb.; (c) 192.
2. (a) 331.5 ft. per sec.; (b) 0.84; (c) 4,330 ft.-lb.
3. 500 ft. per sec.; 10.23 B.Th.U. per lb.; 80.5 per cent.
4. 53° 4'; 683.
5. 1.06 in.; 51.1 B.Th.U. per lb.
6. 27.8 per cent; 14.14 B.Th.U. per lb.; 86.1 per cent.

IX (page 185)

1. (a) 0.039 h.p.; (b) 0.174 h.p.
2. (a) 65.4 h.p.; (b) 49.0 h.p.
3. 1.77 h.p.
4. 0.316 lb. per sec.; 195.2; 190.3; 185.2 lb. per sq. in. abs.
5. 218 lb.

X (page 200)

1. 310.7 B.Th.U. per lb.; 0.8373 dry; 12.03 per cent; 0.2214.
2. 25,100 ft.-lb.
3. 1,925; 1,465; 76.2 per cent.
4. (1) 482 ft. per sec.; (2) 4.64 B.Th.U. per lb.; (3) 0.98 per cent;
(4) 987.60 B.Th.U. per lb.
6. 1.056.
8. $m = 1.091$; 1.086.
9. 1.055.
10. $m = 1.209$; 1.0805; 1.0725.

XI (page 211)

	Rankine Cycle	Regenerative Cycle
1. (a)	32.3	33.95 per cent.
(b)	8.40	9.10 lb. per kWh
(c)	8.40	7.80 lb. per kWh
2.	0.165 lb.; 38.6 per cent.	
3.	5,290 h.p.	
4.	0.115 lb. at 70 lb.; 0.0796 lb. at 12 lb. pressure; 37.5 per cent.	
5. (a)	$w_1 = 0.0834$, $w_2 = 0.0766$, $w_3 = 0.0525$ lb.;	(b) 37.9 per cent;
(c)	34.9 per cent; (d) 8.6 per cent; (e) 8.53 and 7.62 lb.	
(f)	335,000 and 381,000 lb. per hour; (g) 5,600 and 6,350 B.Th.U.	
6.	227.94 B.Th.U. per lb.; 8,136 lb.	

XII (page 235)

1. (a) 6,040 h.p.; 77.87 per cent. (b) 5,196 h.p.; 74.57 per cent.
2. 0.826; 35.18 per cent; 0.939; 36.07 per cent.
3. 32.5 per cent; 0.13; 33.3 per cent; 0.098.
4. 40.88 per cent.
5. 1732.1; 1186.0; 546.1 B.Th.U. per lb.; 31.5 per cent.
6. 8.18 lb.; 10,940.
7. 40.8; 40.6 per cent.
8. 35.9 per cent; 60 and 6.63 lb. per sq. in. abs.; 234° F.; 35.18 per cent.
9. Superheat before reheating = 56° F., and after reheating = 307.7° F. 34.12 per cent.

XIII (page 260)

1. 68 per cent; 390 h.p.
2. 219 kW.
3. 67.1; 75.4; 77.6 per cent.
4. 8.26 lb. per sec.
5. 1,261 B.Th.U. per lb.

XIV (page 270)

1. 1,935 h.p.
2. 23.15; 18.37; 11.41 lb. per h.p. hr.
3. 26.6; 13.99 lb. per kWh; 9.48 lb. per sq. in. abs.
4. 634 kW.
5. 755 h.p.; 12,100 lb.
6. 114.6 lb.; 6,950 lb.
7. 818.2 lb.; 1,170 h.p.
8. 33,700 lb.

XV (page 285)

1. 1,930 h.p.; 43,700,000 B.Th.U. per hour. 61,600,000 B.Th.U. per hour.
2. 2,175 kW.
3. 23,380,000 B.Th.U. per hour. 351 kW., 12,170,000 B.Th.U. per hour.
4. 21,860; 11,180 lb. per hour.
5. 1,610 kW.
6. 26.6 tons per day of 24 hours.
7. 3,540; 2,690 lb. per hour.

XVII (page 335)

1. 2.126; 2.392; 2.660; 2.920 lb. per sec.
2. 1.346; 1.276; 1.202; 1.133 ft. per sec.
- 0.366; 0.379; 0.398; 0.411 lb.
- 0.361; 1.121; 1.962; 2.840 lb. in.
- 6.28 lb. in. inclined at 4 degrees to wheel plane.

3. (a) 0.121 sq. in.; (b) 0.26 in.; (c) 0.000842 in.⁴; (d) 0.0059 in.⁴
4. $M_x = 6.276$ lb. in. $M_y = 0.219$ lb. in.
 Due to M_x 1,940 lb. per sq. in.
 Due to M_y 20 lb. per sq. in.
 11,610 lb. per sq. in.
 13,570 lb. per sq. in.
5. (a) 55.8; (b) 79.3; (c) 34.6; (d) 88.4 lb. in.; (e) 12,400; (f) 8,950;
 (g) 9,840; (h) 21,350; (i) 22,240 lb. per sq. in.

XIX (page 390)

1. 790 ft. per sec.; 0.088 in.
2. 4,520 lb. per sq. in.
3. 8,200 lb. per sq. in.; 0.00232 in.
4. 11,200 lb. per sq. in.; 0.00372 in.
5. (a) 22,790; (b) 7,386 lb. per sq. in.; (c) 0.00304; (d) 0.00432 in.
6. (a) 1.174; (b) 0.7188 in.
7. 1.116 in.
8. 18,275 lb. per sq. in.; 0.00647 in.; 7,720; 8,750 lb. per sq. in.

XX (page 414)

1. (a) 2,480 r.p.m.; (b) 2,428 r.p.m.
2. 294.4; 2,649 r.p.m.
4. 3.145 in.
5. 0.755 in.
6. 3,215 r.p.m.
7. 5,360 r.p.m.
8. 2.55 in.

XXIV (page 507)

1. $K = 302,000$; 16.
2. $K = 256,000$; 589; 0.474.
3. (a) 22,500; (b) 893; (c) 1,000.
4. (a) 38.8 in.; (b) 2,360 r.p.m.
5. 34.8 in.; 2,960 r.p.m.
6. 7.65 in.
7. 18,200 kW.
9. 10,200 h.p.

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